ELECTROMAGNETIC WAVES AND TRANSMISSION LINES

B.TECH

(II YEAR – II SEM)

Department of Electronics and Communication Engineering

SVR ENGINEERING COLLEGE
NANDYAL.

AYYALURU METTA, NANDYAL– 518 503 (A.P)
(Affiliated to JNTUA Anantapur, Approved by AICTE, New Delhi)

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR

II B.Tech II-Sem (E.C.E)	T	Tu	C
	3	1	3

(15A04403) ELECTROMAGNETIC THEORY & TRANSMISSION LINES

***LEARNING OUTCOMES*:**

This course provides the foundational education in static electromagnetic fields, and time varying electromagnetic waves. Through lecture, and out-of-class assignments, students are provided learning experiences that enable them to:

a. Analyze and solve the problems of electric and magnetic fields that vary with three dimensional spatial co-ordinates as well as with time.
b. Become proficient with analytical skills for understanding propagation of electromagnetic waves in different media.
c. Understand the concept of transmission lines & their applications.
d. Develop technical & writing skills important for effective communication.
e. Acquire team-work skills for working effectively in groups.

UNIT-I

Electrostatics: Review of Vector algebra, Co-ordinate systems & transformation, Vector calculus, Coulomb's Law, Electric Field Intensity – Fields due to Different Charge Distributions, Electric Flux Density, Gauss Law and Applications, Electric Potential, Relations Between E and V, Maxwell's Two Equations for Electrostatic Fields, Electric dipole, Energy Density, Convection and Conduction Currents, Dielectric Constant, Isotropic and Homogeneous Dielectrics, Continuity Equation, Relaxation Time, Poisson's and Laplace's Equations, Capacitance – Parallel Plate, Coaxial, Spherical Capacitors, Illustrative Problems.

UNIT-II

Magnetostatics: Biot-Savart Law, Ampere's Circuital Law and Applications, Magnetic Flux Density, Maxwell's Two Equations for Magnetostatic Fields, Magnetic Scalar and Vector Potentials, Forces due to Magnetic Fields, Magnetic torque and moment, Magnetic dipole, Inductances and Magnetic Energy, Illustrative Problems.

UNIT-III

Maxwell's Equations (for Time Varying Fields): Faraday's Law and Transformer e.m.f, Inconsistency of Ampere's Law and Displacement Current Density, Maxwell's Equations in Different Final Forms and Word Statements. Boundary Conditions of Electromagnetic fields: Dielectric-Dielectric and Dielectric-Conductor Interfaces, Illustrative Problems.

UNIT-IV

EM Wave Characteristics: Wave Equations for Conducting and Perfect Dielectric Media, Uniform Plane Waves – Definition, All Relations between E & H, Sinusoidal Variations, Wave Propagation in Lossless and Conducting Media, Conductors & Dielectrics – Characterization, Wave Propagation in Good Conductors and Good Dielectrics, Polarization,Reflection and Refraction of Plane Waves – Normal and Oblique Incidences, for both Perfect Conductor and Perfect Dielectrics, Brewster Angle, Critical Angle and Total Internal Reflection, Surface Impedance, Poynting Vector, and Poynting Theorem – Applications, Power Loss in a Plane Conductor, Illustrative Problems.

UNIT-V

Transmission Lines: Types, Transmission line parameters (Primary and Secondary), Transmission line equations, Input impedance, Standing wave ratio & power, Smith chart & its applications, Applications of transmission lines of various lengths, Micro-strip transmission lines – input impedance, Illustrative Problems.

TEXT BOOKS:

1. Matthew N.O. Sadiku, "Elements of Electromagnetics," Oxford Univ. Press, 4th ed., 2008.
2. William H. Hayt Jr. and John A. Buck, "Engineering Electromagnetics," TMH, 7th ed., 2006.

REFERENCES:

1. Electromagnetic Waves and Radiating Systems – E.C. Jordan and K.G. Balmain, PHI, 2nd Ed., 2000.
2. Electromagnetics – John D. Krauss, McGraw- Hill publications, 3rd ed., 1988.
3. John D. Ryder, "Networks, Lines, and Fields," PHI publications, Second Edition, 2012.
4. Schaum's out – lines, "Electromagnetics," Second Edition, Tata McGraw-Hill publications, 2006.
5. G. S. N. Raju, "Electromagnetic Field Theory and Transmission Lines," Pearson Education, 2013
6. N. NarayanaRao, "Fundamentals of Electromagnetics for Engineering," Pearson Edu. 2009.

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR

B.Tech – II-II Sem	**L**	**T**	**P**	**C**
	3	**0**	**0**	**3**

19A04401 ELECTROMAGNETIC WAVES AND TRANSMISSION LINES

Course Objectives:

- To introduce fundamentals of static and time varying electromagnetic fields.
- To teach problem solving in Electromagnetic fields using vector calculus.
- To demonstrate wave concept with the help of Maxwell's equations.
- To introduce concepts of polarization and fundamental theory of electromagnetic waves in transmission lines and their practical applications.
- To analyze reflection and refraction of electromagnetic waves propagated in normal and oblique incidences.

Unit I

Vector Analysis: Coordinate systems and transformation-Cartesian, Cylindrical and Spherical coordinates
Vector Calculus: Differential length area and volume, line surface and volume integrals, del operator, gradient, divergent and curl operations.
Coulomb's Law, Electric Field Intensity – Fields due to Different Charge Distributions, Electric Flux Density, Gauss Law and Applications, Divergence Theorem, Electric Potential, Relations Between E and V, Maxwell's Two Equations for Electrostatic Fields, Energy Density, Convection and Conduction Currents, Dielectric Constant, Isotropic and Homogeneous Dielectrics, Continuity Equation, Relaxation Time, Poisson's and Laplace's Equations, Capacitance – Parallel Plate, Coaxial, Spherical Capacitors, Illustrative Problems.

Unit Outcomes:

- Understand basic laws of static electric field. (L1)
- Derive the Maxwell's equations for electrostatic fields. (L3)
- Solve problems applying laws of electrostatics. (L3)

Unit II

Biot-Savart Law, Ampere's Circuital Law and Applications, Magnetic Flux Density, Maxwell's Two Equations for Magneto static Fields, Magnetic Scalar and Vector Potentials, Forces due to

Magnetic Fields, Magnetic dipole, Ampere’s Force Law, Inductances and Magnetic Energy, Illustrative Problems.

Faraday’s Law and Transformer e.m.f, Inconsistency of Ampere’s Law and Displacement Current Density, Maxwell’s equations for time varying fields, Maxwell’s Equations in Different Final Forms and Word Statements, Illustrative Problems

Unit Outcomes:

- Understand basic laws of static magnetic field. (L1)
- Derive the Maxwell’s equations for magnetic fields. (L3)
- Solve problems applying laws of magneto statics. (L3)
- Derive the Maxwell’s equations for electromagnetic fields. (L3)
- Apply the boundary conditions of electromagnetic fields at the interface of different media. (L2)

Unit III

Boundary Conditions of Electromagnetic fields: Dielectric-Dielectric and Dielectric-Conductor Interfaces, Wave Equations for Conducting and Perfect Dielectric Media, Uniform Plane Waves – Definition, All Relations between E & H, Sinusoidal Variations, Wave Propagation in Lossless and Conducting Media, Conductors & Dielectrics – Characterization, Wave Propagation in Good Conductors and Good Dielectrics, Polarization, Illustrative Problems.

Unit Outcomes:

- Understand concept of wave propagation through the Maxwell’s equations .(L1)
- Derive wave equations for different media. (L3)
- Explain concept of polarization of electromagnetic wave. (L2)

Unit IV

Reflection and Refraction of Plane Waves – Normal and Oblique Incidences, for both Perfect Conductor and Perfect Dielectrics, Brewster Angle, Critical Angle and Total Internal Reflection, Surface Impedance, Poynting Vector, and Poynting Theorem – Applications, Power Loss in a Plane Conductor, Illustrative Problems.

Unit Outcomes:

- Understand principles of reflections and refraction for different incidences. (L1)
- State concept of power flow using Poynting vector. (L2)

- Calculate Brewster angle, power flow and surface impedance. (L3)

Unit V

Transmission Lines: Introduction, Transmission line parameters, Transmission line equivalent circuit, Transmission line equations and their solutions in their phasor form, input impedance, standing wave ratio, Transmission of finite length- half wave, quarter wave transmission line, Smith chart, graphical analysis of transmission lines using Smith chart, stub matching- single and double stub matching, Illustrative Problems.

Unit Outcomes:

- Understand the principles of transmission lines and concept of smith chart.(L1)
- Derive the input impedance of transmission line.(L3)
- Finding the line parameters through problem solving.(l4)
- Study the applications of different lengths of transmission lines.(L2)

Course Outcomes:

After completion of the course, student will be able to
CO1: Explain basic laws of electromagnetic fields and know the wave concept. (L2)
CO2: Solve problems related to electromagnetic fields. (L3)
CO3: Analyze electric and magnetic fields at the interface of different media. (L3)
CO4: Derive Maxwell’s equations for static and time varying fields. (L3)
CO5: Analogy between electric and magnetic fields. (L5)
C06: Describes the transmission lines with equivalent circuit and explain their characteristic with various lengths. (L2)

TEXT BOOKS:

1. Matthew N.O. Sadiku, “Elements of Electromagnetics”, 4th edition. Oxford Univ. Press, 2008.
2. William H. Hayt Jr. and John A. Buck, “Engineering Electromagnetics”, 7th edition., TMH, 2006.

REFERENCES:

1. E.C. Jordan and K.G. Balmain, “Electromagnetic Waves and Radiating Systems”, 2nd Edition, PHI, 2000.
2. John D. Krauss, “Electromagnetics”, 4th Edition,McGraw- Hill publication1999.

3. Electromagnetics, Schaum’s outline series, 2nd Edition, Tata McGraw-Hill publications, 2006.

UNIT – I- Electrostatics

Contents

- Basics of coordinate system
- Coulomb’s Law
- Electric Field Intensity - Fields due to Different Charge Distributions
- Electric Flux Density
- Gauss Law and Applications,
- Electric Potential,
- Relations between E and V
- Maxwell’s Equations for Electrostatic Fields
- Energy Density
- Dielectric Constant
- Isotropic and Homogeneous Dielectrics,
- Continuity Equation
- Relaxation Time
- Poisson's and Laplace's Equations
- Capacitance - Parallel plate
- Problems.

INTRODUCTION

VECTOR ALGEBRA

Vector Algebra is a part of algebra that deals with the theory of vectors and vector spaces.

Most of the physical quantities are either scalar or vector quantities.

SCALAR QUANTITY:

Scalar is a number that defines magnitude. Hence a scalar quantity is defined as a quantity that has magnitude only. A scalar quantity does not point to any direction i.e. a scalar quantity has no directional component.

For example when we say, the temperature of the room is 30o C, we don‘t specify the direction.

Hence examples of scalar quantities are mass, temperature, volume, speed etc.

A scalar quantity is represented simply by a letter – A, B, T, V, S.

VECTOR QUANTITY:

A Vector has both a magnitude and a direction. Hence a vector quantity is a quantity that has both magnitude and direction.

Examples of vector quantities are force, displacement, velocity, etc.

$\vec{\mathbf{A}}, \vec{\mathbf{V}}, \vec{\mathbf{B}}, \vec{\mathbf{F}}$

A vector quantity is represented by a letter with an arrow over it or a bold letter.

UNIT VECTORS:

When a simple vector is divided by its own magnitude, a new vector is created known as the unit vector. A unit vector has a magnitude of one. Hence the name - unit vector.

A unit vector is always used to describe the direction of respective vector.

$$\mathbf{a}_A = \frac{\vec{\mathbf{A}}}{|\mathbf{A}|} \Rightarrow \vec{\mathbf{A}} = |\mathbf{A}|\,\mathbf{a}_A$$

Hence any vector can be written as the product of its magnitude and its unit vector. Unit Vectors along the co-ordinate directions are referred to as the base vectors. For example unit vectors along X, Y and Z directions are ax, ay and az respectively.

Position Vector / Radius Vector ($\overline{OP}$):

A Position Vector / Radius vector define the position of a point(P) in space relative to the origin(O).Hence Position vector is another way to denote a point in space.

$$\overline{OP} = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$

Displacement Vector

Displacement Vector is the displacement or the shortest distance from one point to another.

Vector Multiplication

When two vectors are multiplied the result is either a scalar or a vector depending on how they are multiplied. The two important types of vector multiplication are:

- Dot Product/Scalar Product (A.B)
- Cross product (A x B)

1. DOT PRODUCT (A. B):

Dot product of two vectors A and B is defined as:

$$\bar{A}.\bar{B} = \left|\bar{A}\right| \left|\bar{B}\right| \cos\theta_{AB}$$

Where θ_{AB} is the angle formed between A and B.
Also θ_{AB} ranges from 0 to π i.e. $0 \leq \theta_{AB} \leq \pi$
The result of A.B is a scalar, hence dot product is also known as Scalar Product.

Properties of Dot Product:

1. If A = (Ax, Ay, Az) and B = (Bx, By, Bz) then

$\bar{A}.\bar{B}$= AxBx + AyBy + AzBz

2. $\bar{A}.\bar{B}$= |A| |B|, if cosθ_{AB}=1 which means $\theta_{AB} = 0^0$

This shows that A and B are in the same direction or we can also say that A and B are parallel to each other.

3. $\bar{A}.\bar{B}$ = - |A| |B|, if cos θ_{AB}=-1 which means θ_{AB} = 180^0.
This shows that A and B are in the opposite direction or we can also say that A and B are antiparallel to each other.

4. $\bar{A}.\bar{B}$ = 0, if cos θ_{AB}=0 which means θ_{AB} = 90^0.
This shows that A and B are orthogonal or perpendicular to each other.

5. Since we know the Cartesian base vectors are mutually perpendicular to each other, we have

$$\bar{a}_x.\bar{a}_x = \bar{a}_y.\bar{a}_y = \bar{a}_z.\bar{a}_z = 1$$

$$\bar{a}_x.\bar{a}_y = \bar{a}_y.\bar{a}_z = \bar{a}_z.\bar{a}_x = 0$$

2. Cross Product (A X B):

Cross Product of two vectors A and B is given as:

$$\bar{A} X \bar{B} = |\bar{A}| \, |\bar{B}| \sin\theta_{AB} \, \bar{a}_N$$

Where θ_{AB} is the angle formed between A and B and $\bar{a}_N$ is a unit vector normal to both A and B. Also θ ranges from 0 to π i.e. $0 \leq \theta_{AB} \leq \pi$

The cross product is an operation between two vectors and the output is also a vector.

Properties of Cross Product:

1. If A = (Ax, Ay, Az) and B = (Bx, By, Bz) then,

$$\mathbf{A} * \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z \\ \mathbf{B}_x & \mathbf{B}_y & \mathbf{B}_z \end{vmatrix}$$

The resultant vector is always normal to both the vector A and B.

2. $\bar{A} X \bar{B}$ = 0, if $\sin\theta_{AB}$ = 0 which means θ_{AB} = 0^0 or 180^0;
This shows that A and B are either parallel or antiparallel to each other.

63. $\bar{A} X \bar{B} = |\bar{A}| \, |\bar{B}| \bar{a}_N$, if $\sin\theta_{AB}$ = 0 which means θ_{AB} = 90^0.
This shows that A and B are orthogonal or perpendicular to each other.

4. Since we know the Cartesian base vectors are mutually perpendicular to each other, we have

$$\bar{a}_x X \, \bar{a}_x = \bar{a}_y \, X \, \bar{a}_y = \bar{a}_z X \bar{a}_z = 0$$

$$\bar{a}_x X \, \bar{a}_y = \bar{a}_z \ , \bar{a}_y \, X \, \bar{a}_z = \ \bar{a}_x \ , \ \ \bar{a}_z X \, \bar{a}_x = \bar{a}_y$$

CO-ORDINATE SYSTEMS

Co-Ordinate system is a system of representing points in a space of given dimensions by coordinates, such as the Cartesian coordinate system or the system of celestial longitude and latitude.

In order to describe the spatial variations of the quantities, appropriate coordinate system is required. A point or vector can be represented in a curvilinear coordinate system that may be orthogonal or non-orthogonal. An orthogonal system is one in which the coordinates are mutually perpendicular to each other.

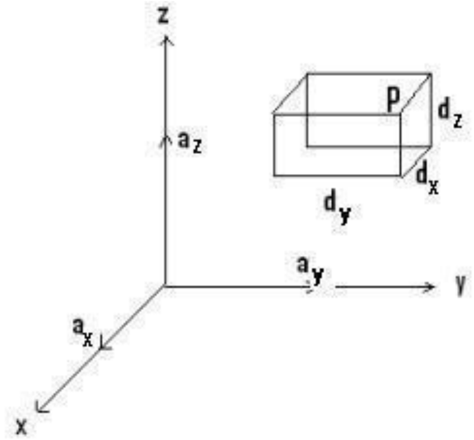
The different co-ordinate system available are:

- Cartesian or Rectangular co-ordinate system.(Example: Cube, Cuboid)
- Circular Cylindrical co-ordinate system.(Example : Cylinder)
- Spherical co-ordinate system. (Example: Sphere)

The choice depends on the geometry of the application.

A set of 3 scalar values that define position and a set of unit vectors that define direction form a co-ordinate system. The 3 scalar values used to define position are called co-ordinates. All coordinates are defined with respect to an arbitrary point called the origin.

1. Cartesian Co-ordinate System / Rectangular Co-ordinate System (x,y,z)



A Vector in Cartesian system is represented as (Ax, Ay, Az) Or

$$\bar{A} = A_x\bar{a}_x + A_y\bar{a}_y + A_z\bar{a}_z$$

Where$\bar{a}_x$,$\bar{a}_y$ and $\bar{a}_z$are the unit vectors in x, y, z direction respectively.

Range of the variables:

It defines the minimum and the maximum value that x, y and z can have in Cartesian system.
-∞ ≤ x,y,z ≤ ∞

Differential Displacement / Differential Length (dl):

It is given as

$$\overline{dl} = dx\bar{a}_x + dy\bar{a}_y + dz\bar{a}_z$$

Differential length for a line parallel to x, y and z axis are respectively given as:

dl = $dx\bar{a}_x$---(For a line parallel to x-axis).
dl = $dy\bar{a}_y$ ---(For a line Parallel to y-axis).
dl = $dz\bar{a}_z$ ---(For a line parallel to z-axis).

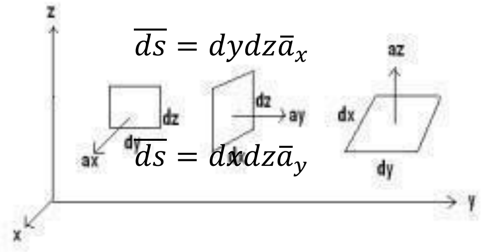
If there is a wire of length L in z-axis, then the differential length is given as dl = dz az. Similarly if the wire is in y-axis then the differential length is given as dl = dy ay.

Differential Normal Surface (ds):

Differential surface is basically a cross product between two parameters of the surface.
The differential surface (area element) is defined as

$$\overline{ds} = ds\bar{a}_N$$

Where$\bar{a}_N$, is the unit vector perpendicular to the surface.



For the 1st figure,

$$\overline{ds} = dydz\bar{a}_x$$

2nd figure,

$$\overline{ds} = dxdz\bar{a}_y$$

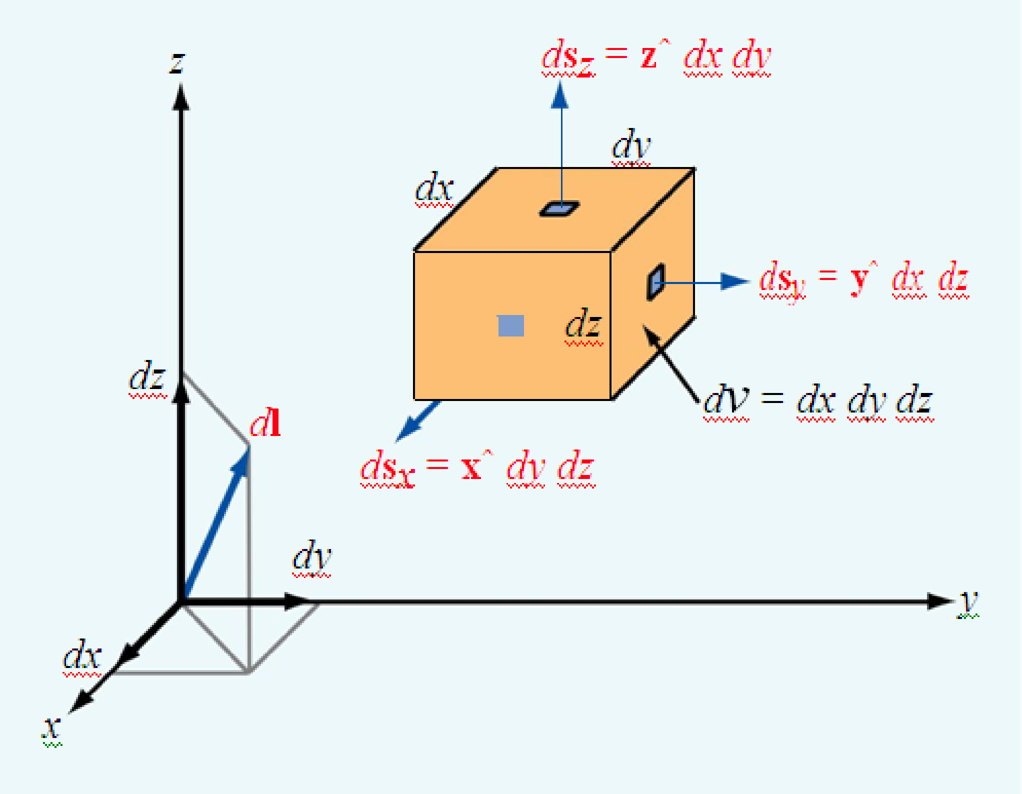
3rd figure,

$$\overline{ds} = dxdy\bar{a}_z$$

Differential Volume:

The differential volume element (dv) can be expressed in terms of the triple product.

$$dv = dxdydz$$



Differential length, area, and volume in Cartesian coordinates.

2. Circular Cylindrical Co-ordinate System

A Vector in Cylindrical system is represented as (A_r, A_{ϕ}, A_z) or

$$\bar{A} = A_r\bar{a}_r + A_{\emptyset}\bar{a}_{\emptyset} + A_z\bar{a}_z$$

Where$\bar{a}_r$, $\bar{a}_{\emptyset}$ and $\bar{a}_z$ are the unit vectors in r, Φ and z directions respectively.

The physical significance of each parameter of cylindrical coordinates:

1. The value r indicates the distance of the point from the z-axis. It is the radius of the cylinder.
2. The value Φ, also called the azimuthal angle, indicates the rotation angle around the z-axis. It is basically measured from the x axis in the x-y plane. It is measured anti clockwise.
3. The value z indicates the distance of the point from z-axis. It is the same as in the Cartesian system. In short, it is the height of the cylinder.

Range of the variables:

It defines the minimum and the maximum values of r, Φ and z.

0 ≤ r ≤ ∞
0 ≤ Φ ≤ 2π
-∞ ≤ z ≤ ∞

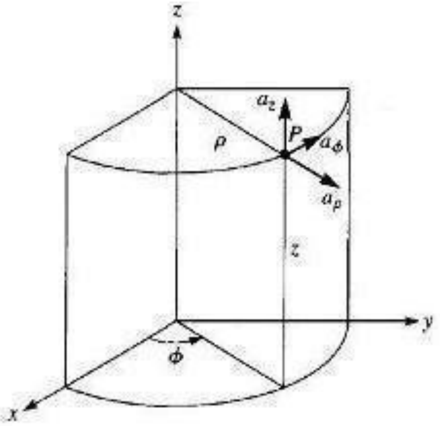


Figure shows Point P and Unit vectors in Cylindrical Co-ordinate System.

Differential Displacement / Differential Length (dl):

It is given as

$$\overline{dl} = dr\bar{a}_r + rd\varphi\bar{a}_\varphi + dz\bar{a}_z$$

Differential length for a line parallel to r, Φ and z axis are respectively given as:

dl = $dr\bar{a}_r$---(For a line parallel to r-direction).
dl = $rd\varphi\bar{a}_\varphi$ ---(For a line Parallel to Φ-direction).
dl = $dz\bar{a}_z$ ---(For a line parallel to z-axis).

Differential Normal Surface (ds):

Differential surface is basically a cross product between two parameters of the surface. The differential surface (area element) is defined as

$$\overline{ds} = ds\bar{a}_N$$

Where$\bar{a}_N$, is the unit vector perpendicular to the surface.

This surface describes a circular disc. Always remember- To define a circular disk we need two parameter one distance measure and one angular measure. An angular parameter will always give a curved line or an arc.

In this case dΦ is measured in terms of change in arc.

Arc is given as:
Arc= radius * angle

$$\overline{ds} = rdrd\varphi\bar{a}_z$$
$$\overline{ds} = drdz\bar{a}_\varphi$$
$$\overline{ds} = rdrd\varphi\bar{a}_r$$

Differential Volume:

The differential volume element (dv) can be expressed in terms of the triple product.

$$dv = rdrd\varphi dz$$

3. **Spherical coordinate System**:

Spherical coordinates consist of one scalar value (r), with units of distance, while the other two scalarvalues (θ, Φ) have angular units (degrees or radians).

A Vector in Spherical System is represented as (A_r ,A_Θ, A_Φ) or

$$\bar{A} = A_r\bar{a}_r + A_\theta\bar{a}_\theta + A_\varphi\bar{a}_\varphi$$

Where$\bar{a}_r$,$\bar{a}_\theta$ and $\bar{a}_\varphi$ are the unit vectors in r, θ and Φ direction respectively.

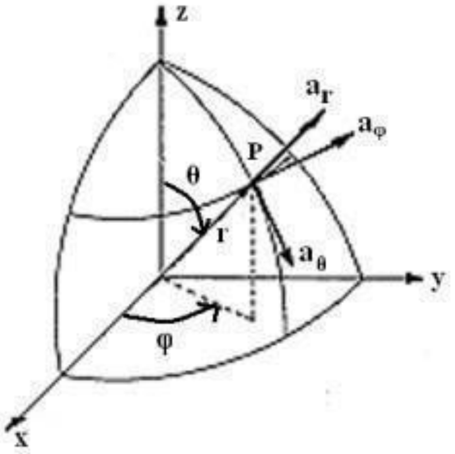
The physical significance of each parameter of spherical coordinates:

1. The value r expresses the distance of the point from origin (i.e. similar to altitude). It is the radius of the sphere.
2. The angle θ is the angle formed with the z- axis (i.e. similar to latitude). It is also called the co-latitude angle. It is measured clockwise.
3. The angle Φ, also called the azimuthal angle, indicates the rotation angle around the z-axis (i.e. similar to longitude). It is basically measured from the x axis in the x-y plane. It is measured counter-clockwise.

Range of the variables:

It defines the minimum and the maximum value that r, θ and υ can have in spherical co-ordinate system.

$0 \leq r \leq \infty$
$0 \leq \theta \leq \pi$
$0 \leq \Phi \leq 2\pi$



Differential length:
It is given as
$\overline{dl} = dr\bar{a}_r + rd\theta\bar{a}_\theta + r\sin\theta\, d\varphi\bar{a}_\varphi$

Differential length for a line parallel to r, θ and Φ axis are respectively given as:

dl = $dr\bar{a}_r$--(For a line parallel to r axis)

dl = $rd\theta\bar{a}_\theta$---(For a line parallel to θ direction)

dl = $r\sin\theta\, d\varphi\bar{a}_\varphi$ --(For a line parallel to Φ direction)

Differential Normal Surface (ds):

Differential surface is basically a cross product between two parameters of the surface.
The differential surface (area element) is defined as

$$\overline{ds} = ds\bar{a}_N$$

Where$\bar{a}_N$, is the unit vector perpendicular to the surface.

$$\overline{ds} = rdrd\theta\bar{a}_\varphi$$
$$\overline{ds} = r^2\sin\theta\, d\varphi d\theta\bar{a}_r$$
$$\overline{ds} = r\sin\theta\, drd\varphi\bar{a}_\theta$$

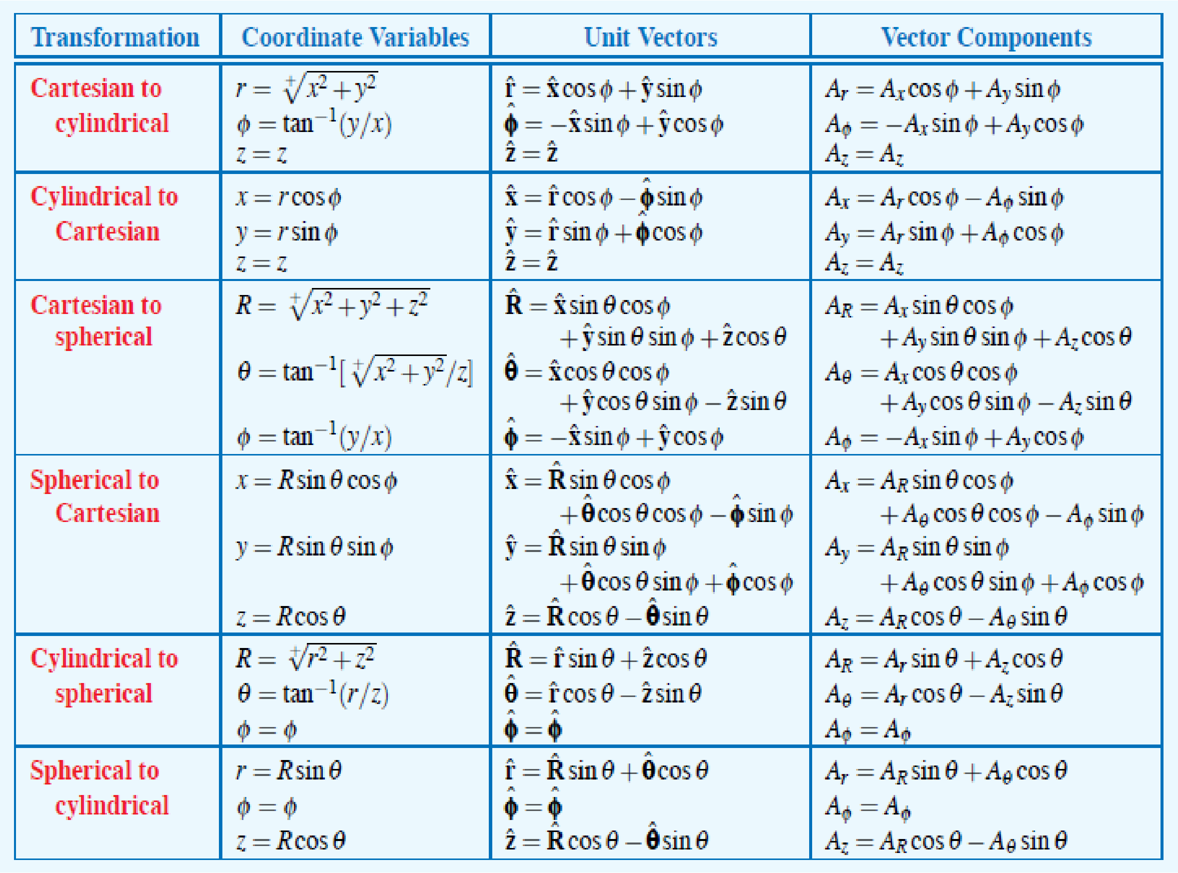
Differential Volume:

The differential volume element (dv) can be expressed in terms of the triple product.

$$dv = r^2\sin\theta\, drd\varphi d\theta$$

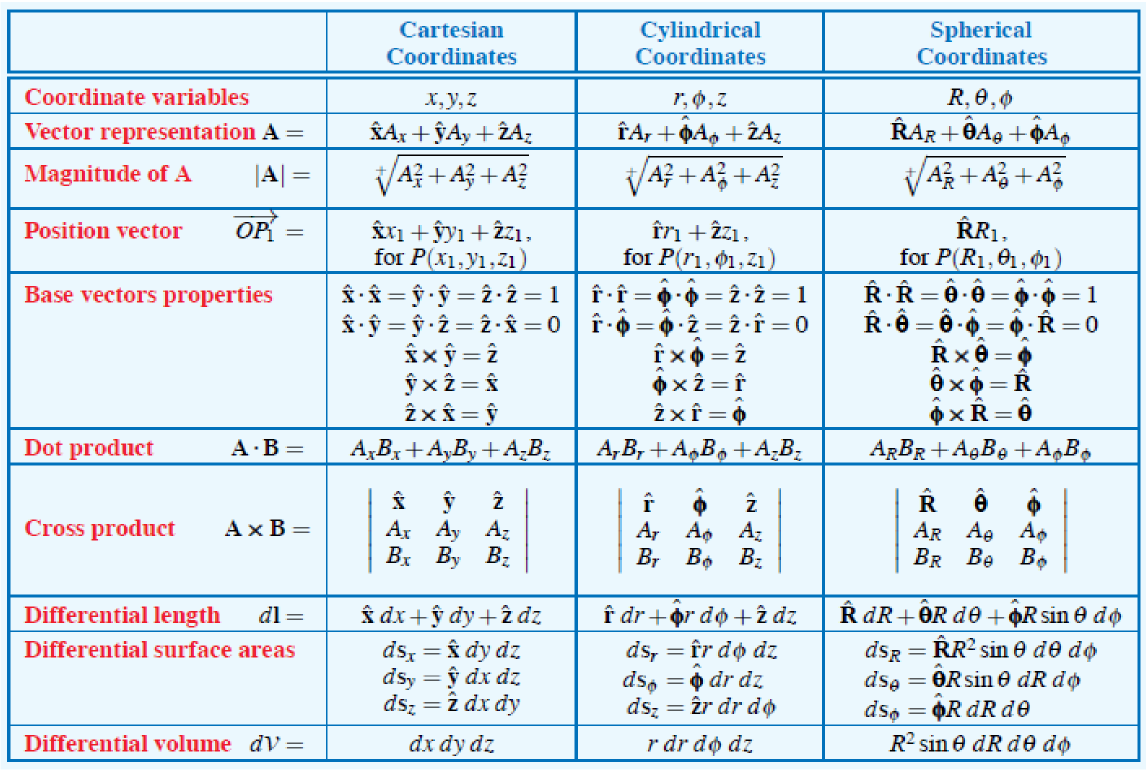
Coordinate transformations:

Coordinate transformations



Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi$ $+\hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi$ $+\hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Vector relations in the three common coordinate systems.



	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}} \cdot \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A} \cdot \mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A} \times \mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$ds_x = \hat{\mathbf{x}}\,dy\,dz$ $ds_y = \hat{\mathbf{y}}\,dx\,dz$ $ds_z = \hat{\mathbf{z}}\,dx\,dy$	$ds_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $ds_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $ds_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$ds_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $ds_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $ds_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

DIVERGENCE THEOREM:

It states that the net outward flux of a vector field A through a closed surface S is equal to the volume integral of the divergence of the field A inside the surface.

STOKES THEOREM:

It states that the circulation of a vector field A around a closed path L is equal to the surface integral of the curl of A over the open surface S bounded by L.

Electrostatics:

Electrostatics is a branch of science that involves the study of various phenomena caused by electric charges that are slow-moving or even stationary. Electric charge is a fundamental property of matter and charge exist in integral multiple of electronic charge. Electrostatics as the study of electric charges at rest.

The two important laws of electrostatics are

- Coulomb‘s Law.
- Gauss‘s Law.

Both these laws are used to find the electric field due to different charge configurations.

Coulomb‘s law is applicable in finding electric field due to any charge configurations where as Gauss‘s law is applicable only when the charge distribution is symmetrical.

Coulomb's Law

Coulomb's Law states that the force between two point charges Q1and Q2 is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

A point charge is a charge that occupies a region of space which is negligibly small compared to the distance between the point charge and any other object.
Point charge is a hypothetical charge located at a single point in space. It is an idealized model of a particle having an electric charge.

Mathematically, $F = \frac{kQ_1Q_2}{R^2}$, where k is the proportionality constant.

In SI units, Q1 and Q2 are expressed in Coulombs(C) and R is in meters.

Force F is in Newtons (N) and $k = \frac{1}{4\pi\varepsilon_0}$, ε_0 is called the permittivity of free space.

(We are assuming the charges are in free space. If the charges are any other dielectric medium, we will use $\varepsilon = \varepsilon_0 \varepsilon_r$ instead where ε_r is called the relative permittivity or the dielectric constant of the medium).

Therefore $F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1Q_2}{R^2}$ .... (1)

As shown in the Figure 1 let the position vectors of the point charges Q1and Q2 are given by $\vec{r}_1$ and $\vec{r}_2$. Let $\overrightarrow{F_{12}}$ represent the force on Q1 due to charge Q2.

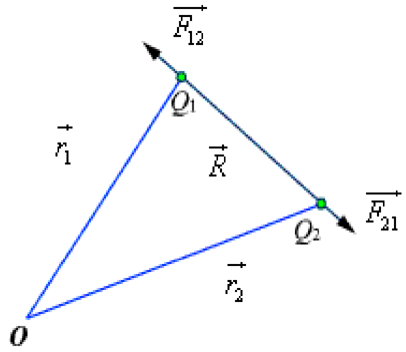


Fig 1: Coulomb's Law

The charges are separated by a distance of $R = \left|\vec{r}_1 - \vec{r}_2\right| = \left|\vec{r}_2 - \vec{r}_1\right|$. We define the unit vectors as

$\widehat{a_{12}} = \frac{\left(\vec{r}_2 - \vec{r}_1\right)}{R}$ and $\widehat{a_{21}} = \frac{\left(\vec{r}_1 - \vec{r}_2\right)}{R}$

$\overrightarrow{F_{12}}$ can be defined as $\overrightarrow{F_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\widehat{a_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\frac{(\vec{r}_2 - \vec{r}_1)}{\left|\vec{r}_2 - \vec{r}_1\right|^3}$.

Similarly the force on Q_1 due to charge Q_2 can be calculated and if $\overrightarrow{F_{21}}$ represents this force then we can write $\overrightarrow{F_{21}} = -\overrightarrow{F_{12}}$

When we have a number of point charges, to determine the force on a particular charge due to all other charges, we apply principle of superposition. If we have N number of charges $Q_1, Q_2, \ldots\ldots\ldots Q_N$ located respectively at the points represented by the position vectors $\vec{r}_1$, $\vec{r}_2$,...... $\overrightarrow{r_N}$, the force experienced by a charge Q located at $\vec{r}$ is given by,

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{Q_i(\vec{r} - \vec{r}_i)}{\left|\vec{r} - \vec{r}_i\right|^3}$$

Electric Field:

Electric field due to a charge is the space around the unit charge in which it experiences a force. Electric field intensity or the electric field strength at a point is defined as the force per unit charge.

Mathematically,

E = F / Q

OR

F = E Q

The force on charge Q is the product of a charge (which is a scalar) and the value of the electric field (which is a vector) at the point where the charge is located. That is

$$\vec{E} = \lim_{Q\to 0} \frac{\vec{F}}{Q} \quad \text{or,} \quad \vec{E} = \frac{\vec{F}}{Q}$$

The electric field intensity *E* at a point *r* (observation point) due a point charge *Q* located at $\vec{r}\,'$ (source point) is given by:

$$\vec{E} = \frac{Q(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}\,'\right|^3}$$

For a collection of *N* point charges $Q_1, Q_2, ...Q_N$ located at $\vec{r}_1, \vec{r}_2, ...\vec{r}_N$, the electric field intensity at point $\vec{r}\,'$ is obtained as

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q_k(\vec{r} - \vec{r}_i)}{\left|\vec{r} - \vec{r}_i\right|^3}$$

The expression (6) can be modified suitably to compute the electric filed due to a continuous distribution of charges.

In figure 2 we consider a continuous volume distribution of charge *(t)* in the region denoted as the source region.

For an elementary charge $dQ = \rho(\vec{r}\,')dv'$, i.e. considering this charge as point charge, we can write the field expression as:

$$d\vec{E} = \frac{dQ(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}\,'\right|^3} = \frac{\rho(\vec{r}\,')dv'(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}\,'\right|^3}$$

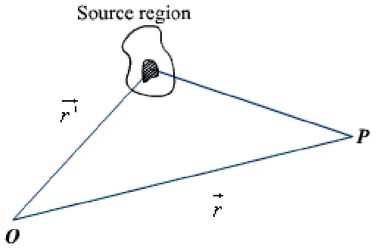


Fig 2: Continuous Volume Distribution of Charge

When this expression is integrated over the source region, we get the electric field at the point *P* due to this distribution of charges. Thus the expression for the electric field at *P* can be written as:

$$\overrightarrow{E(r)} = \int_V \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}'\right|^3} dv'$$ ....volume charge...........................

Similar technique can be adopted when the charge distribution is in the form of a line charge density or a surface charge density.

$$\overrightarrow{E(r)} = \int_L \frac{\rho_L(\vec{r}')(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}'\right|^3} dl'$$ ....line charge ....

$$\overrightarrow{E(r)} = \int_S \frac{\rho_s(\vec{r}')(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 \left|\vec{r}-\vec{r}'\right|^3} ds'$$ ....surface charge......................

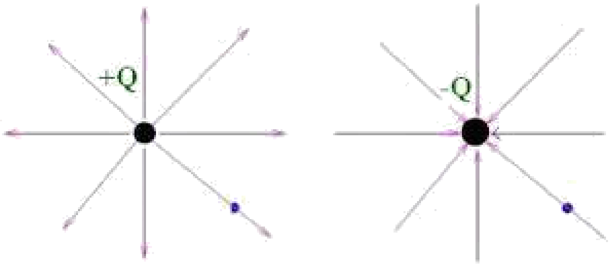
Electric Lines of Forces:

Electric line of force is a pictorial representation of the electric field.

Electric line of force (also called Electric Flux lines or Streamlines) is an imaginary straight or curved path along which a unit positive charge tends to move in an electric field.

Properties Of Electric Lines Of Force:

1. Lines of force start from positive charge and terminate either at negative charge or move to infinity.
2. Similarly lines of force due to a negative charge are assumed to start at infinity and terminate at the negative charge.



3. The number of lines per unit area, through a plane at right angles to the lines, is proportional to the magnitude of E. This means that, where the lines of force are close together, E is large and where they are far apart E is small.
4. If there is no charge in a volume, then each field line which enters it must also leave it.
5. If there is a positive charge in a volume then more field lines leave it than enter it.
6. If there is a negative charge in a volume then more field lines enter it than leave it.
7. Hence we say Positive charges are sources and Negative charges are sinks of the field.
8. These lines are independent on medium.
9. Lines of force never intersect i.e. they do not cross each other.
10. Tangent to a line of force at any point gives the direction of the electric field E at that point.

Electricfluxdensity:

As stated earlier electric field intensity or simply ‘Electric field' gives the strength of the field at a particular point. The electric field depends on the material media in which the field is being considered. The flux density vector is defined to be independent of the material media (as we'll see that it relates to the charge that is producing it).For a linear isotropic medium under consideration; the flux density vector is defined as:

$$\vec{D} = \varepsilon \vec{E}$$

We define the electric flux as

$$\psi = \int_S \vec{D} . d\vec{s}$$

Gauss's Law:

Gauss's law is one of the fundamental laws of electromagnetism and it states that the total electric flux through a closed surface is equal to the total charge enclosed by the surface.

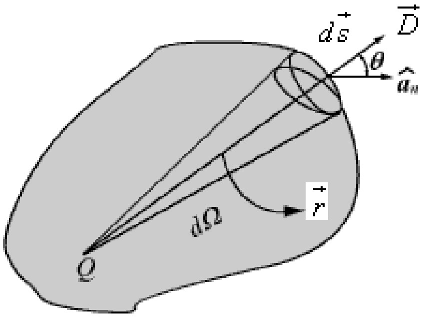


Fig 3: Gauss's Law

Let us consider a point charge *Q* located in an isotropic homogeneous medium of dielectric constant . The flux density at a distance *r* on a surface enclosing the charge is given by

$$\vec{D} = \varepsilon\vec{E} = \frac{Q}{4\pi r^2}\hat{a}_r$$

If we consider an elementary area ***ds***, the amount of flux passing through the elementary area is given by

$$d\psi = \vec{D}.ds = \frac{Q}{4\pi r^2} ds\cos\theta$$

But $\frac{ds\cos\theta}{r^2} = d\Omega$, is the elementary solid angle subtended by the area $d\vec{s}$ at the location of *Q*.

Therefore we can write $d\psi = \frac{Q}{4\pi} d\Omega$

For a closed surface enclosing the charge, we can write $\psi = \oint_s d\psi = \frac{Q}{4\pi}\oint_s d\Omega = Q$

which can seen to be same as what we have stated in the definition of Gauss's Law.

Hence we have,

$$Q_{enc} = \oint_S D \, . \, ds = \int_V \rho_V \, dv$$

Applying Divergence theorem we have,

$$\oint_S D \, . \, ds = \int_V \nabla \cdot D \, dv$$

Comparing the above two equations, we have

$$\int_V \nabla \cdot D \, dv = \int_V \rho_V \, dv$$

This equation is called the 1st Maxwell's equation of electrostatics.

Application of Gauss's Law:

Gauss's law is particularly useful in computing $\vec{E}$ or $\vec{D}$ where the charge distribution has some symmetry. We shall illustrate the application of Gauss's Law with some examples.

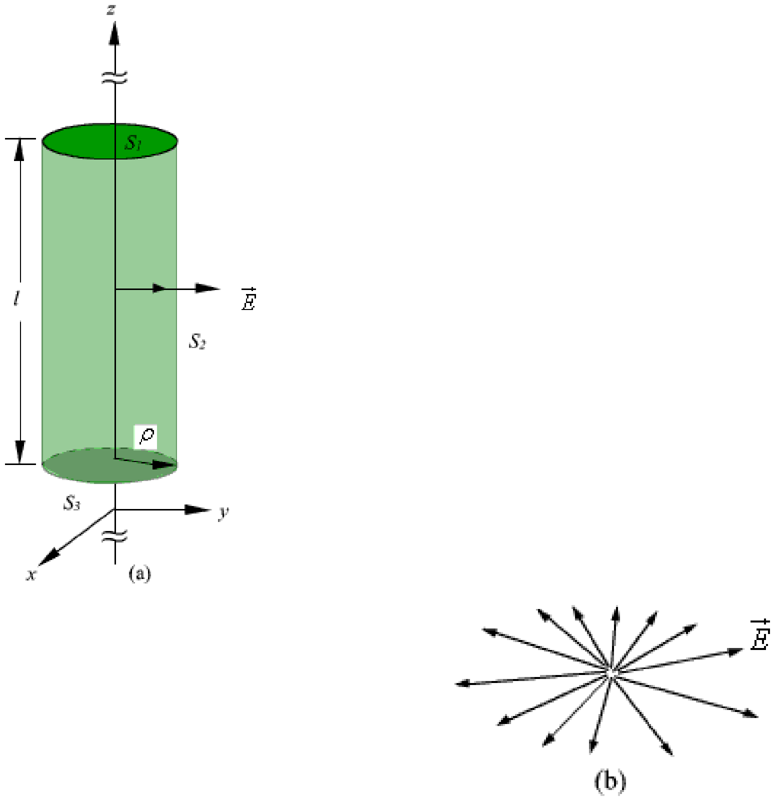
1. **$\vec{E}$ due to an infinite line charge**

As the first example of illustration of use of Gauss's law, let consider the problem of determination of the electric field produced by an infinite line charge of density $_L$C/m. Let us consider a line charge positioned along the *z*-axis as shown in Fig. 4(a) (next slide). Since the line charge is assumed to be infinitely long, the electric field will be of the form as shown in Fig. 4(b) (next slide).

If we consider a close cylindrical surface as shown in Fig. 2.4(a), using Gauss's theorm we can write,

$$\rho_L l = Q = \oint_S \varepsilon_0 \vec{E} . d\vec{s} = \int_{S_1} \varepsilon_0 \vec{E} . d\vec{s} + \int_{S_2} \varepsilon_0 \vec{E} . d\vec{s} + \int_{S_3} \varepsilon_0 \vec{E} . d\vec{s}$$

Considering the fact that the unit normal vector to areas S_1 and S_3 are perpendicular to the electric field, the surface integrals for the top and bottom surfaces evaluates to zero. Hence we can write, $\rho_l l = \varepsilon_0 E.2\pi\rho l$



(a)

(b)

Fig 4: Infinite Line Charge

$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0\rho}\hat{a}_\rho$$

2. Infinite Sheet of Charge

As a second example of application of Gauss's theorem, we consider an infinite charged sheet covering the *x-z* plane as shown in figure 5. Assuming a surface charge density of ρ_s for the infinite surface charge, if we consider a cylindrical volume having sides Δs placed symmetrically as shown in figure 5, we can write:

$$\oint_s \vec{D}\cdot d\vec{s} = 2D\Delta s = \rho_s \Delta s$$

$$\therefore \qquad \vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_y$$

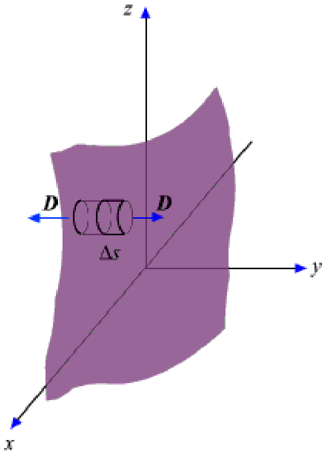


Fig 5: Infinite Sheet of Charge

It may be noted that the electric field strength is independent of distance. This is true for the infinite plane of charge; electric lines of force on either side of the charge will be perpendicular to the sheet and extend to infinity as parallel lines. As number of lines of force per unit area gives the strength of the field, the field becomes independent of distance. For a finite charge sheet, the field will be a function of distance.

3. Uniformly Charged Sphere

Let us consider a sphere of radius r0 having a uniform volume charge density of rv C/m3. To determine $\vec{D}$ everywhere, inside and outside the sphere, we construct Gaussian surfaces of radius r < r0 and r > r0 as shown in Fig. 6 (a) and Fig. 6(b).

For the region $r \leq r_0$; the total enclosed charge will be

$$Q_{en} = \rho_v \frac{4}{3} \pi r^3$$

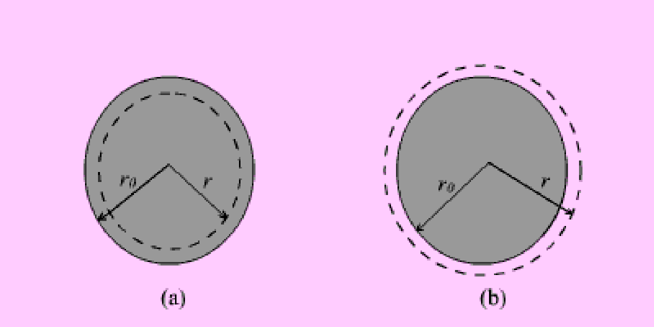


Fig 6: Uniformly Charged Sphere

By applying Gauss's theorem,

$$\oint_s \vec{D} \cdot d\vec{s} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} D_r r^2 \sin\theta d\theta d\phi = 4\pi r^2 D_r = Q_{en}$$

Therefore

$$\vec{D} = \frac{r}{3} \rho_v \hat{a}_r \qquad 0 \le r \le r_0$$

For the region $r \ge r_0$; the total enclosed charge will be

$$Q_{en} = \rho_v \frac{4}{3} \pi r_0^3$$

By applying Gauss's theorem,

$$\vec{D} = \frac{r_0^3}{3r^2} \rho_v \hat{a}_r \qquad r \ge r_0$$

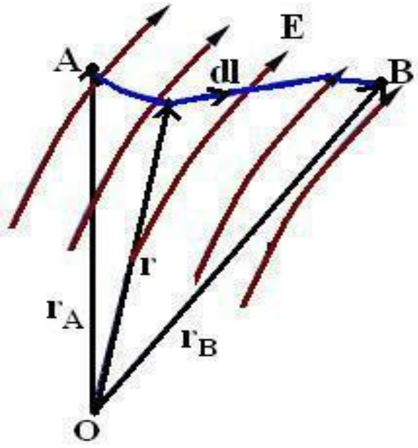
Electric Potential / Electrostatic Potential (V):

If a charge is placed in the vicinity of another charge (or in the field of another charge), it experiences a force. If a field being acted on by a force is moved from one point to another, then work is either said to be done on the system or by the system.

Say a point charge Q is moved from point A to point B in an electric field E, then the work done in moving the point charge is given as:

WA→B = - ∫AB (F . dl) = - Q ∫AB(E . dl)

where the – ve sign indicates that the work is done on the system by an external agent.



The work done per unit charge in moving a test charge from point A to point B is the electrostatic potential difference between the two points(VAB).

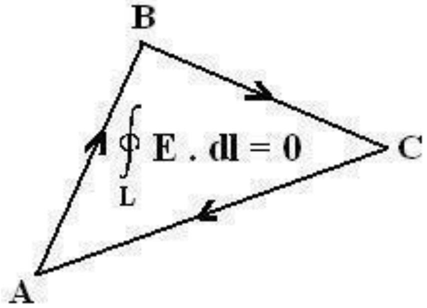
VAB = WA→B / Q

- ∫AB(E . dl)

- ∫InitialFinal (E . dl)

If the potential difference is positive, there is a gain in potential energy in the movement, external agent performs the work against the field. If the sign of the potential difference is negative, work is done by the field.

The electrostatic field is conservative i.e. the value of the line integral depends only on end points and is independent of the path taken.



- Since the electrostatic field is conservative, the electric potential can also be written as:

$$V_{AB} = -\int_A^B \bar{E}\,.\bar{dl}$$

$$V_{AB} = -\int_A^{p_0} \bar{E}\,.\bar{dl} - \int_{p_0}^{B} \bar{E}\,.\bar{dl}$$

$$V_{AB} = -\int_{p_0}^{B} \bar{E}\,.\bar{dl} + \int_{p_0}^{A} \bar{E}\,.\bar{dl}$$

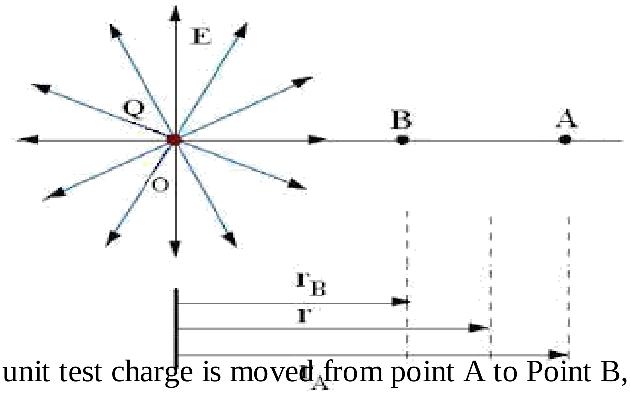
$$V_{AB} = V_B \; - V_A$$

Thus the potential difference between two points in an electrostatic field is a scalar field that is defined at every point in space and is independent of the path taken.

- The work done in moving a point charge from point A to point B can be written as:

WA→B = - Q [V_B – V_A] = $-Q\int_A^B \bar{E}\,.\,\bar{dl}$

- Consider a point charge Q at origin O.



Now if a unit test charge is moved from point A to Point B, then the potential difference between them is given as:

$$V_{AB} = -\int_A^B \mathbf{E}\,.\,\mathbf{dl} = -\int_{r_A}^{r_B} \mathbf{E}\,.\,\mathbf{dl} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon r^2}\,\mathbf{a}_r \cdot \mathbf{dr}\,\mathbf{a}_r$$

$$= \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) = V_B - V_A$$

- Electrostatic potential or Scalar Electric potential (V) at any point P is given by:

$$V = -\int_{P_0}^{P} \bar{E}\,.\,\bar{dl}$$

The reference point Po is where the potential is zero (analogues to ground in a circuit). The reference is often taken to be at infinity so that the potential of a point in space is defined as

$$V = -\int_{\infty}^{P} \bar{E} \,.\, \bar{dl}$$

Basically potential is considered to be zero at infinity. Thus potential at any point (rB = r) due to a point charge Q can be written as the amount of work done in bringing a unit positive charge frominfinity to that point (i.e. rA → ∞)

Electric potential (V) at point r due to a point charge Q located at a point with position vector r1 is given as:

$$V = \frac{Q}{4\pi\varepsilon|r - r_1|}$$

Similarly for N point charges Q1, Q2 ….Qn located at points with position vectors r1, r2, r3…..rn, theelectric potential (V) at point r is given as:

$$V = \frac{1}{4\pi\varepsilon}\sum_{k=1}^{N}\frac{Q_k}{|r - r_k|} \qquad V = \frac{Q}{4\pi\varepsilon r}$$

The charge element dQ and the total charge due to different charge distribution is given as:

dQ = ρldl → Q = ∫L (ρldl) → (Line Charge)

dQ = ρsds → Q = ∫S (ρsds) → (Surface Charge)

dQ = ρvdv → Q = ∫V (ρvdv) → (Volume Charge)

$$V = \int_L \frac{\rho_L\, dl}{4\pi\varepsilon\,|r - r_1|} \qquad \text{(Line Charge)}$$

$$V = \int_S \frac{\rho_S\, ds}{4\pi\varepsilon\,|r - r_1|} \qquad \text{(Surface Charge)}$$

$$V = \int_V \frac{\rho_V\, dv}{4\pi\varepsilon\,|r - r_1|} \qquad \text{(Volume Charge)}$$

Second Maxwell’s Equation of Electrostatics:

The work done per unit charge in moving a test charge from point A to point B is the electrostatic potential difference between the two points(V_{AB}).

$V_{AB} = V_B – V_A$

Similarly,

$V_{BA} = V_A – V_B$

Hence it‘s clear that potential difference is independent of the path taken. Therefore

$V_{AB} = - V_{BA}$

$V_{AB} + V_{BA} = 0$

$\int_{AB} (E . dl) + [- \int_{BA} (E . dl)] = 0$

$$\oint_L \mathbf{E} . \mathbf{dl} = 0$$

The above equation is called the second Maxwell‘s Equation of Electrostatics in integral form.. The above equation shows that the line integral of Electric field intensity (E) along a closed path is equal to zero.

In simple words—No work is done in moving a charge along a closed path in an electrostatic field.

Applying Stokes‘ Theorem to the above Equation, we have:

$$\oint_L \mathbf{E} . \mathbf{dl} = \int_S (\nabla \mathrm{x} \mathbf{E}) \cdot \mathbf{ds} = 0$$

$$---> \nabla \mathrm{x} \mathbf{E} = 0$$

If the Curl of any vector field is equal to zero, then such a vector field is called an Irrotational or Conservative Field. Hence an electrostatic field is also called a conservative field.

The above equation is called the second Maxwell‘s Equation of Electrostatics in differential form.

Relationship Between Electric Field Intensity (E) and Electric Potential (V):

Since Electric potential is a scalar quantity, hence dV (as a function of x, y and z variables) can be written as:

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$\left(\frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z\right)\cdot\left(dx\,a_x + dy\,a_y + dz\,a_z\right) = -\,E\,.\,dl$$

$$\nabla V\,.\,dl = -\,E\,.\,dl \quad \text{--->} \quad \boxed{E = -\nabla V}$$

Hence the Electric field intensity (E) is the negative gradient of Electric potential (V).
The negative sign shows that E is directed from higher to lower values of V i.e. E is opposite to the direction in which V increases.

Energy Density In Electrostatic Field / Work Done To Assemble Charges:

In case, if we wish to assemble a number of charges in an empty system, work is required to do so. Also electrostatic energy is said to be stored in such a collection.

Let us build up a system in which we position three point charges Q1, Q2 and Q3 at position r1, r2 and r3 respectively in an initially empty system.

Consider a point charge Q1 transferred from infinity to position r1 in the system. It takes no work to bring the first charge from infinity since there is no electric field to fight against (as the system is empty i.e. charge free).

Hence, W1 = 0 J

Now bring in another point charge Q2 from infinity to position r2 in the system. In this case we have to do work against the electric field generated by the first charge Q1.

Hence, W2 = Q2 V21

where V21 is the electrostatic potential at point r2 due to Q1.

- Work done W2 is also given as:

$$W_2 = \frac{Q_2 Q_1}{4\pi\varepsilon\ |r_2 - r_1|}$$

Now bring in another point charge Q3 from infinity to position r3 in the system. In this case we have to do work against the electric field generated by Q1 and Q2.

Hence, W3 = Q3 V31 + Q3 V32 = Q3 (V31 + V32)

where V31 and V32 are electrostatic potential at point r3 due to Q1 and Q2 respectively.

The work done is simply the sum of the work done against the electric field generated by point charge Q1 and Q2 taken in isolation:

$$W_3 = \frac{Q_3 Q_1}{4\pi\varepsilon\ |r_3 - r_1|} + \frac{Q_3 Q_2}{4\pi\varepsilon\ |r_3 - r_2|}$$

- Thus the total work done in assembling the three charges is given as:

WE = W1 + W2 + W3

0 + Q2 V21 + Q3 (V31 + V32)

Also total work done (WE) is given as:

$$W_E = \frac{1}{4\pi\varepsilon}\left[\frac{Q_2 Q_1}{|r_2 - r_1|} + \frac{Q_3 Q_1}{|r_3 - r_1|} + \frac{Q_3 Q_2}{|r_3 - r_2|}\right]$$

If the charges were positioned in reverse order, then the total work done in assembling them is given as:

WE = W3 + W2+ W1
= 0 + Q2V23 + Q3(V12+ V13)

Where V23 is the electrostatic potential at point r2 due to Q3 and V12 and V13 are electrostatic potential at point r1 due to Q2 and Q3 respectively.

- Adding the above two equations we have,

2WE = Q1 (V12 + V13) + Q2 (V21 + V23) + Q3 (V31 + V32)

= Q1 V1 + Q2 V2 + Q3 V3

Hence

WE =1 / 2 [Q1V1 + Q2V2 + Q3V3]

where V1, V2 and V3 are total potentials at position r1, r2 and r3 respectively.

- The result can be generalized for N point charges as:

$$W_E = \frac{1}{2} \sum_{k=1}^{N} Q_k V_k$$

The above equation has three interpretation: This equation represents the potential energy of the system.This is the work done in bringing the static charges from infinity and assembling them in the required system. This is the kinetic energy which would be released if the system gets dissolved i.e. the charges returns back to infinity.

In place of point charge, if the system has continuous charge distribution (line, surface or volume charge), then the total work done in assembling them is given as:

$$W_E = \frac{1}{2} \int_L \rho_L V \, dl \quad \text{(Line Charge)}$$

$$W_E = \frac{1}{2} \int_S \rho_S V \, ds \quad \text{(Surface Charge)}$$

$$W_E = \frac{1}{2} \int_V \rho_V V \, dv \quad \text{(Volume Charge)}$$

Since ρv = ∇ . D and E = - ∇ V,

Substituting the values in the above equation, work done in assembling a volume charge distribution in terms of electric field and flux density is given as:

$$W_E = \frac{1}{2}\int_V D . E \, dv = \frac{1}{2}\int_V \varepsilon E^2 \, dv$$

The above equation tells us that the potential energy of a continuous charge distribution is stored in an electric field.

The electrostatic energy density wE is defined as:

$$w_E = \frac{1}{2}\varepsilon E^2 \quad ; \quad W_E = \int_V w_E \, dv$$

ELECTROSTATICS-II

Properties of Materials and Steady Electric Current:

Electric field can not only exist in free space and vacuum but also in any material medium. When an electric field is applied to the material, the material will modify the electric field either by strengthening it or weakening it, depending on what kind of material it is.

Materials are classified into 3 groups based on conductivity / electrical property:

- Conductors (Metals like Copper, Aluminum, etc.) have high conductivity ($\sigma >> 1$).
- Insulators / Dielectric (Vacuum, Glass, Rubber, etc.) have low conductivity ($\sigma << 1$).
- Semiconductors (Silicon, Germanium, etc.) have intermediate conductivity.

Conductivity (σ) is a measure of the ability of the material to conduct electricity. It is the reciprocal of resistivity (ρ). Units of conductivity are Siemens/meter and mho.

The basic difference between a conductor and an insulator lies in the amount of free electrons available for conduction of current. Conductors have a large amount of free electrons where as insulators have only a few number ofelectrons for conduction of current. Most of the conductors obey ohm‘s law. Such conductors are also called ohmic conductors.

Due to the movement of free charges, several types of electric current can be caused. The different types of electric current are:

- Conduction Current.
- Convection Current.
- Displacement Current.

Electric current:

Electric current (I) defines the rate at which the net charge passes through a wire of cross sectional surface area S.

Mathematically,

If a net charge ΔQ moves across surface S in some small amount of time Δt, electric current(I) is defined as:

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

How fast or how speed the charges will move depends on the nature of the material medium.

Current density:

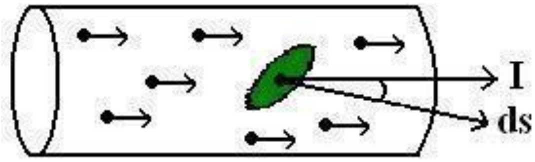
Current density (J) is defined as current ΔI flowing through surface ΔS.

Imagine surface area ΔS inside a conductor at right angles to the flow of current. As the area approaches zero, the current density at a point is defined as:

$$J = \lim_{\Delta s \to 0} \frac{\Delta I}{\Delta S}$$

The above equation is applicable only when current density (J) is normal to the surface.

In case if current density(J) is not perpendicular to the surface, consider a small area ds of the conductor at an angle θ to the flow of current as shown:



In this case current flowing through the area is given as:

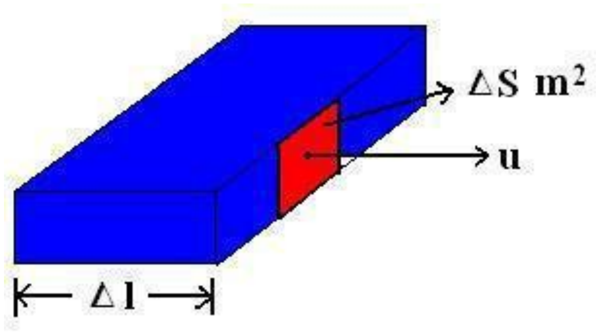
$dI = J\ dS \cos\theta = J\ .\ dS$ and $I = \int_S \bar{J}.\overline{ds}$

Where angle θ is the angle between the normal to the area and direction of the current.

From the above equation it‘s clear that electric current is a scalar quantity.

CONVECTION CURRENT DENSITY:

Convection current occurs in insulators or dielectrics such as liquid, vacuum and rarified gas. Convection current results from motion of electrons or ions in an insulating medium. Since convection current doesn‘t involve conductors, hence it does not satisfy ohm‘s law. Consider a filament where there is a flow of charge ρv at a velocity u = uy ay.



- Hence the current is given as:

$$\Delta I = \frac{\Delta Q}{\Delta t}$$

But we know $\boxed{\Delta Q = \rho_V \Delta V}$

Hence

$$\Delta I = \frac{\Delta Q}{\Delta t} = \frac{\rho_V \Delta V}{\Delta t} = \rho_V \Delta S \frac{\Delta l}{\Delta t}$$

$$= \rho_V \Delta S\, u_y$$

Again, we also know that $\boxed{J_y = \frac{\Delta I}{\Delta S}}$

Hence $J_y = \frac{\Delta I}{\Delta S} = \rho_V u_y$

Where uy is the velocity of the moving electron or ion and ρ_v is the free volume charge density.

- Hence the convection current density in general is given as:

$$J = \rho_v\, u$$

Conduction Current Density:

Conduction current occurs in conductors where there are a large number of free electrons. Conduction current occurs due to the drift motion of electrons (charge carriers). Conduction current obeys ohm‘s law.

When an external electric field is applied to a metallic conductor, conduction current occurs due to the drift of electrons.

The charge inside the conductor experiences a force due to the electric field and hence should accelerate but due to continuous collision with atomic lattice, their velocity is reduced. The net effect is that the electrons moves or drifts with an average velocity called the drift velocity (υd) which is proportional to the applied electric field (E).

Hence according to Newton‘s law, if an electron with a mass m is moving in an electric field E with anaverage drift velocity υd, the the average change in momentum of the free electron must be equal to the applied force ($F = - e E$).

$$\frac{m \upsilon_d}{\tau} = - eE$$

where τ is the average time interval between collision.

$$\upsilon_d = \left(- \frac{e \tau}{m} \right) E$$

The drift velocity per unit applied electric field is called the mobility of electrons (μe).

$\upsilon d = - \mu e\, E$

where μe is defined as:

$$\mu_e = \left(- \frac{e \tau}{m} \right)$$

Consider a conducting wire in which charges subjected to an electric field are moving with drift velocity υd.
Say there are Ne free electrons per cubic meter of conductor, then the free volume charge density(ρv)within the wire is

ρ_v= - e Ne

The charge ΔQ is given as:

$\Delta Q = \rho_v \, \Delta V = - e \, Ne \, \Delta S \, \Delta l = - e \, Ne \, \Delta S \, \upsilon d \, \Delta t$

- The incremental current is thus given as:

$$\Delta I = \frac{\Delta Q}{\Delta t} = - N_e \, e \, \Delta S \, \upsilon_d$$

Now since $\boxed{\upsilon_d = - \mu_e E}$

Therefore

$$\Delta I = N_e \, e \, \Delta S \, \mu_e E$$

The conduction current density is thus defined as:

$$J_c = \frac{\Delta I}{\Delta S} = N_e \, e \, \mu_e E = \sigma E$$

where σ is the conductivity of the material.

The above equation is known as the Ohm‘s law in point form and is valid at every point in space.

In a semiconductor, current flow is due to the movement of both electrons and holes, hence conductivity is given as:

$\sigma = (Ne \, \mu e + Nh \, \mu h)e$

DIELECTRC CONSTANT:

It is also known as Relative permittivity.

If two charges q 1 and q 2 are separated from each other by a small distance r. Then by using the coulombs law of forces the equation formed will be

$$\mathbf{F}_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\mathbf{r}^2}$$

In the above equation ε_0 is the electrical permittivity or you can say it, Dielectric constant.

If we repeat the above case with only one change i.e. only change in the separation medium between the charges. Here some material medium must be used. Then the equation formed will be.

$$\mathbf{F}_\mathrm{m} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\mathbf{r}^2}$$

Now after division of above two equations

$$\frac{\mathbf{F}_\mathrm{o}}{\mathbf{F}_\mathrm{m}} = \frac{\varepsilon}{\varepsilon_\mathrm{o}} = \varepsilon_\mathrm{r} \text{ Or k}$$

In the above figure

$\varepsilon_\mathbf{r}$ is the Relative Permittivity. Again one thing to notice is that the dielectric constant is represented by the symbol (K) but permittivity by the symbol $\varepsilon_\mathbf{r}$

CONTINUITY EQUATION:

The continuity equation is derived from two of Maxwell's equations. It states that the divergence of the current density is equal to the negative rate of change of the charge density,

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}.$$

Derivation

One of Maxwell's equations, Ampère's law, states that

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

Taking the divergence of both sides results in

$$\nabla \cdot \nabla \times \mathbf{H} = \nabla \cdot \mathbf{J} + \frac{\partial \nabla \cdot \mathbf{D}}{\partial t},$$

but the divergence of a curl is zero, so that

$$\nabla \cdot \mathbf{J} + \frac{\partial \nabla \cdot \mathbf{D}}{\partial t} = 0. \qquad (1)$$

Another one of Maxwell's equations, Gauss's law, states that

$$\nabla \cdot \mathbf{D} = \rho.$$

Substitute this into equation (1) to obtain

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0,$$

which is the continuity equation.

1.13 RELAXATION TIME:

- Let us consider that a charge is introduced at some interior point of a given material (conductor or dielectric).
- From, continuity of current equation, we have

$$\overline{J} = \frac{-rf_v}{rt} \textbf{----}(1)$$

- We have, the point form of Ohm's law as,

$$\overline{J} = 6\overline{E} \textbf{---}(2)$$

- From Gauss's law, we have,

$$\nabla\overline{D} = f_v \Rightarrow \in \nabla.\overline{E} = f_v \left[\because \overline{D} = \in .\overline{E}\right]$$

$$\therefore \nabla.\overline{E} = \frac{f_v}{\in} ------(1)$$

- Substitute equations (2) and (3) in equation (1), we get

$$\nabla.6\overline{E}f = 6.\nabla.\overline{E} = 6.\frac{f_v}{\in} = \frac{-\partial f_v}{\partial t}$$

$$\Rightarrow \frac{\partial f_v}{\partial t} + \frac{6}{\in}.f_v = 0 ------(4)$$

- The above equation is a homogeneous linear ordinary differential equation. By separating variable in eq (4), we get,

$$\frac{\partial f_v}{\partial t} = \frac{-6}{\in}.f_v$$

$$\Rightarrow \frac{\partial f_v}{\partial t} = \frac{-6}{\in}.\partial t$$

- Now integrate on both sides of above equation

$$\int \frac{\partial f_v}{\partial t} = -\frac{6}{\in}.\int \partial t$$

$$\Rightarrow \ln f_v = -\frac{6}{\in}.t + \ln f_{v0}$$

Where ln p_{v0} is a constant of integration.

Thus,

$$\boxed{f_v = f_{v0} e^{-t/Tr}} \quad -----(5)$$

$$\boxed{T_r = \frac{\in}{6}}$$

- In eq (5), f_{v0} is the initial charge density (i.e fv at t=0).
- We can see from the equation that as a result of introducing charge at some interior point of the material there is a decay of volume charge density f_v.
- The time constant "T_r" is known as the relaxation time or rearrangement time.
- Relaxation time is the time it takes a charge placed in the interior of a material to drop to e^{-1} = 36.8 percent f its initial value.
- The relation time is short for good conductors and long for good dielectrics.

LAPLACE'S AND POISSON'S EQUATIONS:

A useful approach to the calculation of electric potentials is to relate that potential to the charge density which gives rise to it. The electric field is related to the charge density by the divergence relationship

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

E = electric field
ρ = charge density
ε_0 = permittivity

and the electric field is related to the electric potential by a gradient relationship

$$E = -\nabla V$$

Therefore the potential is related to the charge density by Poisson's equation

$$\nabla \cdot \nabla V = \nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

In a charge-free region of space, this becomes LaPlace's equation

$$\nabla^2 V = 0$$

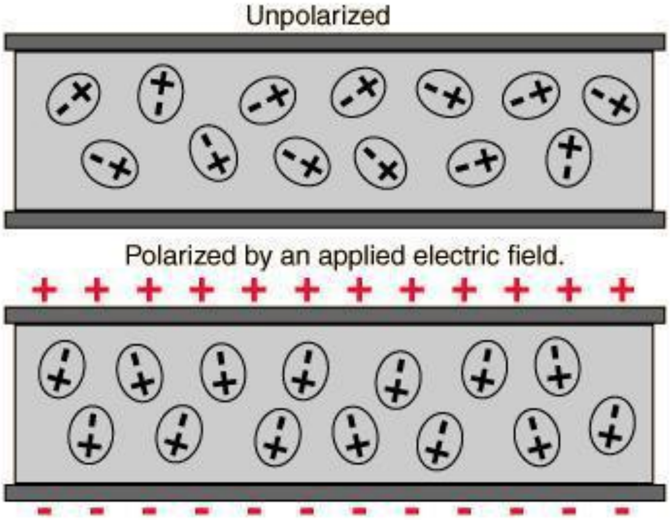
This mathematical operation, the divergence of the gradient of a function, is called the LaPlacian. Expressing the LaPlacian in different coordinate systems to take advantage of the symmetry of a charge distribution helps in the solution for the electric potential V. For example, if the charge distribution has spherical symmetry, you use the LaPlacian in spherical polar coordinates.

Since the potential is a scalar function, this approach has advantages over trying to calculate the electric field directly. Once the potential has been calculated, the electric field can be computed by taking the gradient of the potential.

Polarization of Dielectric:

If a material contains polar molecules, they will generally be in random orientations when no electric field is applied. An applied electric field will polarize the material by orienting the dipole moments of polar molecules.

This decreases the effective electric field between the plates and will increase the capacitance of the parallel plate structure. The dielectric must be a good electric insulator so as to minimize any DC leakage current through a capacitor.



The presence of the dielectric decreases the electric field produced by a given charge density.

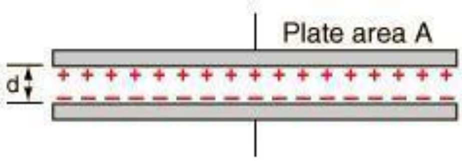
$$E_{effective} = E - E_{polarization} = \frac{\sigma}{k\varepsilon_0}$$

The factor k by which the effective field is decreased by the polarization of the dielectric is called the dielectric constant of the material.

Capacitance:

The capacitance of a set of charged parallel plates is increased by the insertion of adielectric material. The capacitance is inversely proportional to the electric field between the plates, and the presence of the dielectric reduces the effective electric field. The dielectric is characterized by a dielectric constant k, and the capacitance is multiplied by that factor.

Parallel Plate Capacitor



$$C = \frac{\varepsilon A}{d} = \frac{k\varepsilon_0 A}{d}$$

Show

The capacitance of flat, parallel metallic plates of area A and separation d is given by the expression above where:

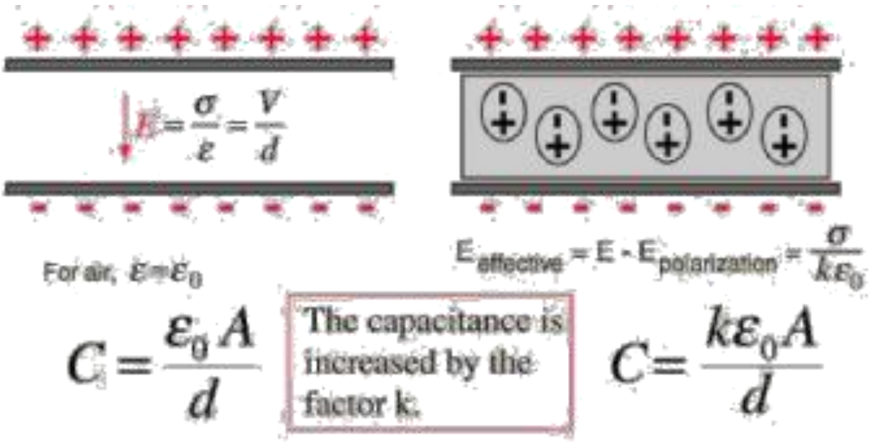
$$\varepsilon_0 = 8.854 \ x \ 10^{-12} \ F/m$$

= permittivity of space and

k = relative permittivity of the dielectric material between the plates.

k=1 for free space, k>1 for all media, approximately =1 for air.

The Farad, F, is the SI unit for capacitance and from the definition of capacitance is seen to be equal to a Coulomb/Volt.



Series and parallel Connection of capacitors

Capacitors are connected in various manners in electrical circuits; series and parallel connections are the two basic ways of connecting capacitors. We compute the equivalent capacitance for such connections.

Series Case: Series connection of two capacitors is shown in the figure 1. For this case we can write,

$$V=V_1+V_2=\frac{Q}{C_1}+\frac{Q}{C_2}$$

$$\frac{V}{Q}=\frac{1}{C_{eqs}}=\frac{1}{C_1}+\frac{1}{C_2}$$ ....(1)

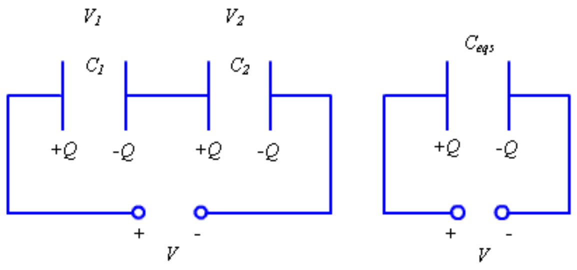


Fig 1.: Series Connection of Capacitors

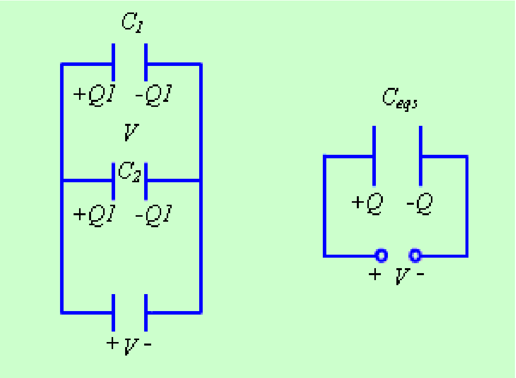


Fig 2: Parallel Connection of Capacitors

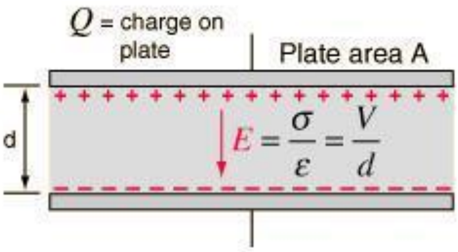
The same approach may be extended to more than two capacitors connected in series.

Parallel Case: For the parallel case, the voltages across the capacitors are the same.

The total charge $Q = Q_1 + Q_2 = C_1V + C_2V$

Therefore, $$C_{eqp} = \frac{Q}{V} = C_1 + C_2$$

Capacitance of Parallel Plates:



The electric field between two large parallel plates is given by

$$E = \frac{\sigma}{\varepsilon} \quad \text{where} \quad \begin{array}{l} \sigma = \text{charge density} \\ \varepsilon = \text{permittivity} \end{array}$$

$$\text{and} \quad \sigma = \frac{Q}{A}$$

The voltage difference between the two plates can be expressed in terms of the work done on a positive test charge q when it moves from the positive to the negative plate.

$$V = \frac{work\ \ done}{charge} = \frac{Fd}{q} = Ed$$

It then follows from the definition of capacitance that

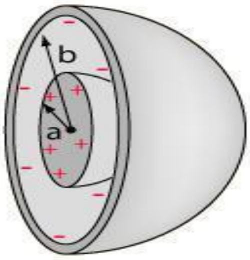
$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{Q\varepsilon}{\sigma d} = \frac{QA\varepsilon}{Qd} = \frac{A\varepsilon}{d}$$

Spherical Capacitor:

The capacitance for spherical or cylindrical conductors can be obtained by evaluating the voltage difference between the conductors for a given charge on each.

By applying Gauss' law to an charged conducting sphere, the electric field outside it is **found to be**

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$



The voltage between the spheres can be found by integrating the electric field along a radial line:

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

From the definition of capacitance, the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$

Isolated Sphere Capacitor:

An isolated charged conducting sphere has capacitance. Applications for such a capacitor may not be immediately evident, but it does illustrate that a charged sphere has stored some energy as a result of being charged. Taking the concentric sphere capacitance expression:

$$C = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$

$$C = 4\pi\varepsilon_0 R$$

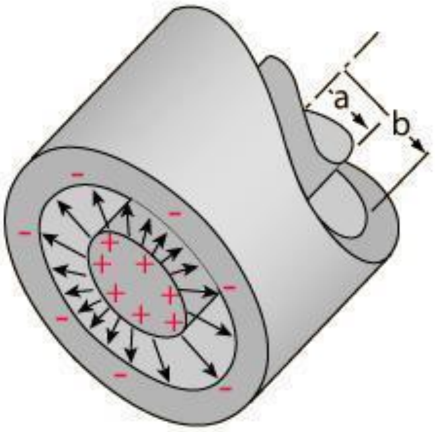
and taking the limits $a \rightarrow R$ and $b \rightarrow \infty$ gives

Further confirmation of this comes from examining the potential of a charged conducting sphere:

$$V = \frac{Q}{4\pi\varepsilon_0 r} \quad \text{so at the surface} \quad C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$

Cylindrical Capacitor:

For a cylindrical geometry like a coaxial cable, the capacitance is usually stated as a capacitance per unit length. The charge resides on the outer surface of the inner conductor and the inner wall of the outer conductor. The capacitance expression is



$$\frac{C}{L} = \frac{2\pi k\varepsilon_0}{\ln\left[\frac{b}{a}\right]}$$

The capacitance for cylindrical orspherical conductors can be obtained by evaluating the voltage difference between the conductors for a given charge on each. By applying Gauss' law to an infinite cylinder in a vacuum, the electric field outside a charged cylinder is found to be

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

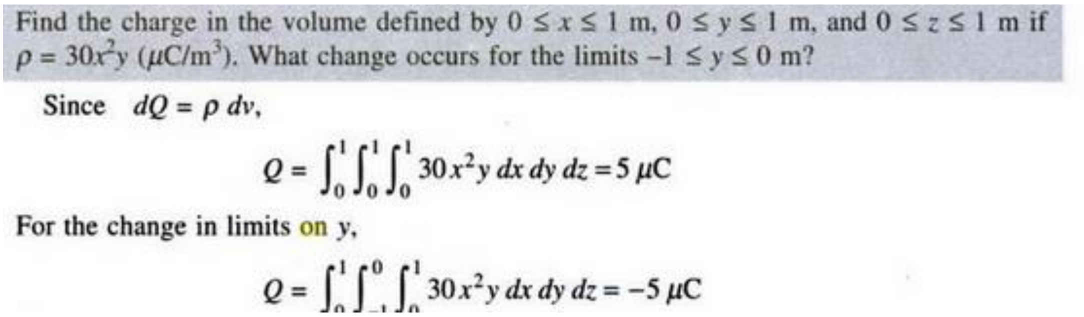
The voltage between the cylinders can be found by integrating the electric field along a radial line:

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{1}{r}dr = \frac{\lambda}{2\pi\varepsilon_0}\ln\left[\frac{b}{a}\right] \qquad \frac{C}{L} = \frac{\lambda}{\Delta V} = \frac{2\pi k\varepsilon_0}{\ln\left[\frac{b}{a}\right]}$$

From the definition of capacitance and including the case where the volume is filled by a dielectric of dielectric constant k, the capacitance per unit length is defined above.

Solved problems:

Problem1:



Find the charge in the volume defined by $0 \le x \le 1$ m, $0 \le y \le 1$ m, and $0 \le z \le 1$ m if $\rho = 30x^2y$ (μC/m^3). What change occurs for the limits $-1 \le y \le 0$ m?

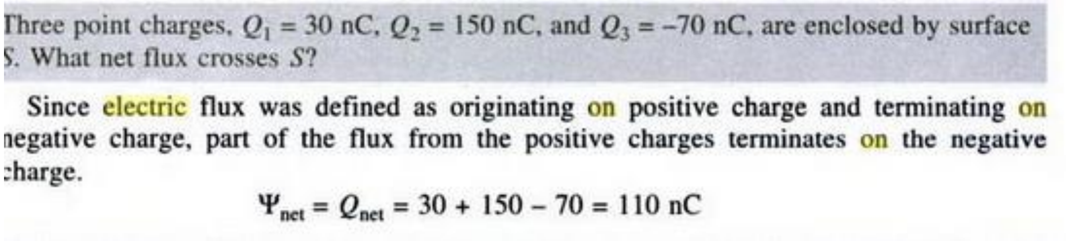
Since $dQ = \rho\, dv$,

$$Q = \int_0^1\int_0^1\int_0^1 30x^2y\,dx\,dy\,dz = 5\ \mu\text{C}$$

For the change in limits on y,

$$Q = \int_0^1\int_{-1}^0\int_0^1 30x^2y\,dx\,dy\,dz = -5\ \mu\text{C}$$

Problem-2

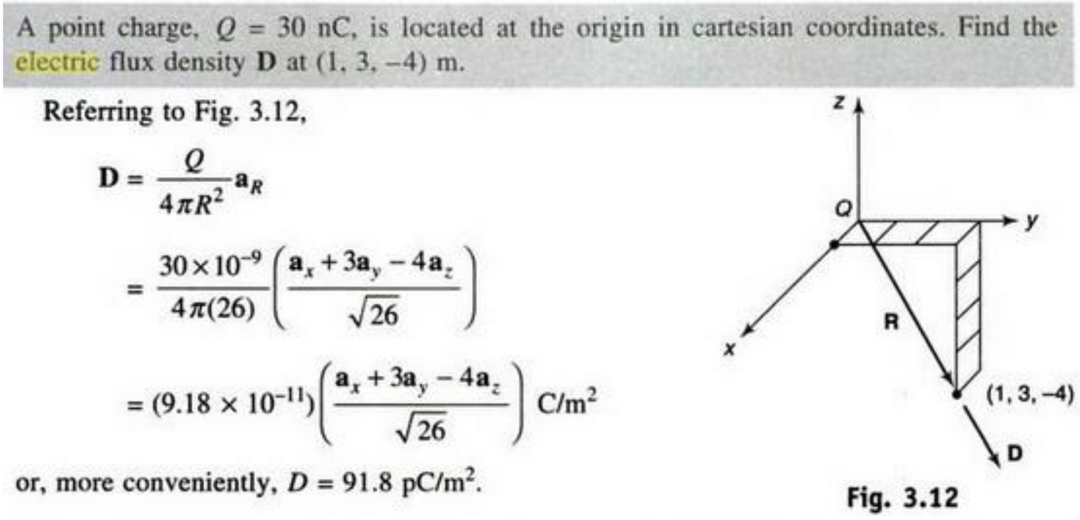


Three point charges, $Q_1 = 30$ nC, $Q_2 = 150$ nC, and $Q_3 = -70$ nC, are enclosed by surface S. What net flux crosses S?

Since electric flux was defined as originating on positive charge and terminating on negative charge, part of the flux from the positive charges terminates on the negative charge.

$$\Psi_{\text{net}} = Q_{\text{net}} = 30 + 150 - 70 = 110\ \text{nC}$$

Problem-3



A point charge, $Q = 30$ nC, is located at the origin in cartesian coordinates. Find the electric flux density **D** at (1, 3, −4) m.

Referring to Fig. 3.12,

$$\mathbf{D} = \frac{Q}{4\pi R^2}\mathbf{a}_R$$

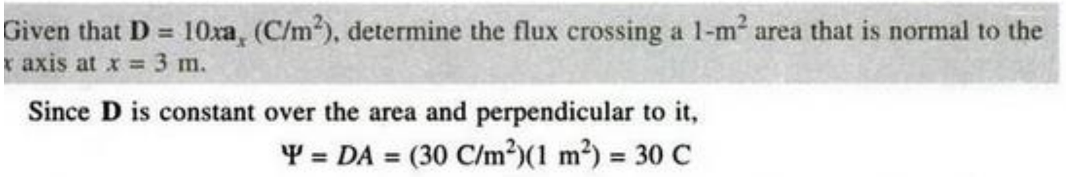
$$= \frac{30\times10^{-9}}{4\pi(26)}\left(\frac{\mathbf{a}_x + 3\mathbf{a}_y - 4\mathbf{a}_z}{\sqrt{26}}\right)$$

$$= (9.18\times10^{-11})\left(\frac{\mathbf{a}_x + 3\mathbf{a}_y - 4\mathbf{a}_z}{\sqrt{26}}\right)\ \text{C/m}^2$$

or, more conveniently, $D = 91.8$ pC/m^2.

Fig. 3.12

Problem-4

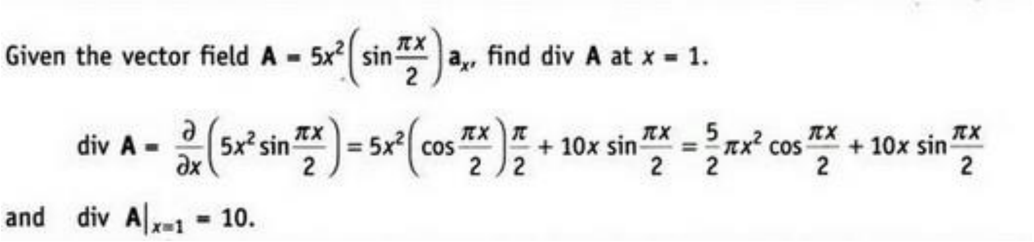


Given that $\mathbf{D} = 10x\mathbf{a}_x$ (C/m^2), determine the flux crossing a 1-m^2 area that is normal to the x axis at $x = 3$ m.

Since **D** is constant over the area and perpendicular to it,

$$\Psi = DA = (30\ \text{C/m}^2)(1\ \text{m}^2) = 30\ \text{C}$$

Problem-5

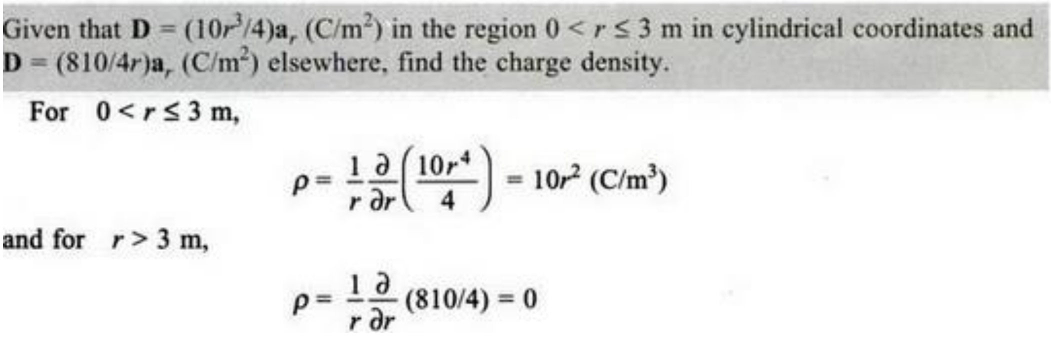


Given the vector field $\mathbf{A} = 5x^2\left(\sin\frac{\pi x}{2}\right)\mathbf{a}_x$, find div **A** at $x = 1$.

$$\text{div } \mathbf{A} = \frac{\partial}{\partial x}\left(5x^2\sin\frac{\pi x}{2}\right) = 5x^2\left(\cos\frac{\pi x}{2}\right)\frac{\pi}{2} + 10x\sin\frac{\pi x}{2} = \frac{5}{2}\pi x^2\cos\frac{\pi x}{2} + 10x\sin\frac{\pi x}{2}$$

and $\quad \text{div } \mathbf{A}\big|_{x=1} = 10.$

Problem-6



Given that $\mathbf{D} = (10r^3/4)\mathbf{a}_r$ (C/m^2) in the region $0 < r \le 3$ m in cylindrical coordinates and $\mathbf{D} = (810/4r)\mathbf{a}_r$ (C/m^2) elsewhere, find the charge density.

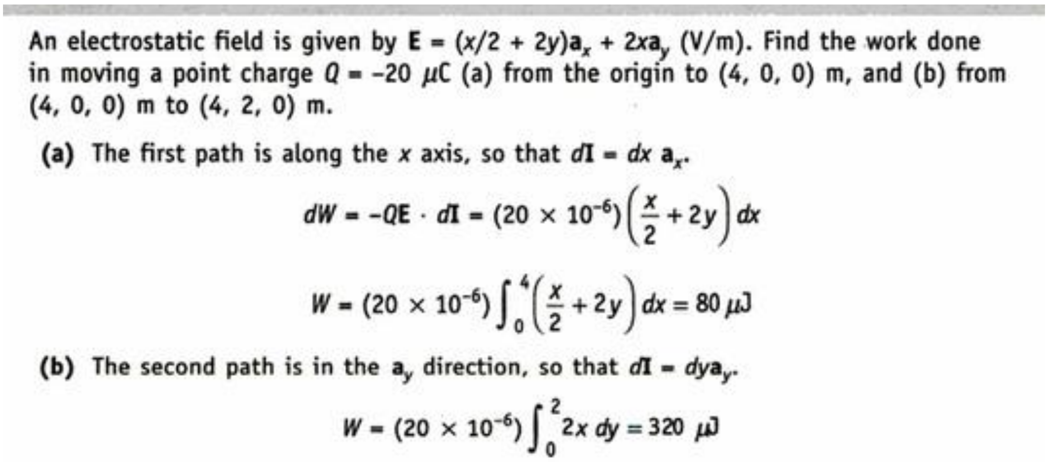
For $\quad 0 < r \le 3$ m,

$$\rho = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{10r^4}{4}\right) = 10r^2 \text{ (C/m}^3\text{)}$$

and for $\quad r > 3$ m,

$$\rho = \frac{1}{r}\frac{\partial}{\partial r}(810/4) = 0$$

Problem-7



An electrostatic field is given by $\mathbf{E} = (x/2 + 2y)\mathbf{a}_x + 2x\mathbf{a}_y$ (V/m). Find the work done in moving a point charge $Q = -20\ \mu$C (a) from the origin to (4, 0, 0) m, and (b) from (4, 0, 0) m to (4, 2, 0) m.

(a) The first path is along the x axis, so that $d\mathbf{l} = dx\,\mathbf{a}_x$.

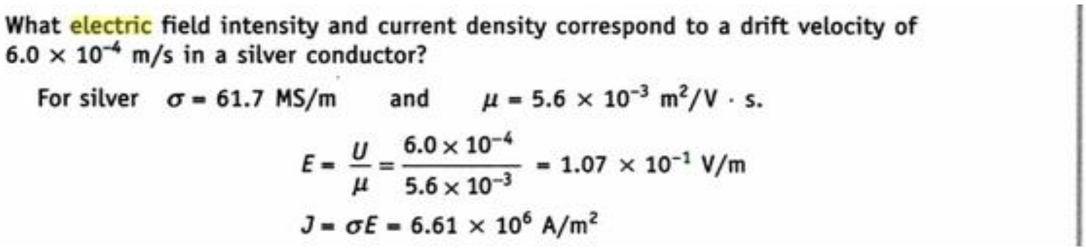
$$dW = -Q\mathbf{E}\cdot d\mathbf{l} = (20 \times 10^{-6})\left(\frac{x}{2} + 2y\right)dx$$

$$W = (20 \times 10^{-6})\int_0^4\left(\frac{x}{2} + 2y\right)dx = 80\ \mu\text{J}$$

(b) The second path is in the $\mathbf{a}_y$ direction, so that $d\mathbf{l} = dy\mathbf{a}_y$.

$$W = (20 \times 10^{-6})\int_0^2 2x\,dy = 320\ \mu\text{J}$$

Problem-8



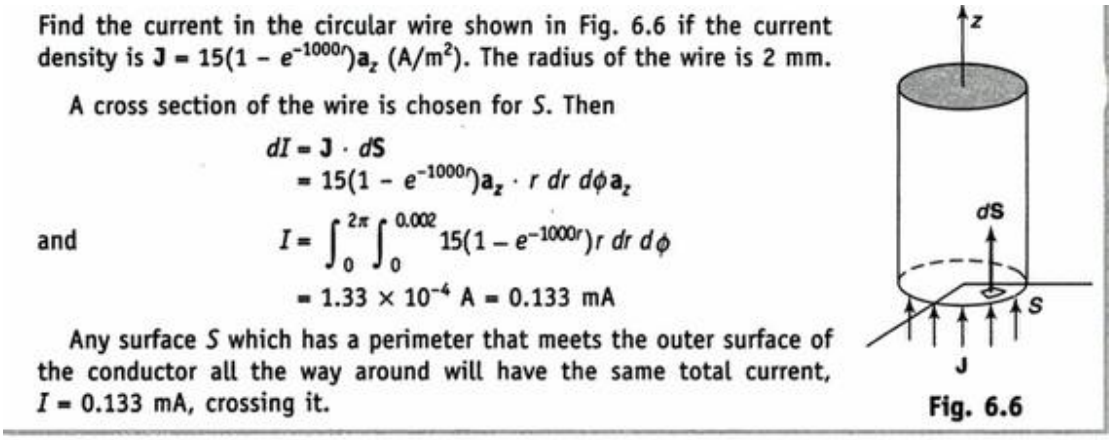
What electric field intensity and current density correspond to a drift velocity of 6.0×10^{-4} m/s in a silver conductor?

For silver $\quad \sigma = 61.7$ MS/m $\quad$ and $\quad \mu = 5.6 \times 10^{-3}$ m^2/V · s.

$$E = \frac{U}{\mu} = \frac{6.0 \times 10^{-4}}{5.6 \times 10^{-3}} = 1.07 \times 10^{-1} \text{ V/m}$$

$$J = \sigma E = 6.61 \times 10^6 \text{ A/m}^2$$

Problem-9



Find the current in the circular wire shown in Fig. 6.6 if the current density is $\mathbf{J} = 15(1 - e^{-1000r})\mathbf{a}_z$ (A/m^2). The radius of the wire is 2 mm.

A cross section of the wire is chosen for S. Then

$$dI = \mathbf{J} \cdot d\mathbf{S}$$
$$= 15(1 - e^{-1000r})\mathbf{a}_z \cdot r\,dr\,d\phi\,\mathbf{a}_z$$

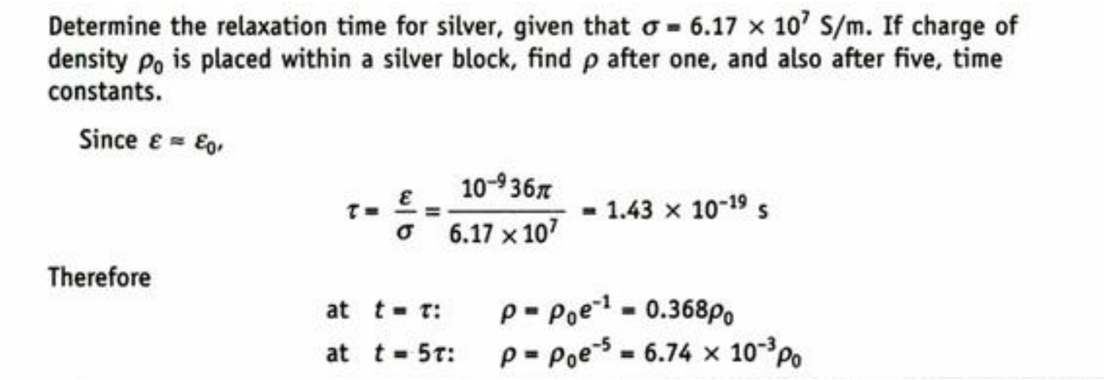
and

$$I = \int_0^{2\pi}\int_0^{0.002} 15(1 - e^{-1000r})r\,dr\,d\phi$$
$$= 1.33 \times 10^{-4}\ \text{A} = 0.133\ \text{mA}$$

Any surface S which has a perimeter that meets the outer surface of the conductor all the way around will have the same total current, $I = 0.133$ mA, crossing it.

Fig. 6.6

Problem-10



Determine the relaxation time for silver, given that $\sigma = 6.17 \times 10^7$ S/m. If charge of density ρ_0 is placed within a silver block, find ρ after one, and also after five, time constants.

Since $\varepsilon \approx \varepsilon_0$,

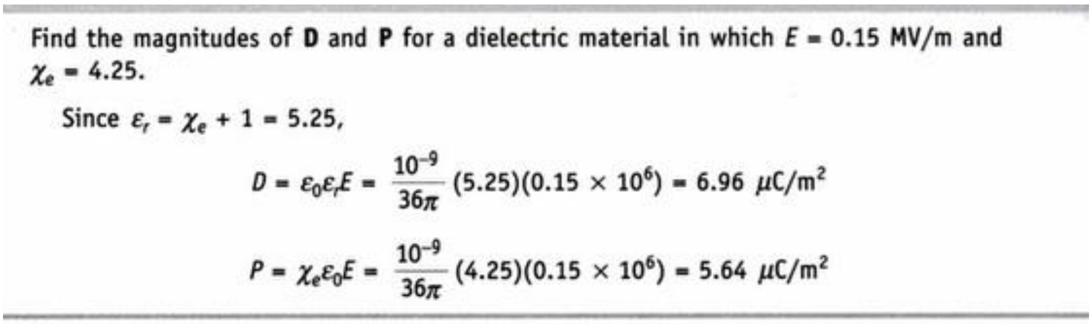
$$\tau = \frac{\varepsilon}{\sigma} = \frac{10^{-9}36\pi}{6.17 \times 10^7} = 1.43 \times 10^{-19}\ \text{s}$$

Therefore

at $t = \tau$: $\rho = \rho_0 e^{-1} = 0.368\rho_0$

at $t = 5\tau$: $\rho = \rho_0 e^{-5} = 6.74 \times 10^{-3}\rho_0$

Problem-11



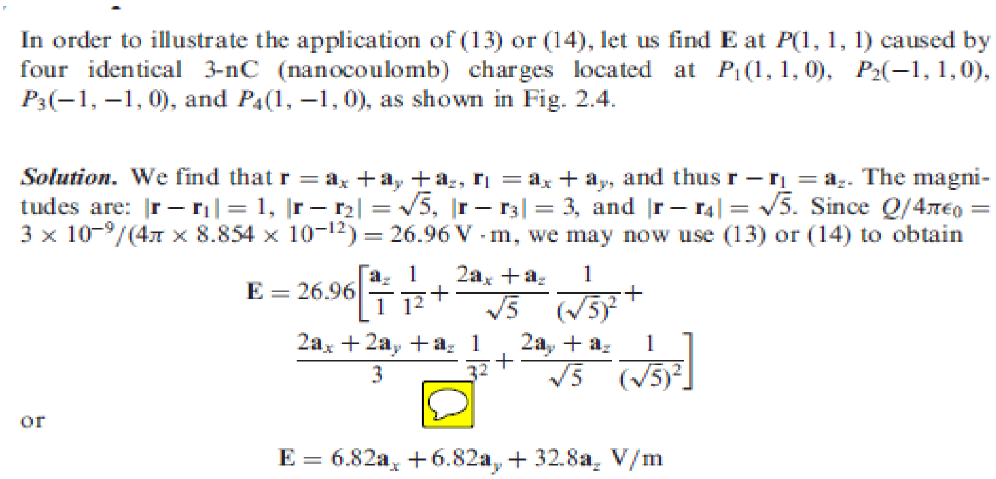
Find the magnitudes of **D** and **P** for a dielectric material in which $E = 0.15$ MV/m and $\chi_e = 4.25$.

Since $\varepsilon_r = \chi_e + 1 = 5.25$,

$$D = \varepsilon_0\varepsilon_r E = \frac{10^{-9}}{36\pi}(5.25)(0.15 \times 10^6) = 6.96\ \mu\text{C/m}^2$$

$$P = \chi_e\varepsilon_0 E = \frac{10^{-9}}{36\pi}(4.25)(0.15 \times 10^6) = 5.64\ \mu\text{C/m}^2$$

Problem-12



In order to illustrate the application of (13) or (14), let us find **E** at $P(1,1,1)$ caused by four identical 3-nC (nanocoulomb) charges located at $P_1(1,1,0)$, $P_2(-1,1,0)$, $P_3(-1,-1,0)$, and $P_4(1,-1,0)$, as shown in Fig. 2.4.

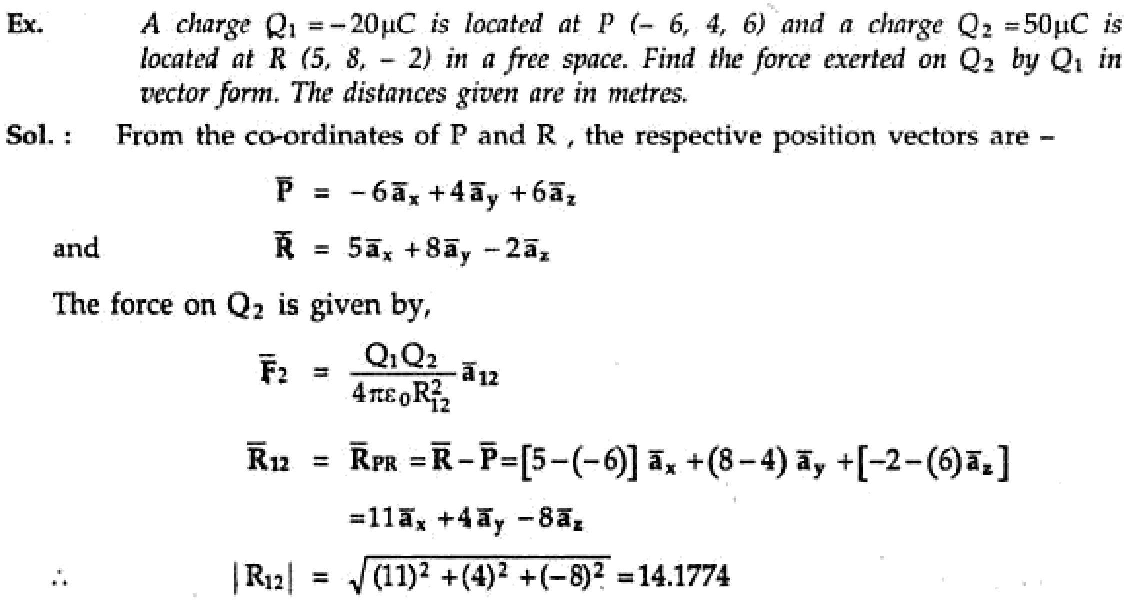
Solution. We find that $\mathbf{r} = \mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$, $\mathbf{r}_1 = \mathbf{a}_x + \mathbf{a}_y$, and thus $\mathbf{r} - \mathbf{r}_1 = \mathbf{a}_z$. The magnitudes are: $|\mathbf{r} - \mathbf{r}_1| = 1$, $|\mathbf{r} - \mathbf{r}_2| = \sqrt{5}$, $|\mathbf{r} - \mathbf{r}_3| = 3$, and $|\mathbf{r} - \mathbf{r}_4| = \sqrt{5}$. Since $Q/4\pi\epsilon_0 = 3 \times 10^{-9}/(4\pi \times 8.854 \times 10^{-12}) = 26.96\ \mathrm{V \cdot m}$, we may now use (13) or (14) to obtain

$$\mathbf{E} = 26.96\left[\frac{\mathbf{a}_z}{1}\,\frac{1}{1^2} + \frac{2\mathbf{a}_x + \mathbf{a}_z}{\sqrt{5}}\,\frac{1}{(\sqrt{5})^2} + \frac{2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{3}\,\frac{1}{3^2} + \frac{2\mathbf{a}_y + \mathbf{a}_z}{\sqrt{5}}\,\frac{1}{(\sqrt{5})^2}\right]$$

or

$$\mathbf{E} = 6.82\mathbf{a}_x + 6.82\mathbf{a}_y + 32.8\mathbf{a}_z\ \mathrm{V/m}$$

Problem-13



Ex. *A charge $Q_1 = -20\mu C$ is located at $P(-6, 4, 6)$ and a charge $Q_2 = 50\mu C$ is located at $R(5, 8, -2)$ in a free space. Find the force exerted on Q_2 by Q_1 in vector form. The distances given are in metres.*

Sol. : From the co-ordinates of P and R , the respective position vectors are –

$$\overline{P} = -6\bar{a}_x + 4\bar{a}_y + 6\bar{a}_z$$

and

$$\overline{R} = 5\bar{a}_x + 8\bar{a}_y - 2\bar{a}_z$$

The force on Q_2 is given by,

$$\overline{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\,\bar{a}_{12}$$

$$\overline{R}_{12} = \overline{R}_{PR} = \overline{R} - \overline{P} = [5-(-6)]\,\bar{a}_x + (8-4)\,\bar{a}_y + [-2-(6)\bar{a}_z]$$

$$= 11\bar{a}_x + 4\bar{a}_y - 8\bar{a}_z$$

$$\therefore \quad |R_{12}| = \sqrt{(11)^2 + (4)^2 + (-8)^2} = 14.1774$$

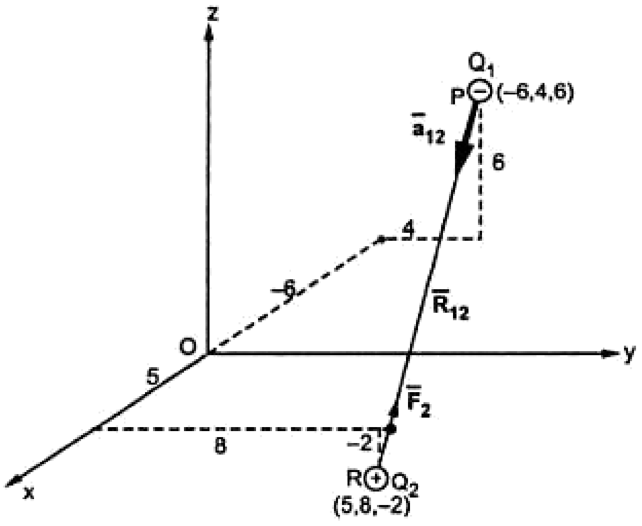


Fig. 2.5

$$\therefore \qquad \bar{a}_{12} = \frac{\bar{R}_{12}}{\left|\bar{R}_{12}\right|} = \frac{11\,\bar{a}_x + 4\,\bar{a}_y - 8\,\bar{a}_z}{14.1774}$$

$$\therefore \qquad \bar{a}_{12} = 0.7758\ \bar{a}_x + 0.2821\ \bar{a}_y - 0.5642\ \bar{a}_z$$

$$\therefore \qquad \bar{F}_2 = \frac{-20\times 10^{-6}\times 50\times 10^{-6}}{4\pi\times 8.854\times 10^{-12}\times (14.1774)^2}\left[\bar{a}_{12}\right]$$

$$= -0.0447\left[0.7758\ \bar{a}_x + 0.2821\ \bar{a}_y - 0.5642\ \bar{a}_z\right] \qquad \text{... (A)}$$

$$= -0.0346\ \bar{a}_x - 0.01261\ \bar{a}_y + 0.02522\ \bar{a}_z\ \text{N} \qquad \text{... (B)}$$

This is the required force exerted on Q_2 by Q_1.

UNIT-2.

4/2/17

Magneto Statistics :-

Differences between electroStatics & Magnetostatics.

Electrostatics	Magnetostatics.
1) charge is at rest	1) charge is moving with Const Velocity ($\frac{dv}{dt} = 0$)
2) $\bar{E}$ v/m or N/c $\bar{D}$ c/m^2 ($\bar{D} = \frac{d\psi}{ds}$)	2) $\bar{H}$ A/m $\bar{B}$ wb/m^2 ($\bar{B} = \frac{d\phi}{ds}$)
3) $\bar{D} = \epsilon \bar{E}$	3) $\bar{B} = \mu \bar{H}$
4) Coloumb's law Guass law (Guassian surface)	4). Biot-Savart's law (Ampere's ckt law (Amperian path)
5) Q, ρ_l, ρ_s, ρ_v	5) I dl/ k ds ▱, J dv
6) Electric dipole moment	6) Magnetic dipole
7) $\omega_E = \frac{1}{2} \bar{D} \bar{E}$ J/m^3	7) $\omega_E = \frac{1}{2} \bar{H} \bar{B}$ J/m^3
8) $W_E = \frac{1}{2} CV^2$ J	8) $W_E = \frac{1}{2} L I^2$ J
9) $\nabla \cdot \bar{D} = \rho_v$ $\nabla \times \bar{E} = 0$	9) $\nabla \times \bar{H} = \bar{J}$ $\nabla \cdot \bar{B} = 0.$

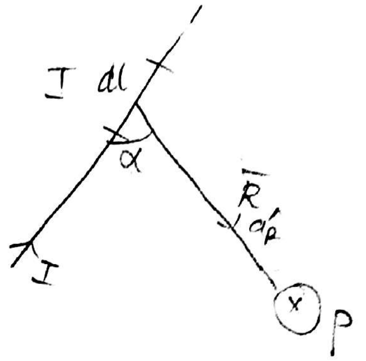
Biot-Savart's law:-

. An esf is produced by stationary charges.
. If charges are moving with constant velocity then ~~magneto static field~~ / static magnetic field is produced, 2 major laws governing magneto static field
1. Biot Savart's Law 2. Ampere's circuit Law.

Biot-Savart's law States that the magnetic field in insteusity $\bar{H}$ dH at a point p by the diffeiential current element $I\bar{dl}$ is
i) proportional to product of $I\bar{dl}$ & sine of angle 'α' between the elements and line joining the point p to

the element. and

ii) Inversely propotional to the square of the distance between the element and point 'p'.



$\uparrow$ I ; $\bar{H}$ is into the paper So 'x'

$\downarrow$ I, $\bar{H}$ is coming out of the paper (.)

$$d\bar{H} \propto \frac{I\, d\bar{l} \sin\alpha}{|R^2|}$$

$$d\bar{H} = K \frac{I\, d\bar{l} \sin\alpha}{|R|^2}$$

(SI units)

$$K = \frac{1}{4\pi}, \quad d\bar{H} = K \frac{I\, d\bar{l} \sin\alpha}{|R|^2}$$

$$d\bar{H} = \frac{1}{4\pi} \frac{I\, d\bar{l} \sin\alpha}{|R|^2}.$$

$$\bar{A} \times \bar{B} = |\bar{A}||\bar{B}| \sin\alpha$$

$$\sin\alpha = \frac{\bar{A} \times \bar{B}}{|\bar{A}|\,|\bar{B}|}$$

$$\bar{A} = d\bar{l}, \quad |\bar{A}| = dl$$

$$\bar{B} = \hat{a}_R, \quad |\bar{B}| = 1.$$

$$\sin\alpha = \frac{d\bar{l} \times \hat{a}_R}{dl}$$

$$dl \sin\alpha = d\bar{l} \times \hat{a}_R.$$

$$d\bar{H} = \frac{1}{4\pi} \frac{I\, d\bar{l} \times \hat{a}_R}{|R|^2}$$

$$\hat{a}_R = \frac{\bar{R}}{|R|}$$

$$\boxed{d\bar{H} = \frac{1}{4\pi} \frac{I\, d\bar{l} \times \bar{R}}{|R|^3} \quad A/m}$$

$$\bar{H} = \frac{1}{4\pi}\int_{l} \frac{I\,\bar{dl} \times \bar{R}}{|R|^3}$$

Current distribution elements:

i) Current line element:

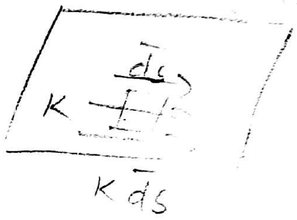
$$\bar{H} = \frac{1}{4\pi}\int_{l} \frac{I\,\bar{dl} \times \bar{R}}{|R|^3}$$

$$I = \rho_l\,\bar{u}$$

$$= \frac{c}{m}\cdot\frac{m}{s} = \frac{c}{s} = A$$

$$I\,dl = A\text{-}m$$

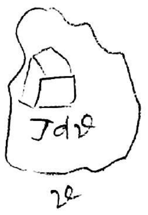
ii) Surface charge element:



$$\bar{H} = \frac{1}{4\pi}\int_{s} \frac{K\,\bar{ds} \times \bar{R}}{|R|^3} \qquad K\,\bar{ds} = A-m$$

iii) Volume element:

Consider the Volume within the Volume & consider a Small Cube



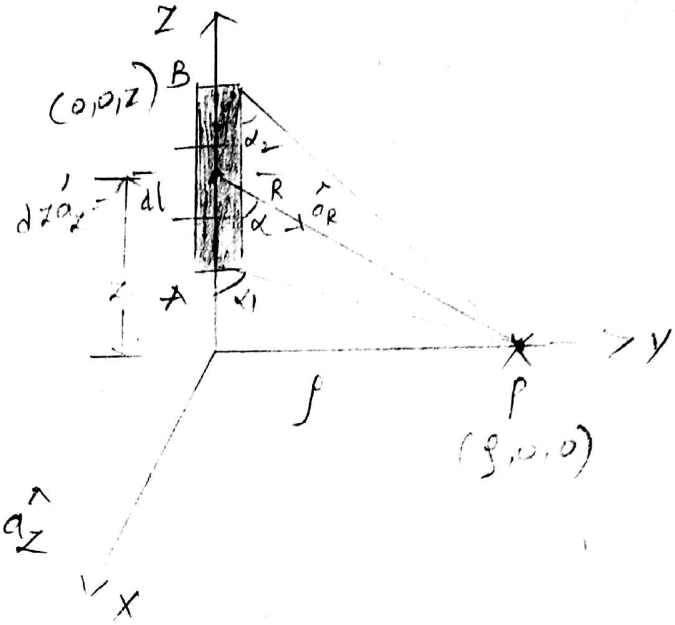
$$\bar{H} = \frac{1}{4\pi}\int_{v} \frac{\bar{J}\,dv \times \bar{R}}{|R|^3} \qquad \bar{J} = \rho_v\,\bar{u} = \frac{c}{m^3}\cdot\frac{m}{s} =$$

$$\bar{J}\cdot dv = \frac{A}{m^3}\cdot m^3 = A-m \qquad \bar{J} = A/m^2.$$

$$\therefore \boxed{I\,\bar{dl} = K\,\bar{ds} = \bar{J}\,dv.}$$

Magnetic field due to straight current carrying conductor of finite length:

Assume that conductor is placed along z-axis with its upper end & lower end making an angles of α_1 & α_2 respective from Biot Savarats law



$$\bar{H} = \frac{1}{4\pi}\int_l \frac{I\,\bar{dl} \times \bar{R}}{|R|^3}$$

$\bar{R} =$ f.pt − Src point

$= (\rho,0,0) - (0,0,z)$

$\bar{R} = (\rho, 0, -z)$; $\bar{R} = \rho\hat{a}_\rho - z\hat{a}_z$

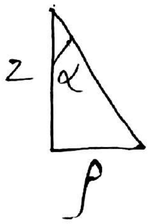
$|\bar{R}| = \sqrt{\rho^2 + z^2}$

$$\bar{dl} \times \bar{R} = \begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ 0 & 0 & dz \\ \rho & 0 & -z \end{vmatrix} = \hat{a}_\rho(0) - \hat{a}_\phi(-\rho\,dz) + \hat{a}_z(0)$$

$$= \rho\,dz\,\hat{a}_\phi .$$

$\therefore \bar{dl} \times \bar{R} = \rho\,dz\,\hat{a}_\phi .$

$$\bar{H} = \frac{1}{4\pi}\int_l \frac{I(\rho\,dz\,\hat{a}_\phi)}{(\rho^2+z^2)^{3/2}} .$$



$\cot\alpha = \frac{z}{\rho} = z = \rho\cot\alpha$

diff on bS

$dz = \rho[-\mathrm{cosec}^2\alpha\,d\alpha]$

$$= \frac{I}{4\pi}\int_l \frac{\rho(-\rho\,\mathrm{cosec}^2\alpha\,d\alpha)\,\hat{a}_\phi}{(\rho^2 + \rho^2\cot^2\alpha)^{3/2}}$$

$$= -\frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2\,\mathrm{cosec}^2\alpha\,d\alpha\,\hat{a}_\phi}{(\rho^2\,\mathrm{cosec}^2\alpha)^{3/2}}$$

$$= \frac{-I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2\,\mathrm{cosec}^2\alpha\,d\alpha\,\hat{a}_\phi}{\rho^3\,\mathrm{cosec}^3\alpha}$$

$$= \frac{-I}{4\pi\rho}\int^{\alpha_2} \frac{d\alpha\,\hat{a}_\phi}{}$$

$$\bar{H} = \frac{-I}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} \sin\alpha \, d\alpha \, \hat{a}_\phi$$

$$\bar{H} = \frac{-I}{4\pi\rho}\left(-\cos\alpha\right)_{\alpha_1}^{\alpha_2} \hat{a}_\phi$$

$$\boxed{\bar{H} = \frac{+I}{4\pi\rho}\left(\cos\alpha_2 - \cos\alpha_1\right) \hat{a}_\phi} \; .. A/m$$

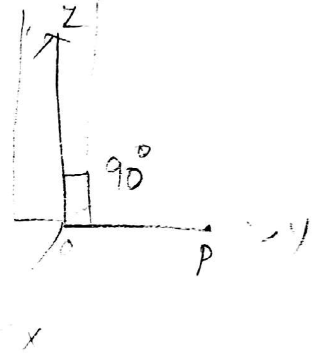
Case-(i):-

If the Conductor is of Semiinfinite length then $A \rightarrow 0, \quad B \rightarrow \infty$

$\alpha_1 = 90^\circ, \quad \alpha_2 = 0^\circ$

$$\bar{H} = \frac{I}{4\pi\rho}\left[1-0\right] \hat{a}_\phi$$

$$\boxed{\bar{H} = \frac{I}{4\pi\rho} \hat{a}_\phi} \quad A/m$$



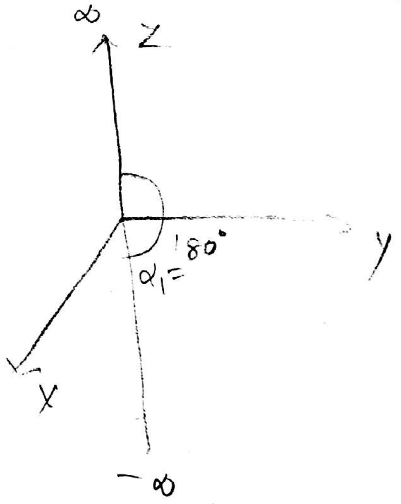
Case-ii). If the length of the conductor is of infinite length

i.e $A \rightarrow -\infty, \; B \rightarrow \infty$

$\alpha_1 = 180^\circ, \; \alpha_2 = 0$

$$\bar{H} = \frac{I}{4\pi\rho}\left[1-(-1)\right] \hat{a}_\phi$$

$$\boxed{\bar{H} = \frac{I}{2\pi\rho} \hat{a}_\phi} \quad A/m$$



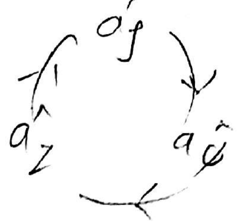
Here $\hat{a}_\phi$ can be written as $\hat{a}_l \times \hat{a}_\rho$.

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho .$$

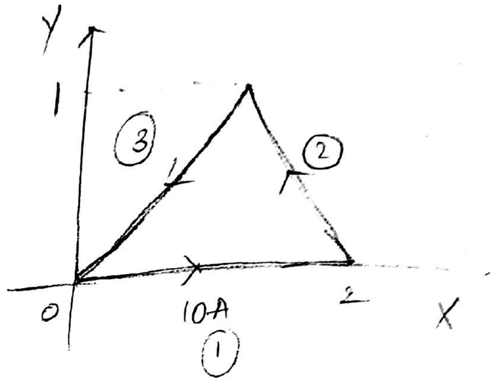
Where $\hat{a}_l$ = unit vector which is lying along the direction of current line element and ~~$\hat{a}_\phi$~~ $\hat{a}_\rho$ = unit vector perpendicular ($\perp^{r}$) to the field pt under current line element.

$$\hat{a_\phi} = \hat{a_l} \times \hat{a_\rho}$$

$$\hat{a_\phi} = \hat{a_z} \times \hat{a_\rho}$$



A conducting triangle loop of figure ① as shown below Carries a current of 10A. find H at (0,0,5) due to side one of the loop.



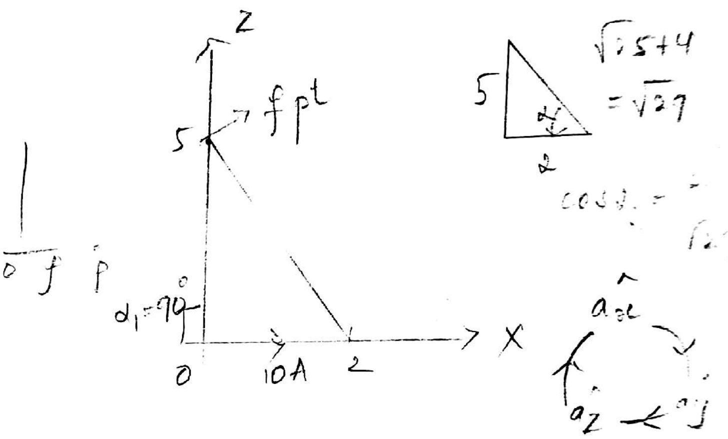
$$\bar{H} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right] \hat{a_\phi}$$

$$\hat{a_\phi} = \hat{a_l} \times \hat{a_\rho}$$

$$-\hat{a_y} = \hat{a_x} \times \hat{a_z}$$

$$\bar{H} = \frac{10}{4\pi \times 5}\left[\frac{2}{\sqrt{29}} - 0\right] -\hat{a_y}$$

$$\bar{H} = -59 \hat{a_y} \text{ m A/m}$$

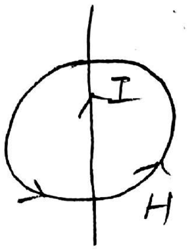


Ampere's Circuit law (or) Maxwell Third Equation:

Statement:-

Ampere's circuit law states that line integral of H ~~over~~ around a closed path is equal to net current enclosed by the path.

$$\boxed{\oint_l \bar{H} \cdot \bar{dl} = I_{enc}}$$



w.k.T $I_{enc} = \iint_S \bar{J} \cdot \bar{ds}$

$$\oint_l \bar{H} \cdot \bar{dl} = \iint_S \bar{J} \cdot \bar{ds}$$

Apply Stokes theorem for the LH's part of above eqn then .

$$\iint_S \nabla \times \bar{H} \ \bar{ds} = \iint_S \bar{J} \cdot \bar{ds}$$

$$\nabla \times \bar{H} = \bar{J}.$$

The above eqn is called Maxwell third eqn in point form.

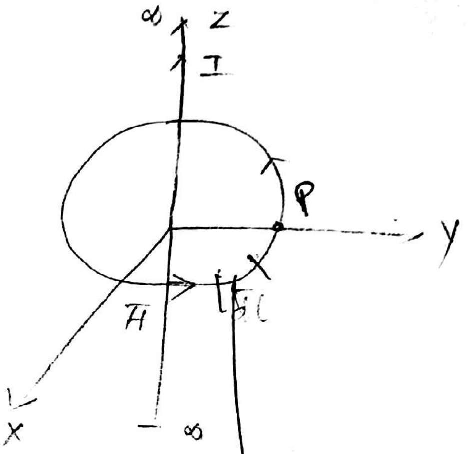
Therefore from this eqn we can conclude that curl product of magnetic field intensity is having some finite value. It implies that magnetic field intensity is a rotational Vector or non-Conservative field.

Maxwells eqns	point form	integral form
1) 1st maxwell's eqn (Guass law)	$\nabla \times \bar{D} = \rho_v$	$\psi = Q_{enc}$ $\iint_S \bar{D} \cdot \bar{ds} = \iiint_v \rho_v dv$
2) 2nd maxwell's eqn (principle of Electric field intensity)	$\nabla \times \bar{E} = 0$ (irrotational)	$\oint_l \bar{E} \cdot \bar{dl} = 0$
3) 3rd Maxwell's (Ampere's ckt law)	$\nabla \times \bar{H} = \bar{J}$ (rotational)	$\oint_l \bar{H} \cdot \bar{dl} = \iint_S \bar{J} \cdot \bar{ds}$

Applications of amperes Circuit law:

1. Infinite Current line ~~element~~ current:

Consider an Infinite Current line element which is placed on Z-axis



$$\oint_l \bar{H} \cdot \bar{dl} = I_{enc}$$

$$\bar{H} = H_\phi \hat{a_\phi}$$

$$\bar{dl} = \rho \, d\phi \, \hat{a_\phi}$$

$$\oint_l \bar{H} \cdot \bar{dl} = \oint_l H_\phi \hat{a_\phi} \cdot \rho \, d\phi \hat{a_\phi}$$

$$\oint_l \bar{H} \cdot \bar{dl} = \oint_l H_\phi \rho \, d\phi$$

$$= H_\phi \rho \int_{\phi=0}^{2\pi} d\phi$$

$$= H_\phi \rho [\phi]_0^{2\pi}$$

$$\oint_l \bar{H} \cdot \bar{dl} = H_\phi \rho \, 2\pi$$

$$I_{enc} = I$$

$$H_\phi \rho \, 2\pi = I$$

$$H_\phi = \frac{I}{2\pi \rho}$$

$$\bar{H} = H_\phi \hat{a_\phi}$$

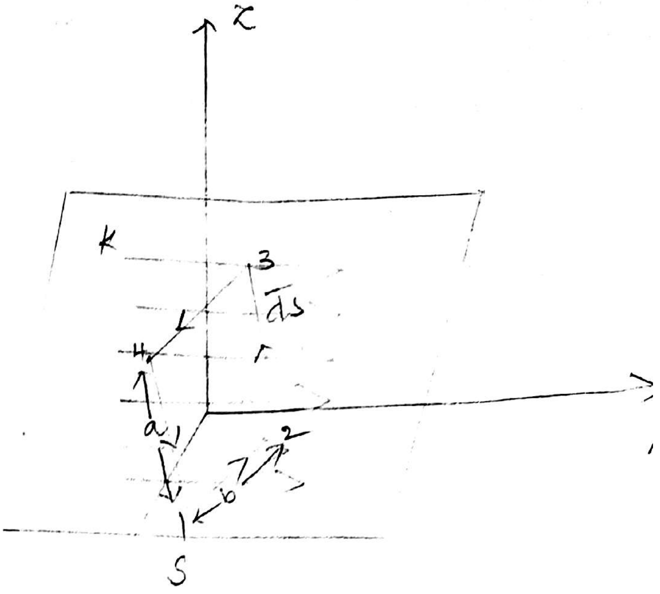
$$\boxed{\bar{H} = \frac{I}{2\pi \rho} \hat{a_\phi}.}$$

Infinite Sheet current ~~element~~:

Infinite Sheet - Current element:

Apply Ampere's Circuit law

$$\oint \bar{H} \cdot \bar{dl} = I_{enc}$$



Z=0, plane ||el to xy

$$\oint_l \bar{H} \cdot \bar{dl} =$$

$$\int_1^2 \bar{H} \cdot \bar{dl} + \int_2^3 \bar{H} \cdot \bar{dl} + \int_3^4 \bar{H} \cdot \bar{dl} + \int_4^1 \bar{H} \cdot \bar{dl}$$

The surface current at the edges becomes as zero $\therefore \int_2^3 \text{ \& } \int_4^1 = 0.$

$$\oint_l \bar{H} \cdot \bar{dl} = \int_1^2 \bar{H} \cdot \bar{dl} + \int_3^4 \bar{H} \cdot \bar{dl} \longrightarrow ①$$

$\bar{H}$ can be written as

$$\bar{H} = \begin{cases} H_0 (-\hat{a_x}) & , z < 0 \quad , \bar{dl} = dx(-\hat{a_x}) \\ H_0 (\hat{a_x}) & , z > 0 \quad , \bar{dl} = dx(\hat{a_x}) \end{cases}$$

$$① \Rightarrow \oint_l \bar{H} \cdot \bar{dl} = \int_1^2 H_0 (-\hat{a_x}) \cdot dx(-\hat{a_x}) + \int_3^4 H_0 \hat{a_x} \cdot dx \hat{a_x}.$$

$$\oint_l \bar{H} \cdot \bar{dl} = H_0 \int_1^2 dx + H_0 \int_3^4 dx.$$

$$\oint_l \bar{H} \cdot \bar{dl} = H_0 b + H_0 b = 2H_0 b$$

$I_{enc} = ?$

Since here the surface current is given as

$$K = \frac{A}{m}$$

$$A = Km$$

$$A = K_y b$$

$$A = I_{enc} = K_y b$$

$$\oint \bar{H} \cdot \bar{dl} = I_{enc}$$

$$2H_0 \cancel{b} = K_y \cancel{b}$$

$$H_0 = \frac{K_y}{2} .$$

$$\bar{H} = \begin{cases} \cancel{K_y}\ \frac{K_y}{2} (-\hat{a}_x), & z<0 \\ \frac{K_y}{2} (\hat{a}_x), & z>0 . \end{cases}$$

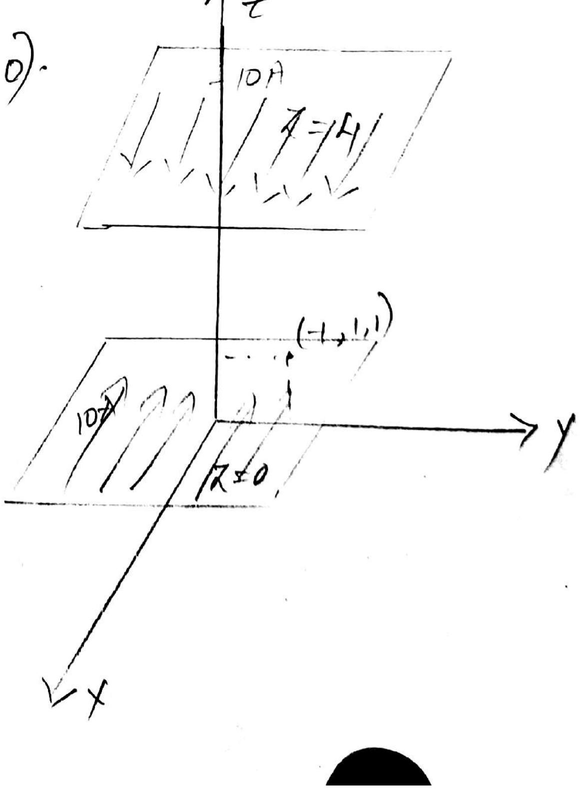
Generally $\bar{H} = \frac{1}{2} K_y \times \hat{a}_n$ A/m

Where $\hat{a}_n$ = unit normal Vector directed from the Current sheet to the point of interest($\underset{\text{field}}{\text{current}}$ point).

problem.

planes $Z=0$ & $Z=4$ Carries current of $\bar{K} = -10\hat{a}_x$ Amp/m & $\bar{K} = 10\hat{a}_x$ A/m respectively. Determine $\bar{H}$ at

(i) (-1, 1, 1) ii) (0, -3, 10).

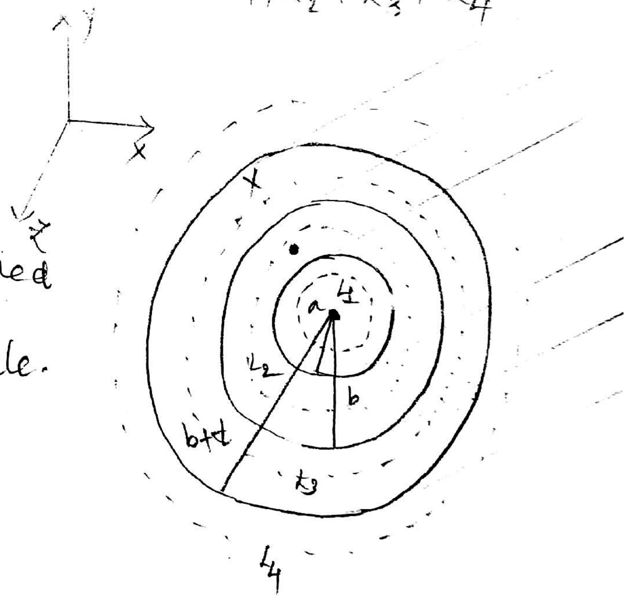


Infinite long Co-axial transmission line:-

let 'a' be the radius of inner conductor and 'b' be Th radius of outer conductor and 't' is the thickness of outer Conductor. Inner Conductor Carries Current along the z-axis whereas outer conductor Carries Current along -ve Z-axis. It means that the Co-axial cable is placed along Z-axis To find $\bar{H}$ we need to assume a path called Amperian pa The Amperian paths are represented as L_1, L_2, L_3, L_4.

— = radius of Conductor

- - - = Amperian path.



dot (.) and cross (x) is explained by flemings Right hand thumb rule.

$L_1 = 0 < \rho < a$

$L_2 = a < \rho < b$

$L_3 = b < \rho < b+t$

$L_4 = \rho > b+t$

Case-(i):- $L_1 = 0 < \rho < a$ (or) Loop 1.

Apply Amperes Circuit law.

$$\oint_l \bar{H} \cdot \bar{dl} = I_{enc.}$$

WKT $I_{enc} = \iint_S \bar{J} \cdot \bar{ds}$

$$\bar{J} = \frac{I}{\text{Area}}$$

$$\bar{dl} = d\rho \hat{a}_\rho + \rho d\phi \hat{a}_\phi + dz \hat{a}_z$$

$$\bar{J} = \frac{I}{\pi a^2} \hat{a}_z .$$

$L_1 = 0 < \rho < a$

$$I_{enc} = \iint_S \frac{I}{\pi a^2} \hat{a}_z \rho d\rho \, d\phi \, \hat{a}_z .$$

$\bar{ds}_z = \rho d\rho d\phi \hat{a}_z$

$$I_{enc} = \frac{I}{\pi a^2} \int_{\rho=0}^{\rho} \int_{\phi=0}^{2\pi} \rho d\rho \, d\phi$$

$$= \frac{I}{\pi a^2} \frac{\rho^2}{2} 2\pi$$

$$\boxed{I_{enc} = \frac{I \rho^2}{a^2}} \text{ Amp.}$$

$$\oint_l \bar{H} \cdot \bar{dl} = \oint_l H_\phi \hat{a}_\phi \cdot \rho \, d\phi \, \hat{a}_\phi$$

$$= \oint_l H_\phi \rho \, d\phi$$

$$\oint_l \bar{H} \cdot \bar{dl} = H_\phi \rho [2\pi]$$

$$\oint_l \bar{H} \cdot \bar{dl} = I_{enc}$$

$$H_\phi \rho (2\pi) = \frac{I \rho^2}{a^2}$$

$$H_\phi = \frac{I \rho}{2\pi a^2} .$$

$$\bar{H} = H_\phi \hat{a}_\phi = \frac{I \rho}{2\pi a^2} \hat{a}_\phi .$$

$$\boxed{\bar{H} = \frac{I \rho}{2\pi a^2} \hat{a}_\phi} \text{ A/m.}$$

Case (ii) :- $\quad d_2 = a < \rho < b$

Apply Ampere's circuit law

$$\oint_l \bar{H} \cdot \bar{dl} = I_{enc}$$

We know that $I_{enc} = I$

$$\oint_l \bar{H} \cdot \bar{dl} = \oint_l H_\phi \hat{a_\phi} \cdot \rho \, d\phi \, \hat{a_\phi}$$

$$\oint_l \bar{H} \cdot \bar{dl} = \oint_l H_\phi \rho \, d\phi$$

$$\oint_l \bar{H} \cdot \bar{dl} = H_\phi \rho [2\pi]$$

$$\oint_l \bar{H} \cdot \bar{dl} = I_{enc}$$

$$H_\phi \rho [2\pi] = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$\bar{H} = H_\phi \hat{a_\phi}$$

$$\boxed{\bar{H} = \frac{I}{2\pi\rho} \hat{a_\phi}} \ A/m$$

Case - iii) $\quad d_3 = b < \rho < b+t$.

Apply Ampere's circuit law

$$\oint_l \bar{H} \cdot \bar{dl} = I_{enc}$$

$$\oint_l \bar{H} \cdot \bar{dl} = \oint_l H_\phi \hat{a_\phi} \cdot \rho \, d\phi \, \hat{a_\phi}$$

$$\oint_l \bar{H} \cdot \bar{dl} = \oint_l H_\phi \rho \, d\phi$$

$$\oint_l \bar{H} \cdot \bar{dl} = H_\phi \rho [2\pi].$$

$$I_{enc} = I_1 + I_2$$

Here $I_1 = I$, $I_2 = ?$

W.k.t $I_2 = \iint_S \overline{J} \cdot \overline{dS}$; $\overline{J} = \frac{I}{A} = \frac{I(-\hat{a_z})}{\pi\left[(b+t)^2 - b^2\right]}$

$$= \frac{-I\hat{a_z}}{\pi\left[b^2 + t^2 + 2bt - b^2\right]}$$

$$= \frac{-I\hat{a_z}}{\pi\left[t^2 + 2bt\right]}$$

$\overline{dS}_z = \rho\, d\rho\, d\phi\, \hat{a_z}$

$$I_2 = \iint_S \frac{-I\hat{a_z}}{\pi\left[t^2+2bt\right]} \rho\, d\rho\, d\phi\, \hat{a_z}$$

$$= \frac{-I}{\pi\left[t^2+2bt\right]} \iint_S \rho\, d\rho\, d\phi$$

$$= \frac{-I}{\pi\left[t^2+2bt\right]} \int_{\rho=b}^{\rho} \rho\, d\rho \int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{-I}{\pi\left[t^2+2bt\right]} \left(\frac{\rho^2}{2}\right)_b^{\rho} 2\pi$$

$$= \frac{-I}{2\cancel{\pi}\left[t^2+2bt\right]} (\rho^2 - b^2)\, 2\cancel{\pi}$$

$$I_2 = \frac{-I\left[\rho^2 - b^2\right]}{\left[t^2 + 2bt\right]}.$$

$\therefore I_{en} = I_1 + I_2$

$$= I - \frac{I}{t^2+2bt}\left[\rho^2 - b^2\right]$$

$$= I\left[1 - \frac{\rho^2 - b^2}{t^2+2bt}\right]$$

$= I\Big[$

$$\oint \overline{H} \cdot \overline{dl} = I_{enc}$$

$$H_\phi\, \rho\, 2\pi = I\left[1 - \frac{1}{t^2+2bt}\left[\rho^2 - b^2\right]\right]$$

$$H_\phi = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2+2bt}\right]$$

$$\overline{H} = H_\phi \hat{a_\phi}$$

$$\overline{H} = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2+2bt}\right]\hat{a_\phi}.$$

Case-(iv) $\rho > b+t$

Apply Ampere's Circuit law.

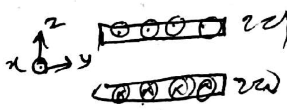
$$\oint_l \bar{H}\cdot \bar{dl} = I_{enc}$$

$$I_{enc} = I_1 + I_2 = I - I = 0.$$

Hence outside the amperian path the magnetic field will be Zero.

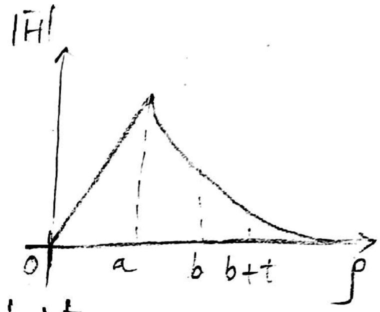
① Planes z=0 & z=4 carry current $K = -10a_x$ A/m and $K = 10a_x$ A/m. Find ~~H~~ @ (4,1,1) (0,-3,10)

$10a_y$, 0



② y=1 plane carries $K = 50a_z$ A/m
H @ (0,0,0) (1,5,-3)
$25a_x$, $-25a_x$

$$\bar{H} = \begin{cases} \frac{I\rho}{2\pi a^2}\hat{a}_\phi , & 0<\rho<a \\ \frac{I}{2\pi\rho}\hat{a}_\phi , & a<\rho<b \\ \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2-b^2}{t^2+2bt}\right]\hat{a}_\phi , & b<\rho<b+t \\ 0 , & \rho > b+t \end{cases}$$



Magnetic flux density: similar to D.

The magnetic flux density represented as $\bar{B}$ is similar to the electric flux density $\bar{D}$.

$\bar{B} = \mu \bar{H}$ where μ = permeability of the medium

$\mu = \mu_o \mu_r$

μ_o → permeability of free space

μ_r → relative permeability

$\mu_r = 1$

$\mu_o = 4\pi \times 10^{-7}$ H/m.

The Magnetic flux through a surface 'S' is given as

$$\bar{B} = \frac{\phi}{S}$$

$$\bar{B} = \frac{d\phi}{\bar{ds}}$$

$$d\phi = \bar{B}\cdot ds$$

$$\phi = \iint \bar{B}\cdot \bar{ds}$$

units : Wb/m² or Teslas (T)

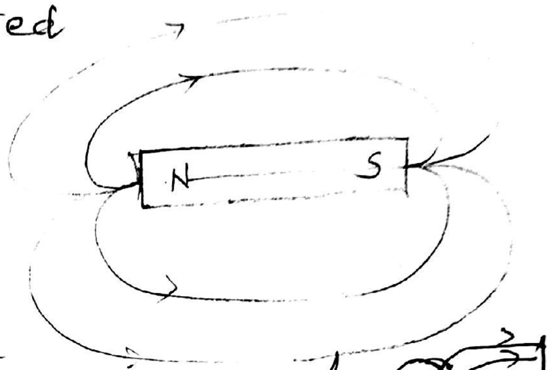
ϕ (Wb)

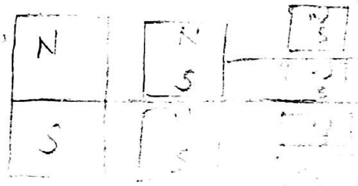
Magnetic flux line is a path to which B is tangential at every point on the line.

Direction of B is directed as N.

Each flux line is closed and has no beginning and or end.

The Magnetic fluxlines is always a closed line and has no beginning or ending point. Magnetic flux lines donot Cross each other. In electro static fields the flux passing through a closed surface is same as the charge enclosed. Thus it is possible to have an isolated electric charge or electric flux lines But if We define to have an isolated magnetic pole by dividing bar magnetic into pieces having north and South poles as shown in fig. It is impossible to sepera the bar magnets.





Thus total flux passing through a closed surface in a magnetic field is Zero. $\phi = \iint_S \bar{B} \cdot \bar{ds} = 0$. Thus thes Eqn is referred to as law of conservation of Magnetic flux or Guass law of magnetostatic field. Magnetostatic field is not conservative but magnetic flux is conservative ($\nabla \times H \neq 0$, $\nabla \cdot B = 0$)

Apply divergence theorem to the above eqn.

$$\phi = \iint_S \bar{B} \cdot \bar{ds} = \iiint_v \nabla \cdot \bar{B} \, dv = 0$$

Magnetic poles cannot be isolated.

$\nabla \cdot \bar{B} = 0$ is called Maxwell 4th equation.

i.e., No monopole of a bar magnet doesnot exist.

Scalar and Vector Magnetic potentials:-

	Scalar	Vector
	V_m	$\bar{A}$
Unit	weber	wb/m

To define V_m & $\bar{A}$, we use two Vector ~~potential~~ identities.

i) $\nabla \times (\nabla V) = 0 \rightarrow$ ① V any scalar field

ii) $\nabla \cdot (\nabla \times \bar{A}) = 0. \rightarrow$ ② A - Vector field

Compare the above two Vector identities with 3rd & 4th Maxwell's Eqn

$$\nabla \times \bar{H} = \bar{J} \qquad \nabla \cdot \bar{B} = 0$$

$$\bar{H} = -\nabla V_m \text{ if } \bar{J} = 0 \qquad \bar{B} = \nabla \times \bar{A}$$

$$\frac{wb}{m^2} = \frac{1}{m} \times \frac{wb}{m}$$

$$\bar{H} = -\nabla V_m \text{ if } \bar{J} = 0$$

from 3rd Maxwell's Eqn $\nabla \times \bar{H} = \bar{J}$

$$\nabla \times (-\nabla V_m) = 0.$$

$$-\nabla^2 V_m = 0.$$

$\nabla^2 V_m = 0 \rightarrow$ Laplacian Eqn of magnetostatic field.

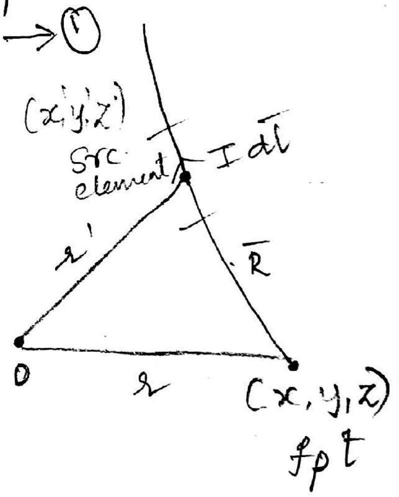
* Expression for Vector magnetic field potential $\bar{A}$:-

we know that $\bar{B} = \nabla \times \bar{A} \rightarrow$ ①

$$\bar{B} = \mu_0 \bar{H}$$

from Biot-Savarts law

$$\bar{H} = \frac{1}{4\pi} \int_l \frac{I\bar{dl} \times \bar{R}}{|\bar{R}|^3}.$$



$\bar{R}$ = field pt - Source pt

$$= (x, y, z) - (x', y', z')$$

$$\bar{R} = (x - x', y - y', z - z')$$

$$\bar{R} = (x - \bar{x}) \hat{a_x} + (y - y') a_y + (z - z') \hat{a_z} \rightarrow ②$$

$$|\bar{R}| = \left(\left[(x - \bar{x})^2 + (y - \bar{y})^2 + (z - z)^2 \right) \right)^{1/2}.$$

$$\frac{1}{|\bar{R}|} = \left(\left((x' - x)^2 + (y - y^2) + (z - z') \right)^2 \right)^{1/2}$$

Magnetic scalar and vector Potentials:-

In electrostatics we relate electric potential V to E as $E = -\nabla V$. similarly, we can define potential ~~associated~~ with magnetostatic field B. Magnetic potential could be V_m scalar or A (vector).

From vector identities, $\nabla\cdot(\nabla V) = 0 \rightarrow$ ① must hold

$\nabla\cdot(\nabla\times A) = 0 \rightarrow$ ② always for scalar V and vect A

As $E = -\nabla V$, $H = -\nabla V_m$. if $J = 0$.

From Amperes law, $J = \nabla\times H = \nabla\times(-\nabla V_m) = 0 \rightarrow$ ③

Thus magnetic scalar potential is defined only in region $J = 0$.

W.k.t $\nabla\cdot B = 0$, to satisfy this and ② simultaneously, we define ~~A such that~~

$$B = \nabla\times A \rightarrow ④$$

As, $V = \int \frac{Q}{4\pi\varepsilon_0 r} \longrightarrow$ ⑤

we can define, $A = \int \frac{\mu_0 I\,dl}{4\pi R}$ for line current $\rightarrow$ ⑥

$A = \int \frac{\mu_0 K\,ds}{4\pi R}$ for surface current $\rightarrow$ ⑧

$A = \int \frac{\mu_0 J\,dv}{4\pi R}$ for volume current $\rightarrow$ ⑧

As $H = \int \frac{I\,dl\times R}{4\pi R^3}$ so $B = \frac{\mu_0}{4\pi}\int \frac{I\,dl\times R}{4\pi R^3} \rightarrow$ ⑨

R is distance vector from line element dl' at (x', y', z') to field point (x, y, z)

$(R) = |r - r'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$

Hence $\nabla\left(\frac{1}{R}\right) = \frac{-1}{R^2} = \frac{-R}{R^3} = \frac{(x-x')a_x + (y-y')a_y + (z-z')a_z}{\left(\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}\right)^3}$

~~a_x~~ a_R

or $\frac{R}{R^3} = -\nabla\left(\frac{1}{R}\right) \left(= \left(\frac{a_R}{R^2}\right)\right) \rightarrow$ ⑩

sub this in ⑨, $B = \frac{-\mu_0}{4\pi}\int I\,dl\times\nabla\left(\frac{1}{R}\right) \rightarrow$ ⑪

From vector identity, $\nabla \times (fF) = f\nabla \times F + (\nabla f) \times F$

f – scalar field ($\frac{1}{R}$); F – vector field (dl')

So, $\nabla \times \frac{dl'}{R} = \underline{\frac{1}{R}\nabla \times dl'} + \nabla\left(\frac{1}{R}\right) \times dl'$

$$dl' \times \nabla\left(\frac{1}{R}\right) = \cancel{\nabla \times \frac{dl'}{R}} \to 0 - \nabla\left(\frac{1}{R}\right) \times dl'$$

$$dl' \times \nabla\left(\frac{1}{R}\right) = -\nabla\left(\frac{1}{R}\right) \times dl'$$

$$\therefore B = \nabla \times \int \frac{\mu_0 I dl}{4\pi R}$$

$$\therefore A = \int \frac{\mu_0 I dl'}{4\pi R} \to (11)$$

$$\phi = \int_s B \cdot ds = \int_s (\nabla \times A) \cdot ds = \oint_l A \cdot dl$$

$$\boxed{\phi = \oint_l A dl}$$

→ A charged particle of m = 2kg, Q = 3 C starts @ (1, -2, 0) with velocity $4\hat{a}_x + 3\hat{a}_z$ m/s in an E $12a_x + 10a_y$ V/m. At t = 1, find (a) a (b) u (c) K.E ~~(d) position~~

Sol: $F = ma = QE$

(a) $a = \frac{QE}{m} = \frac{3}{2}(12a_x + 10a_y)$

$a = 18a_x + 15\hat{a}_y$ m/s²

(b) $a = \frac{du}{dt} = \frac{d}{dt}(u_x, u_y, u_z)$

$\Rightarrow \frac{d}{dt}u_x = 18$, $u_x = 18t + A \to ①$

$u_y = 15t + B \to ②$

$u_z = 0 + C \to ③$

@ t = 0, $u = 4a_x + 3\hat{a}_z$

$u_x(t=0) = 4$

$\frac{du_x}{dt} = 0$

$u_x = 4 = 0 + A \to ④$

$u_y = 0 = 0 + B \to ⑤$

$u_z = 3 = 0 + C \to ⑥$

Sub 4, 5, 6 in 1, 2, 3

$u(t) = (u_x, u_y, u_z) = (18t + 4, 15t, 3)$

$\underline{u(t=1s)} = 22a_x + 15a_y + 3a_z$ m/s

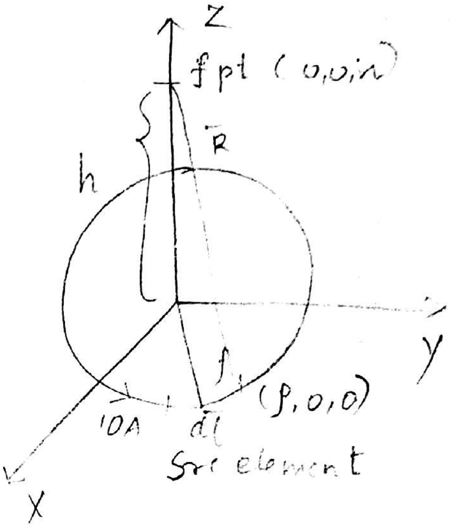
(c) $K.E = \frac{1}{2}m|u|^2 = \frac{1}{2}(2)(22^2 + 15^2 + 3^2)$

$= 718$ J

~~(d) $u = \frac{dl}{dt} = \frac{d}{dt}(4, 4, 0) = 18t$.~~

Problem:-

A circular loop located on $x^2+y^2=9, z=0$ carries a direct current of 10A along $\hat{a_\phi}$ direction. find $\overline{H}$ at (0,0,4) and (0,0,−4).



$$\overline{H} = \frac{1}{4\pi}\int_l \frac{I\,\overline{dl} \times \overline{R}}{|\overline{R}|^3}$$

$$\overline{R} = fpt - Srcpt$$

$$= (0,0,h) - (\rho,0,0)$$

$$= (-\rho,0,h)$$

$$\overline{R} = -\rho\hat{a_\rho} + h\hat{a_z}$$

$$|\overline{R}| = \sqrt{\rho^2+h^2}$$

$$\overline{dl} = \rho\, d\phi\, \hat{a_\phi}$$

$$\overline{dl} \times \overline{R} = \begin{vmatrix} \hat{a_\rho} & \hat{a_\phi} & \hat{a_z} \\ 0 & \rho d\phi & 0 \\ -\rho & 0 & h \end{vmatrix} \Rightarrow \hat{a_\rho}\left[\rho d\phi h - 0\right] - \hat{a_\phi}\left[0\right] + \hat{a_z}\left[0+\rho^2 d\phi\right.$$

$$= \rho d\phi h \hat{a_\rho} + \rho^2 d\phi\, \hat{a_z}.$$

Because of Symmetry condition $\hat{a_\rho}$ component will be zero

$$\overline{dl} \times \overline{R} = \rho^2 d\phi\, \hat{a_z}.$$

$$\overline{H} = \frac{1}{4\pi}\int_l \frac{I\rho^2 d\phi \hat{a_z}}{(\rho^2+h^2)^{3/2}} = \frac{1}{4\pi}\int_l \frac{10\,\rho^2 d\phi \hat{a_z}}{(\rho^2+h^2)^{3/2}}$$

$$\overline{H}_{(0,0,4)} = \frac{10}{4\pi}\int \frac{3^2 d\phi\, \hat{a_z}}{(3^2+4^2)^{3/2}}$$

$$= \frac{90}{4\pi \times 125}\int_{\phi=0}^{2\pi} d\phi\, \hat{a_z}.$$

$$= \frac{\overset{\overset{9}{\cancel{45}}}{\cancel{90}}}{\underset{2}{\cancel{4}}\pi \times \underset{25}{\cancel{125}}}(2\pi)\,\hat{a_z}$$

$$\overline{H}_{(0,0,4)} = \frac{9}{25}\hat{a_z} = 0.36\,\hat{a_z} \text{ Amp/m}$$

Note :-

Here $\overline{dl} \times \overline{R}$ in that term if h is replaced by $-h$ the z-components of $\overline{H}$ will remains same even though $\hat{a_\rho}$ Component becomes to zero.

$$\overline{H}_{(0,0,-4)} = 0.36\, \hat{a_z} \text{ Amp/m}$$

Torque and Magnetic moment :

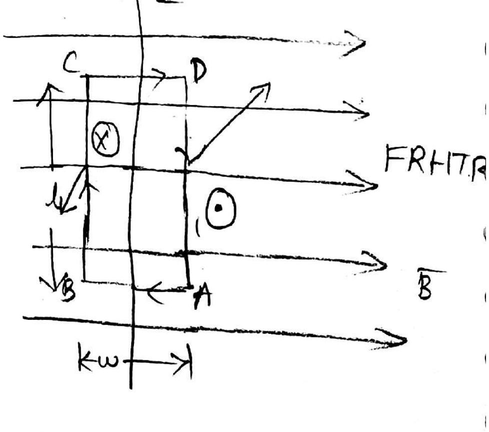
We can determine the torque on a current loop when it is placed in uniform magnetic field $\overline{B}$. The concept of current loop experiencing torque in magnetic field is of very important in understanding the behaviour of orbitting charge particle that is like dc motors and generators.

⇒ If the loop is placed parallel to magnetic field it experien a force that tends to rotate it. Therefore the torque on the loop is the vector product of force and moment.

$$\overline{T} = \overline{F} \times \overline{r} \quad N\text{-}m$$

let us apply this principle to the rectangular loop of length 'l' and width "w" placed in a uniform magnetic field as shown in figure

In arms A to B, C to D the width of the rectangle $\overline{dw}$ is placed parallel to $\overline{B}$ (∥el $\theta = 0°$, $\sin 0° = 0$). It means that there is no force is exerted on those sides. Thus $\overline{F}$ can be written as.



$$\overline{F} = \int_D^A I\, \overline{dl} \times \overline{B} + \int_B^C I\, \overline{dl} \times \overline{B}.$$

$$= I \int_0^0 \overline{dl} \times \overline{B} + I \int^l \overline{dl} \times \overline{B}$$

Where $dl = d\bar{z}\, a_{\bar{z}}$

$$\bar{F} = -I \int_{l}^{0} dz\, \hat{a_z} \times \bar{B} + I \int_{0}^{l} dz\, \hat{a_z} \times \bar{B}$$

$\bar{F} = \bar{F_0} - \bar{F_0}$, where $|\bar{F_0}| = BIl$

$\bar{F} = 0$ newtons

Thus, no force is exerted on a loop as a whole However, $\bar{F_0}$, $-\bar{F_0}$ acts at different points on the loop thereby Creating a couple if the normal to the plane of the loop makes an angle α with $\bar{B}$ as shown in figure. the T on the loop is given as

$|T| = F_0 w \sin\alpha$

$= BIlw \sin\alpha$

$= BIS \sin\alpha.$

let $\bar{M} = IS\hat{a_n}$ units $(A\text{-}m^2)$

$\bar{T} = Bm \sin\alpha\, \hat{a_n}$

we define a new quantity m which is magnetic dipole moment. $T = \bar{B} \times \bar{m}$

Conclusion:- 1) The limitation here, is that $\bar{B}$ should be uniform and T is in the direction of axis of rotation

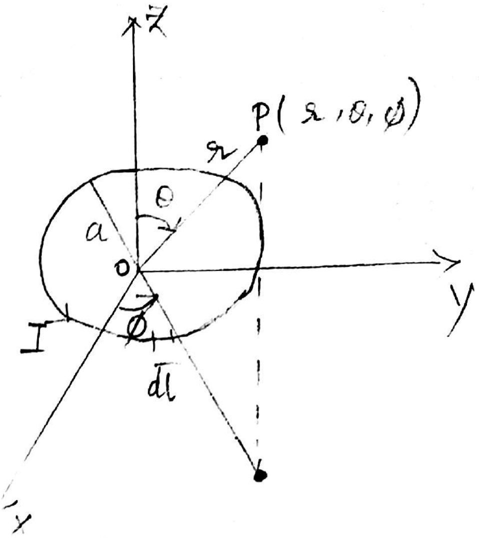
2) If α reduces then m and 'B' will be in same direction it means $T = 0$, it represents loop is placed $\perp^{lr}$ to $\bar{B}$

Magnetic dipole moment:

Consider a bar magnet (having N&S pole) or a small filamentary Current loop is usually referred to as a magnetic dipole.

Here from the above diagram

if $r >> a$ for such a case vector magnetic potential can be written as $\bar{A} = \frac{\mu_0 m \sin\theta \, \hat{a}_\phi}{4\pi r^2}$. $m = IS$



we know that $\bar{B} = \nabla \times \bar{A}$

$$\nabla \times \bar{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{a}_{u_1} & h_2 \hat{a}_{u_2} & h_3 \hat{a}_{u_3} \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_{u_1} & h_2 A_{u_2} & h_3 A_{u_3} \end{vmatrix}$$

$$\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta \, \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 1\times 0 & 0 & r\sin\theta \frac{\mu_0 m \sin\theta}{4\pi r^2} \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta}\left[\hat{a}_r \frac{\partial}{\partial \theta}\left(\frac{r \sin^2\theta \, \mu_0 m}{4\pi r^2}\right)\right] - r\hat{a}_\theta \left[\frac{\partial}{\partial r}\left[r\sin\theta \frac{\mu_0 m \sin\theta}{4\pi r^2}\right.\right.$$

$$= \frac{1}{r^2 \sin\theta}\left\{\left[\hat{a}_r \frac{\mu_0 m}{4\pi r} \frac{\partial}{\partial \theta} \sin^2\theta\right] - \left(r\hat{a}_\theta \frac{\mu_0 m \sin^2\theta}{4\pi} \frac{\partial}{\partial r}\left(\frac{1}{r}\right)\right)\right.$$

$\frac{\partial}{\partial \theta} \sin^2\theta = 2\sin\theta\cos\theta$, $\frac{\partial}{\partial r} \cdot \frac{1}{r} = \frac{\partial}{\partial r} r^{-1} = \frac{-1}{r^2}$.

$$= \frac{1}{r^2 \cancel{\sin\theta}} \frac{\mu_0 m}{4\pi r} 2\cancel{\sin\theta}\cos\theta \, \hat{a}_r - \frac{1}{r^2 \cancel{\sin\theta}} \cancel{r} \frac{\mu_0 m \sin^2\theta}{4\pi}\left(\frac{-1}{r^2}\right) \hat{a}_\theta$$

$$\bar{B} = \frac{\mu_0 m}{4\pi r^3}\left\{2\cos\theta \, \hat{a}_r + \sin\theta \, \hat{a}_\theta\right\} \text{ Wb/m}^2 \text{ or Tesla.}$$

Where magnetic dipole moment $m = IS = Q_m l$.

Q_m = pole strength of magnet

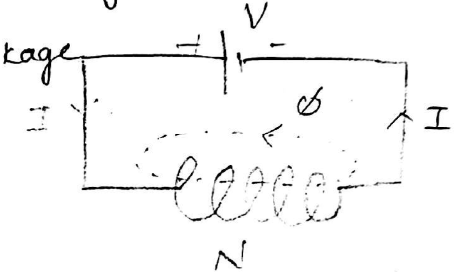
l = length of bar magnet

*2m

Inductors :-

A Circuit carrying current "I" produces $\bar{B}$ that causes a flux $\phi = \iint_S \bar{B} \cdot \bar{ds}$ to pass through each turn of the circuit.

Let 'N' be the no. of turns and each turn is said to be identical. we define the flux linkage. $\lambda = N\phi \rightarrow ①$



If the medium is linear then flux linkage λ is directly proportional to current 'I'.

$$\lambda \propto I$$

$$\lambda = LI \longrightarrow ②$$

$$① = ②$$

$$N\phi = LI$$

$$L = \frac{N\phi}{I} \text{ Henry.}$$

Magnetic energy stored in an inductor from circuit theory is given as $W_m = \frac{1}{2} LI^2$ J.

Energy Stored in magneto Static fields :-

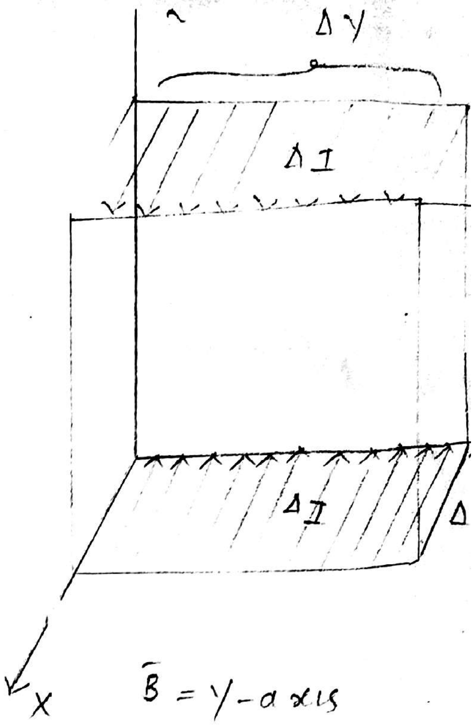
Just as potential energy in an electric field is given as $W_E = \frac{1}{2} \int_v \bar{D} \cdot \bar{E}\, dv$.

Similarly we need to find out the expression for "energy density" in a magneto statistic field. $w_m = \frac{dW_m}{dv}$ J/m^3 $\longrightarrow ①$.

A simple approach by using the magnetic field energy of an inductor is given as $W_m = \frac{1}{2} LI^2$ J.

$$\Delta W_m = \frac{1}{2} \Delta L \, \Delta I^2 \;\; J \longrightarrow ②$$

To consider a differential volume dv in a uniform at top & bottom surfaces with ΔI.



We know that $L = \frac{N\phi}{I}$.

$L = \frac{\phi}{I}$. N=1

$\Delta L = \frac{\Delta \phi}{\Delta I}$

$\phi = \iint_S \bar{B_y}\, \bar{dS_y}$.

$\phi = \bar{B}\, dx\, dz$

$\Delta \phi = \mu \bar{H}\, \Delta x\, \Delta z$

$\Delta L = \frac{\Delta \phi}{\Delta I} = \frac{\mu \bar{H}\, \Delta x\, \Delta z}{\Delta I}$

$\bar{H} = \frac{A}{m} = \frac{I}{m} \Rightarrow I = \bar{H} m$

$\Delta I = \bar{H}\, \Delta y$.

② $\Rightarrow \Delta W_m = \frac{1}{2} \Delta L\, \Delta I^2$

$= \frac{1}{2} \frac{\mu \bar{H}\, \Delta x\, \Delta z}{\Delta I} \cdot \Delta I^2$

$\Delta W_m = \frac{1}{2} \mu \bar{H}\, \Delta x\, \Delta z\, \bar{H}\, \Delta y$

$\Delta W_m = \frac{1}{2} \mu \bar{H}^2 \Delta v$

$w_m = \frac{\Delta W_m}{\Delta v}$

$= \frac{\frac{1}{2} \mu \bar{H}^2 \Delta v}{\Delta v}$

$w_m = \frac{1}{2} \mu \bar{H}^2\ J/m^3$

Problem:

2m ~~Given~~ Given that $\bar{B} = \rho \sin\phi \, \hat{a}_\phi$ find the total flux crossing the surface defined by $\phi = \frac{\pi}{4}$, $1 \le \rho \le 2$, $0 \le z \le 5$.

$$\phi = \iint_S \bar{B}\cdot\bar{ds}.$$

$$= \iint_S \rho \sin\phi \, \hat{a_\phi} = \iint_{S.} \rho \sin\phi \, d\rho \, dz \cdot \hat{a_\phi}\cdot\hat{a_\phi}$$

$$= \sin\phi \int_{\rho=1}^{2} \rho \, d\rho \left(\int_{z=0}^{5} dz\right) \hat{a_\phi}\cdot \hat{a_\phi}$$

$$= \sin\phi \left(\frac{\rho^2}{2}\right)_1^2 (z)_0^5$$

$$= \sin\phi \, \frac{1}{2}(4-1)(5)$$

$$= \frac{15}{2}\sin\phi = \frac{15}{2}\sin\frac{\pi}{4} = \frac{15}{2}\cdot\frac{1}{\sqrt{2}}$$

$\phi = 5.303$ weber.

may-2016
Evaluate Both sides of Stoke's theorem for the field $\bar{H} = 6xy\,\hat{a_x} - 3y^2\,\hat{a_y}$ Amp/metre and the rectangular part around the region $2 \le x \le 5$, $-1 \le y \le 1$, $z=0$.

$$\oint_l \bar{A}\cdot\bar{dl} = \iint_S (\nabla\times\bar{A})\cdot\bar{ds}$$

$$\oint_l \bar{H}\cdot\bar{dl} = \iint_S (\nabla\times\bar{H})\cdot\bar{ds}$$

L.H.S $\oint_l \bar{H}\cdot\bar{dl} = \oint_l (6xy\,a_x - 3y^2\,\hat{a_y})\cdot(dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z})$

$$\oint_l 6xy\,dx - 3y^2 dy. \Rightarrow \int_{x=2}^{5}\int_{y=-1}^{1}(6xy\,dx - 3y^2 dy)$$

$$6y\int_2^5 x\,dx - 3\int_{-1}^{1} y^2 dy \Rightarrow 6y\left[\frac{x^2}{2}\right]_2^5 - 3\left(\frac{y^3}{3}\right)_{-1}^{1}$$

$= 3y[25-4] - [1-(-1)]$

$= 3y[21] - 2$

$= 63y - 2$

$R \cdot H \cdot S = \iint_S \nabla \times \bar{H} \cdot \bar{ds}.$

$$\nabla \times \bar{H} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{a_{u_1}} & h_2 \hat{a_{u_2}} & h_3 \hat{a_{u_3}} \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_{u_1} & h_2 A_{u_2} & h_3 A_{u_3} \end{vmatrix}$$

$$\nabla \times \bar{H} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy & -3y^2 & 0 \end{vmatrix} \Rightarrow \hat{a_x}[0-0] - \hat{a_y}[0] + \hat{a_z}[0-6x]$$

$= -6x\hat{a_z}.$

$R \cdot H \cdot S = \iint_S \nabla \times \bar{H} \cdot \bar{ds} = \iint_S -6x\hat{a_z} (dx\hat{a_x} ~~+ dy\hat{a_y} + dz\hat{a_z}~~)$

$= -6 \iint_S x\hat{a_z} \cdot dxdy\hat{a_z}$

$= -6 \int_{x=2}^{5} x\, dx \int_{-1}^{1} dy$

$= -6 \left[\frac{x^2}{2}\right]_2^5 (y)_{-1}^{1}$

$= -3[25-4][1+1]$

$= -6[21] = -126.$//

A Circular loop Consists of 25 turns of Very fine wire. The average radius of loop is 20cm and it carries a current of 1.6 Amp. find the magnetic flux density at the center of the loop along axial direction.

Scanned by CamScanner

$$\bar{H} = \frac{NI}{2\pi\rho}\hat{a_\phi}$$

$$\bar{B} = \mu\bar{H}$$

$$= 4\pi\times10^{-7}\times\frac{25\times1.6}{[2\pi\times10^{-2}\times4]}\hat{a_\phi}$$

$$25\times1.6\times10^{-7}\times10^{2}$$

$$= 40.0\times10^{-5}$$

$$\bar{B} = 4\times10^{-5}\hat{a_\phi}\ wb/m^2.$$

given the magnetic vector potential $\bar{A} = -0.25\rho^2\hat{a_z}$ wb/m

hat would be the total magnetic flux crossing through the

urface $\phi = \frac{\pi}{2}$, $1 \le \rho \le 2m$, $0 \le z \le 5m$.

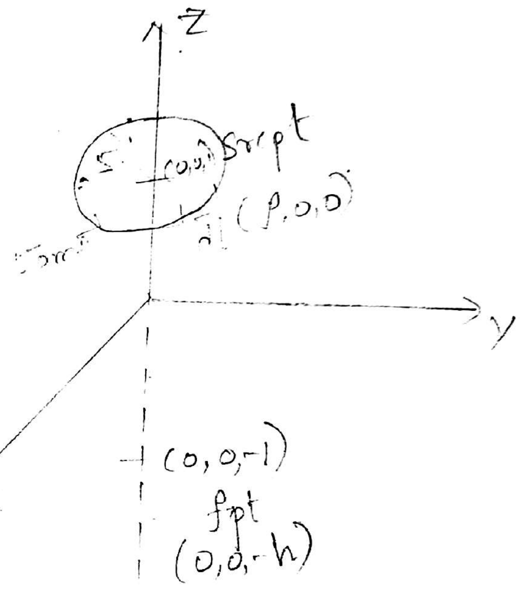
$$\bar{B} = \nabla\times\bar{A}.$$

$$\nabla\times\bar{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{a_{u_1}} & h_2\hat{a_{u_2}} & h_3\hat{a_{u_3}} \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \partial/\partial u_3 \\ h_1A_{u_1} & h_2A_{u_2} & h_3A_{u_3} \end{vmatrix}$$

$$= \frac{1}{\rho}\begin{vmatrix} \hat{a_\rho} & \rho\hat{a_\phi} & \hat{a_z}. \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ 0 & 0 & -0.25\rho^2 \end{vmatrix}$$

A thin ring of radius 5cm is placed on plane z=1cm so that its center is at (0,0,1)cm if the ring carries 50mA along $\hat{a}_\phi$ find the magnitude field intensity at (0,0,-1) cm

from Biot-Savart's law



$$\bar{H} = \frac{1}{4\pi}\int_l \frac{I\,\bar{dl}\times\bar{R}}{|\bar{R}|^3}$$

$$\bar{R} = \text{fpt} - \text{src pt}$$

$$= (0,0,-h) - (\rho,0,0) = (-\rho,0,-h)$$

$$\bar{R} = -\rho\hat{a}_\rho - h\hat{a}_z$$

$$|\bar{R}| = \sqrt{\rho^2+h^2}$$

$$\bar{dl} = \rho\,d\phi\,\hat{a}_\phi$$

$$\bar{dl}\times\bar{R} = \begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ 0 & \rho\,d\phi & 0 \\ -\rho & 0 & -h \end{vmatrix} = \hat{a}_\rho[\rho\,d\phi(-h)] - \hat{a}_\phi[0] + \hat{a}_z[-\rho\,d\phi(-\rho)]$$

$$= -\rho h\,d\phi\,\hat{a}_\rho + \rho^2\,d\phi\,\hat{a}_z.$$

$$\bar{dl}\times\bar{R} = \rho^2\,d\phi\,\hat{a}_z.$$

$$\bar{H} = \frac{I}{4\pi}\int_l \frac{\rho^2\,d\phi\,\hat{a}_z}{(\rho^2+h^2)^{3/2}}, \quad \bar{H} = \frac{50\times10^{-3}}{4\pi}\int \frac{25\times10^{-4}}{(25\times10^{-4} + \times10^{-4}} d \hat{a}_z$$

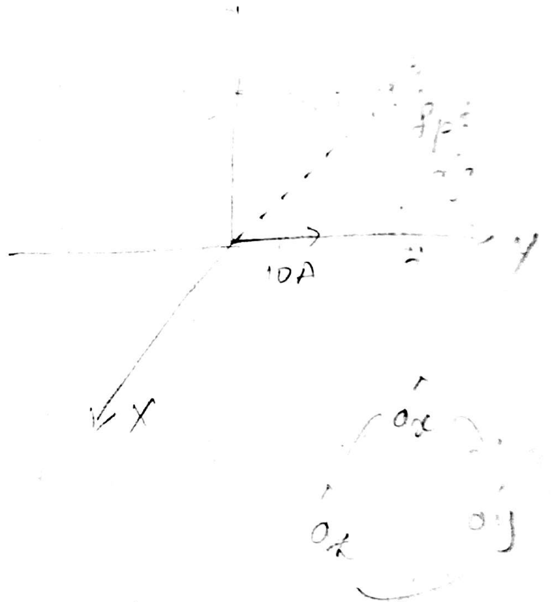
$$= \frac{50\times10^{-3}}{4\pi} \quad \frac{25\times10^{-4}}{\left[25\times10^{-4}+1\times10^{-4}\right]^{3/2}} \int_0^{2\pi} d\phi \, \hat{a}_z .$$

$$= \frac{50\times25\times10^{-7}}{4\pi\left[26\times10^{-4}\right]^{3/2}} \, 2\pi \, \hat{a}_z$$

$$\bar{H} = 0.47 \, \hat{a}_z \text{ A/m}$$

One infinite current filament of 10A lies on y-axis along $\hat{a}_y$ and another infinite current filament of 20 Amp on lies on Z-axis along $-\hat{a}_z$. find the magnetic field intensity H at (-3,4,5)

H due to infinite current line element is given by



$$\bar{H} = \frac{I}{2\pi\rho} \hat{a}_\phi$$

$$\bar{H} = \bar{H}_1 + \bar{H}_2$$

$$\bar{H}_1 = \frac{I_1}{2\pi\rho} \hat{a}_\phi$$

$$\hat{a}_\phi = \hat{a}_y \times \hat{a}_z$$

$$\hat{a}_\phi = \hat{a}_y \times \hat{a}_z$$

$$\hat{a}_\phi = \hat{a}_x$$

$$\bar{H}_1 = \frac{10}{2\pi(5)} \hat{a}_x = \frac{1}{\pi} \hat{a}_x \text{ A/m}$$

$$\bar{H}_1 = \frac{1}{\pi} \hat{a}_x \text{ A/m.}$$

$\bar{H}_2$ due to infinite current line elements.

$$\bar{H}_2 = \frac{I_2}{2\pi\rho} \hat{a}_\phi$$

$$\hat{a}_\phi = -\hat{a}_z \times -\hat{a}_x$$

$$= \hat{a} \times \hat{a} .$$

$\hat{a_\phi} = \hat{a_y}$, $\rho = -3$.

$$\overline{H_2} = \frac{I_2}{2\pi\rho}\hat{a_\phi}$$

$$= \frac{20\,10}{2\pi(-3)} = \frac{-10}{3\pi}\hat{a_y} = -1.06.$$

$$\overline{H} = \overline{H_1} + \overline{H_2} = 0.318\,\hat{a_x} + (-1.06\hat{a_y})\ A/m$$

Given that $\overline{H} = 20\rho^2\hat{a_\phi}$ A/m

i) determine $\overline{J}$, ii) Current enclosed by circular surface

$\rho = 1,\ 0 < \phi < 2\pi,\ z = 0.$

from Maxwells third eqn

$$\nabla \times \overline{H} = \overline{J}.$$

Q. Potential at A is 10V and at B is 15V. If a charge Q = 10μC is moved from ~~A to B~~, what ~~is~~ work required to be done?

Sol: Q, $V_A = 10V$, $V_B = 15V$, $Q = 10\mu C$.

$W = -Q\int_A^B E\,dl = Q V_{AB} = Q[V_B - V_A] = 10\times10^{-6}[15-10]$

$\boxed{W = 50\mu J}$

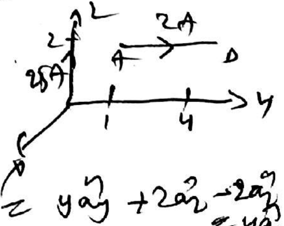
Q1 A point charge of 5nC is located at O. If V = 2V @ (0, 6, -8), find (i) Pot @ (-3, 2, 6) (ii) Pot diff V_{AB}

$V = \frac{Q}{4\pi\epsilon_0 r} + C$; $r = \sqrt{6^2+8^2} = 10$. ⇒ $2 = \frac{5\times10^{-9}}{4\pi\times\frac{10^{-9}}{36\pi}} + C$ ⇒ $C = -2.5$

(i) @ (-3, 2, 6), $V_A = \frac{5\times10^{-9}}{4\pi\frac{10^{-9}}{36\pi}[(-3,2,6)-(0,0,0)]} = 3.92V$

V_B = let $B = (0,6,-8)$ ⇒ $V_B = 2V$; $V_{AB} = V_B - V_A = 2 - 3.92 = -1.92V$

Q2 A st solid wire segment carrying I = 2A A extends from a point (0,1,2) to another point (0,4,2) in free space. This wire is subjected to magnetic field of an infinite current filament lying along z-axis carrying 25A in $\hat{a_z}$. Find Torque on wire about an O at a point (0,0,2)



$H_{AB} = \frac{-I}{2\pi y}\hat{a_x} = \frac{-25}{2\pi y}\hat{a_x}$, $B_{AB} = \frac{-\mu_0 12.5}{\pi y}\hat{a_x}$:

$dF_{AB} = I\,dl \times B = 2a_y dy \times \left(\frac{-\mu_0 12.5}{\pi y}\hat{a_x}\right) = \frac{25\mu_0}{\pi y}\hat{a_z}$

where O is @ (0,0,2) vector from (0,0,2) to (0,y,2) is $B = y\hat{a_y} + 2\hat{a_z} - 2\hat{a_z} = y\hat{a_y}$

$dT = R\times dF = (y\hat{a_y})\times\left(\frac{25\mu_0 dy}{\pi y}\hat{a_z}\right) = \frac{25\mu_0}{\pi} dy\,\hat{a_x}$

$T = \int_1^4 \frac{25\mu_0}{\pi} dy\,\hat{a_x} = 30\hat{a_x}$ μNm

UNIT-III

TIME VARYING FIELDS

⇒ i) Charges at rest condition produces static electric field i.e $E(x,y,z)$

⇒ ii) charges moving with constant velocity produces static magnetic field i.e $\bar{H}(x,y,z)$.

⇒ iii) Charges moving with some acceleration will produces time varying fields i.e $\bar{E}(x,y,z,t)$ & $\bar{H}(x,y,z,t)$.

Here first & second points are independent of each other. Whereas two & three points are depending on each other.

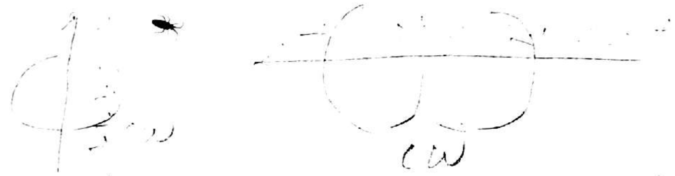
Let the charges are moving with constant velocity as shown in figure then the magnetic field will be in anticlockwise direction.

1)

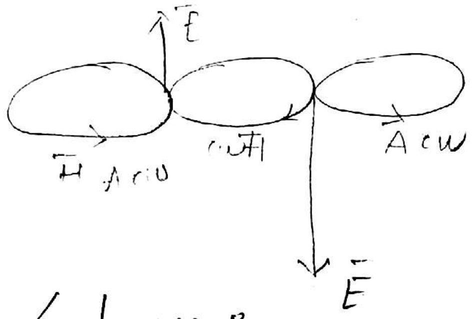
- if we reverse the direction of motion of charges w.r.t time then the field will be in clockwise direction as shown below

Thus we can create a time varying magnetic field by changing the direction of motion of charages.

2)



Consider the positive charge which is moving up and down then direction of electric field will also changes.



E & H are [illegible]

Faraday's Law :-

Statement :- The induced emf (Electro motive force) in any closed circuit is equal to time rate of change of magnetic flux linkages by that circuit.

$$V_{emf} = \frac{-d\lambda}{dt}$$

Where negative sign indicates that induced Voltage in Such a way as to oppose the flux producing it.

where $\lambda = N\phi$, N= no. of turns, ϕ= flux linkage of each turn.

Lenz law:-

The direction of current flows in the circuit such that the induced magnetic field produced by induced Current will impose original magnetic field.

flux can be Varied as follows:-

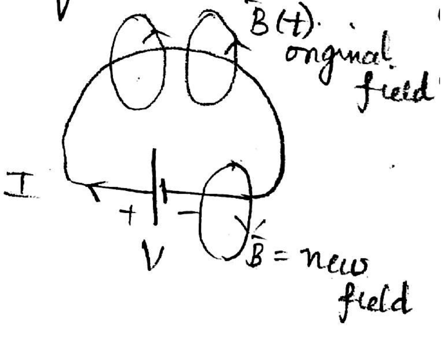
Case(i):- A stationary circuit is kept in a time Varying magnetic field (or) transformer EMF

Case-(ii):- The time Varying circuit placed in static magnetic field (or) Motional Emf.

Case-(iii):- Time Varying circuit placed in Time Varying magnetic field.

Case-(i):- Transformer Emf:-

Transformer emf satisfies the statement of Lenz's law.



Acc to faraday's law

$$V_{emf} = \frac{-d\lambda}{dt} \rightarrow ①, \quad \lambda = N\phi$$

let $N=1$, $\lambda = \phi$

We know that $V_{emf} = \oint_{l} \bar{E}\cdot\bar{dl}$.

W.K.T $\phi = \iint_{s} \bar{B}\cdot\bar{ds}$

Sub above eqns in eqn ①

$$① \Rightarrow V_{emf} = \frac{-d\phi}{dt}$$

$$\oint_{l} \bar{E}\cdot\bar{dl} = \frac{-d}{dt}\iint \bar{B}\cdot\bar{ds}$$

Apply Stokes theorem to L.H.S part of the above Eqn

$$\iint_S \nabla \times \bar{E} \cdot \bar{ds} = -\iint_S \frac{\partial}{\partial t} \bar{B} \cdot \bar{ds}.$$

$$\boxed{\nabla \times \bar{E} = -\frac{\partial}{\partial t} \bar{B}}$$

The above eqn is modified II^{nd} Maxwell equation.
Here the Vector field $\bar{E}$ is called as rotational Vector
r non-conservative field.
se-ii
otional Emf :-

The Magnetic force on a point charge is given by as

$$\bar{F}_m = Q\bar{u} \times \bar{B}$$

$$\bar{E}_m = \frac{\bar{F}_m}{Q}$$

$$\bar{E}_m = \frac{Q\bar{u} \times \bar{B}}{Q}$$

$$V_{emf} = \oint_l \bar{E}_m \cdot \bar{dl} \Rightarrow V_{emf} = \oint_l \bar{u} \times \bar{B} \cdot \bar{dl} = \oint_l \bar{E}_m \cdot \bar{dl}.$$

$$\oint_l \bar{E}_m \cdot \bar{dl} = \oint_l \bar{u} \times \bar{B} \cdot \bar{dl}$$

Apply stokes theorem to both LHS and RHS of above Eqn

$$\iint_S \nabla \times \bar{E}_m \cdot \bar{ds} = \oint_s \nabla \times \bar{u} \times \bar{B} \cdot \bar{ds}$$

$$\boxed{\nabla \times \bar{E}_m = \nabla \times \bar{u} \times \bar{B}.}$$

Case-iii :- Transformer Emf and motional Emf :

Total induced emf = $V_{emf} = V_{trans\ emf} + V_{motional\ Emf}$

$$\boxed{\nabla \times \bar{E}_m = -\frac{\partial \bar{B}}{\partial t} + \nabla \times \bar{u} \times \bar{B}.}$$

** 5m

<u>Displacement Current densities (or) Inconsistency of Amperes Circuit law</u> :-

from static magnetic field. $\nabla \times \bar{H} = \bar{J} \rightarrow$ ①

Since By using the Vector identity $\nabla \cdot (\nabla \times \bar{A}) = 0.$

It means the divergence of curl of any Vector field is equal to zero. apply this vector identity to eqn ①

$$\nabla\cdot(\nabla\times\bar{H}) = \nabla\cdot\bar{J} = 0 \longrightarrow ②$$

But convection current density eqn is given ~~as~~

$$\nabla\cdot\bar{J} = -\frac{\partial}{\partial t}\rho_v \longrightarrow ③$$

Compare eqn 2 with eqn '3'. It indicates there is some inconsistency in the ampere's circuit law to overcome this we need to add one term to the eqn ① ($\bar{J}_d$)

① ⇒ $\nabla\times\bar{H} = \bar{J}+\bar{J}_d \longrightarrow ④$

where $\bar{J}$ = conduction current density, $\bar{J}_d$ = displacement current density.

Apply divergence on both sides.

$$\nabla\cdot(\nabla\times\bar{H}) = \nabla\cdot\bar{J}+\nabla\cdot\bar{J}_d$$

from the vector identity $\nabla\cdot(\nabla\times\bar{H}) = 0$.

$$0 = \nabla\cdot\bar{J}+\nabla\cdot\bar{J}_d$$

$$\nabla\cdot\bar{J} = -\nabla\cdot\bar{J}_d$$

$$+\frac{\partial\rho_v}{\partial t} = -\nabla\cdot\bar{J}_d$$

$$\frac{\partial}{\partial t}(\nabla\cdot D) = \nabla\bar{J}_d$$

from Guass $\nabla\cdot\bar{D} = \rho_v$ ⇒

$$\frac{\partial}{\partial t}(\nabla\cdot\bar{D}) = +\nabla\bar{J}_d$$

$$\nabla\cdot\frac{\partial}{\partial t}\bar{D} = \nabla\cdot\bar{J}_d$$

Comparing b s

$$\frac{\partial}{\partial t}\bar{D} = \bar{J}_d$$

④ ⇒ $\nabla\times\bar{H} = \bar{J}+\frac{\partial\bar{D}}{\partial t}$ → This is called as third Maxwell eqn in time Varying field.

*** Maxwell's Equation in different form :-

Maxwell's Eqn's	pt form (or) differential form	$\int$ form	Word statement
I	$\nabla \cdot \bar{D} = \rho_v$	$\iint_S \bar{D} \cdot \bar{ds} = \iiint_{v} \rho_v \, dv$	Guass law $\Psi = Q_{enc}$
II	$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint \bar{E} \cdot \bar{dl} = \iint_S -\frac{\partial \bar{B}}{\partial t} \bar{ds}$	faraday's law
III	$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$	$\oint \bar{H} \cdot \bar{dl} = \iint_S (\bar{J} + \frac{\partial \bar{D}}{\partial t}) \bar{ds}$	Ampere's circuit law
IV	$\nabla \cdot \bar{B} = 0$	$\iint_S \bar{B} \cdot \bar{ds} = 0$	principle of law of magnetic flux

No monopole of bar magnet does not exist

Maxwell's Eqn is phasor form :-

from third Maxwell's Eqn $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \longrightarrow ①$

Replace $- = \sim$ $\quad \nabla \times \tilde{H} = \tilde{J} + \frac{\partial \tilde{D}}{dt} \longrightarrow ②$

H can be represented as

Here $\tilde{H} = \bar{H} \, e^{j\omega t}$

Sub the above value in ②

$$② \Rightarrow \nabla \times \bar{H} e^{j\omega t} = \bar{J} e^{j\omega t} + \frac{\partial}{\partial t} \bar{D} e^{j\omega t}$$

The above Eqn will represents third maxwell's Eqn in phasor form.

$$\nabla \times Re\{\bar{H} e^{j\omega t}\} = Re\{\bar{J} e^{j\omega t} + \frac{\partial}{\partial t} \bar{D} e^{j\omega t}\}$$

In The above eqn $\frac{\partial}{\partial t} = j\omega$.

$$\nabla\times Re\left\{\bar{H} e^{j\omega t} - \bar{J} e^{j\omega t} - j\omega \bar{D} e^{j\omega t}\right\} = 0$$

$$Re\left\{(\nabla\times\bar{H} - \bar{J} - j\omega\bar{D}) e^{j\omega t}\right\} = 0$$

The above eqn can be written as.

$$\nabla\times\bar{H} - \bar{J} - j\omega\bar{D} = 0$$

$$\boxed{\nabla\times\bar{H} = \bar{J} + j\omega\bar{D}}$$

II → $\nabla\times\bar{E} = \frac{-\partial}{\partial t}\bar{B}$

$$\boxed{\nabla\times\bar{E} = -j\omega\bar{B}}$$

Problems on Maxwells' eqns:-

**

1) Given that $\bar{E} = 10\sin(\omega t - \beta z)\hat{a}_y$ v/m in free space.

Determine $\bar{D}, \bar{B}, \bar{H}$.

from 2nd maxwell eqn

$$\nabla\times\bar{E} = \frac{-\partial\bar{B}}{\partial t} \qquad \bar{B} = \mu_0\bar{H}$$

$$\bar{D} = \epsilon\bar{E} \qquad \epsilon = \epsilon_0\epsilon_r \qquad \bar{H} = \frac{\bar{B}}{\mu_0}. \qquad \mu_0 = 4\pi\times10^{-7}\,H/m$$

$$\bar{D} = \epsilon_0\bar{E} \qquad \epsilon = \epsilon_0$$

Sol:-

$$\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$$

$$\nabla\times\bar{E} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{a}_{u_1} & h_2\hat{a}_{u_2} & h_3\hat{a}_{u_3} \\ \frac{\partial}{\partial u_1} & \partial/\partial u_2 & \partial/\partial u_3 \\ h_1A_{u_1} & h_2A_{u_2} & h_3A_{u_3} \end{vmatrix}$$

$$\nabla\times\bar{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 10\sin(\omega t\beta z) & 0 \end{vmatrix}$$

Scanned by CamScanner

$\nabla \times E = \hat{a_x}[0 - \frac{\partial}{\partial z} 10 \sin(\omega t - \beta z)] - a_y(0) + a_z(0).$

$\nabla \times \overline{E} = \hat{a_x}\left[0 - 10\cos(\omega t - \beta z)(-\beta)\right]$

$\nabla \times \overline{E} = 10\beta\cos(\omega t - \beta z)\,\hat{a_x} = \frac{-\partial \overline{B}}{\partial t}.$

$10\beta\cos(\omega t - \beta z) = \frac{-\partial \overline{B}}{\partial t}.$

$-10\beta\cos(\omega t - \beta z) = \frac{\partial \overline{B}}{\partial t}$

$\overline{B} = -\int 10\beta\cos(\omega t - \beta z)\hat{a_x}\,dt.$

$= -10\beta\int\cos(\omega t - \beta z)\,\hat{a_x}\,dt$

$= -10\beta\,\frac{\sin(\omega t - \beta z)}{\omega t}\,\hat{a_x}.$

$\overline{B} = \frac{-10\beta}{\omega}\sin(\omega t - \beta z)\,\hat{a_x}$. Tesla (or) Wb/m^2

$\overline{D} = \epsilon_0\overline{E}$

$= \frac{10^{-9}}{36\pi}(10\sin(\omega t - \beta z)\hat{a_y}).$

$= \frac{10^{-8}}{36\pi}[\sin(\omega t - \beta z)]\hat{a_y}$

$= 8.84 \times 10^{-11}[\sin(\omega t - \beta z)]\hat{a_y}$ c/m^2

$\overline{H} = \frac{\overline{B}}{\mu_0} = \frac{-10\beta}{\omega(4\pi \times 10^{-7})}\sin(\omega t - \beta z)\hat{a_x}$

$= -0.79 \times 10^{-7}\sin(\omega t - \beta z)\hat{a_x}$ A/m

If the electric field strength of an electro magneto wave in freespace is given by $\overline{E} = 2\cos\omega(t - \frac{z}{V_0})\hat{a_y}$ V/m. V_0=Veloc find $\overline{H}$.

$\nabla \times \overline{E} = \frac{-\partial \overline{B}}{\partial t} \Rightarrow \overline{B} = ?$

$\overline{B} = \mu_0 \overline{H}$

Given $\bar{E} = 2\cos\omega\left(t - \frac{z}{V_0}\right)\hat{a_y}$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{E} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 2\overset{\omega}{\cos}\left(t - \frac{z}{V_0}\right) & 0 \end{vmatrix} \Rightarrow \hat{a_x}\left[0 - \frac{\partial}{\partial z}\left(2\cos\omega\left(t - \frac{z}{V_0}\right)\right)\right] - \hat{a_y}[0] + \hat{a_z}[0]$$

$$\Rightarrow \hat{a_x}\left[-2\sin\left[\omega\left[t - \frac{z}{V_0}\right]\right] \cdot \frac{-1}{V_0}\right]$$

$$\nabla \times \bar{E} \Rightarrow \frac{2\sin\omega\left(t - \frac{z}{V_0}\right)}{V_0}\hat{a_x} = -\frac{\partial \bar{B}}{\partial t}$$

$$\frac{2}{V_0}\sin\omega\left(t - \frac{z}{V_0}\right) = \frac{\partial \bar{B}}{\partial t}$$

$$\bar{B} = \frac{-2}{V_0}\int \sin\omega\left(t - \frac{z}{V_0}\right) = \frac{-2}{V_0}\left[-\cos\omega\left(t - \frac{z}{V_0}\right)\right]\hat{a_x}$$

$$\bar{B} = \frac{2}{V_0}\cos\omega\left(t - \frac{z}{V_0}\right) \text{ tesla or } wb/m^2$$

$$\bar{B} = \mu_0 \bar{H}$$

$$\bar{H} = \frac{\bar{B}}{\mu_0} = \frac{\frac{2}{V_0}\cos\omega\left(t - \frac{z}{V_0}\right)}{4\pi \times 10^{-7}}$$ ~~$\Rightarrow 2.2\pi V_0 \cos\omega\left(t - \frac{Z}{V_0}\right)$~~

$$= 0.159 \times 10^{7}\cos\omega\left(t - \frac{z}{V_0}\right) \text{ A/m}$$

3) In free space the magnetic field of electromagnetic wave is given by $\bar{H} = 0.4\omega\epsilon_0\cos(\omega t - 50x)\hat{a_z}$ A/m find Electric field intensity and displacement current

$$\bar{E} = ?, \quad \frac{\partial \bar{D}}{\partial t} = ?$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

in free space $\sigma = 0 \cdot \bar{J} = \sigma E$

$\bar{J} = 0$

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}.$$

$$\bar{D} = \epsilon_0 \bar{E}.$$

Sol :- $\bar{H} = 0.4\omega\epsilon_0\cos(\omega t - 50x)\hat{a_z}$

$$\nabla \times \bar{H} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & 0.4\omega\epsilon_0\cos(\omega t - 50x) \end{vmatrix}$$

$$= \hat{a_x}\left[0\right] - \hat{a_y}\left[\frac{\partial}{\partial x}\, 0.4\omega\epsilon_0 \cos(\omega t - 50x) - 0\right] - 0$$

$$= -\hat{a_y}\, 0.4\omega\epsilon_0 \sin(\omega t - 50x)\cdot(-50)$$

$$\nabla \times \bar{H} = 0.4 \times 50\, \omega\epsilon_0 \sin(\omega t - 50x)\,\hat{a_y} = \frac{\partial \bar{D}}{\partial t}$$

$$= 20\,\omega\epsilon_0 \sin(\omega t - 50x)\,\hat{a_y}$$

$$\bar{D} = 20\omega\epsilon_0 \int \sin(\omega t - 50x)\,\hat{a_y}$$

$$= 20\omega\epsilon_0\left[-\cos(\omega t - 50x)\right]/\omega$$

$$\frac{\partial \bar{D}}{\partial t},\ C/m^2$$

$$\bar{D} = -20\,\epsilon_0 \cos(\omega t - 50x)\,\hat{a_y}\ C/m^2$$

$$\bar{D} = \epsilon_0 \bar{E}$$

$$\bar{E} = \frac{\bar{D}}{\epsilon_0} = \frac{-20\,\epsilon_0 \cos(\omega t - 50x)}{\epsilon_0}\,\hat{a_y}$$

$$\bar{E} = -20\cos(\omega t - 50x)\,\hat{a_y}\ V/m$$

4) Verify whether the following fields $\bar{E} = 2\sin x \sin t\,\hat{a_y}$, $\bar{H} = \frac{2}{\mu_0}\cos x \cos t\,\hat{a_z}$ cost $\hat{a_x}$ · will satisfy Maxwell's eqn in free space

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}, \quad \bar{B} = \mu_0 \bar{H}$$

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}.$$

$$\nabla \times \bar{E} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 2\sin x \sin t & 0 \end{vmatrix} \Rightarrow +\hat{a_z}(2\cos x \sin t) - \hat{a_z}\,[$$

$$= 2\cos x \sin t\,\hat{a_z}.$$

$$\nabla \times \bar{E} = 2\cos x \sin t\,\hat{a_z} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{B} = \int 2\cos x \sin t\,\hat{a_z}\,dt \Rightarrow 2\cos x \sin$$

5) If the electric field strength of a radio broad casting Signal at a TV reciever is given by $\bar{E} = 5\cos(\omega t - \beta y)\hat{a}_z$ V/m

Determine $\bar{J}_d$, $\bar{J}_c$ Such that $\sigma = 2\times10^{-3}$ v/cm

$\bar{D} = \epsilon_0 \bar{E}$.

$$\bar{J}_d = \frac{\partial \bar{D}}{\partial t},$$

$$\bar{J}_c = \sigma\bar{E} \Rightarrow 2\times10^{-3} \times 5\cos(\omega t - \beta y)\hat{a}_z \Rightarrow 10^{-4}\cos(\omega t - \beta y)\hat{a}_z$$

$$\bar{D} = \epsilon_0\bar{E} = \frac{10^{-9}}{36\pi} \times 5\cos(\omega t - \beta y)\hat{a}_z.$$

$$\bar{D} = 4\cdot42\times10^{-11}\cos(\omega t - \beta y)\hat{a}_z.$$

$$\bar{J}_d = \frac{\partial \bar{D}}{\partial t} = \frac{\partial}{\partial t}\left[4\cdot42\times10^{-11}\cos(\omega t - \beta y)\right]\hat{a}_z$$

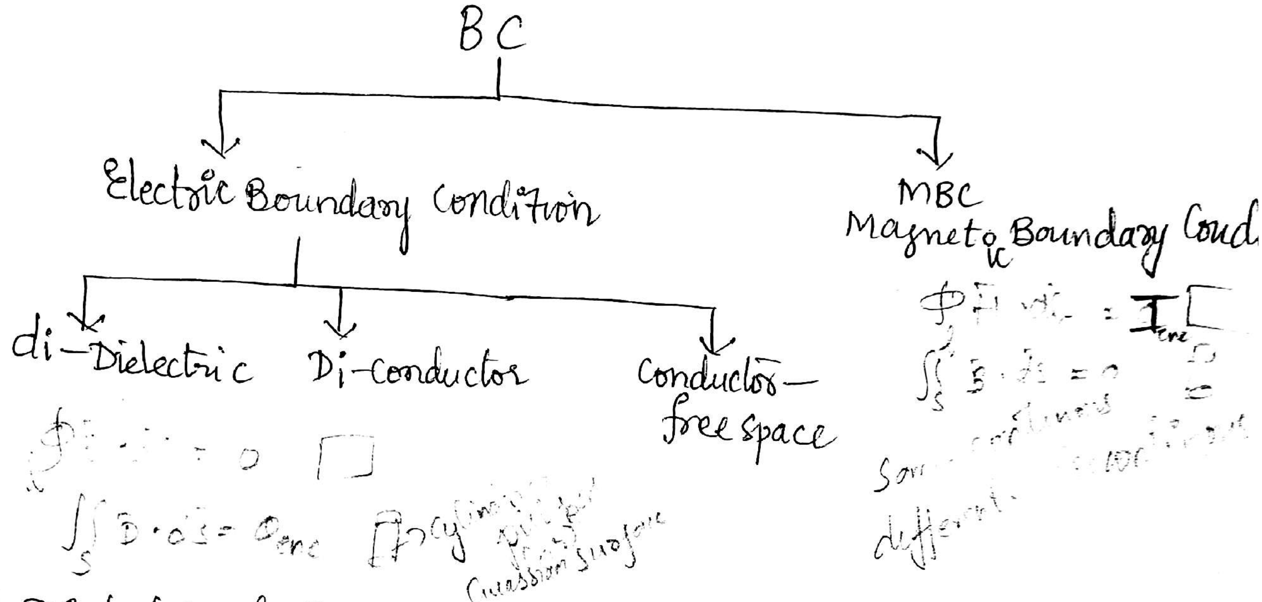
$$= 4\cdot42\times10^{-11}(-\sin\omega t - \beta y)\omega\,\hat{a}_z$$

$$\bar{J}_c = 10^{4}\cos(\omega t - \beta y)\hat{a}_z.$$

Boundary Condition:-

⇒ If the field is existing between two mediums the Condition such that the field must satisfy at the interface (or boundary).

⇒ Boundary Conditions helps us to calculate field components in one region if we know the field components in another region.



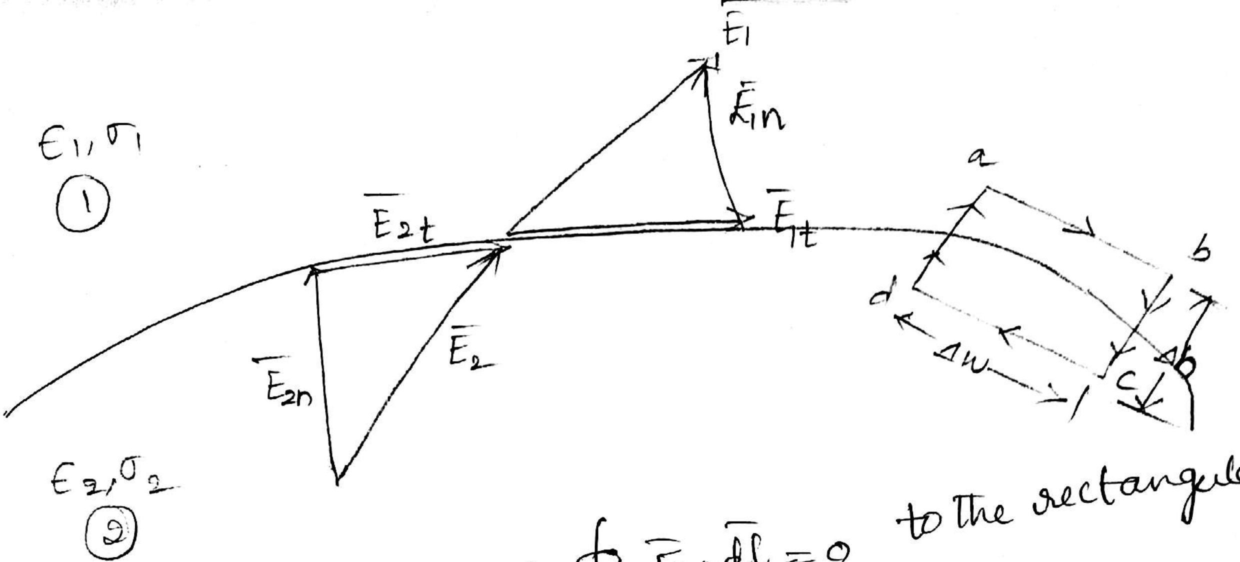
① Dielectric to Dielectric Boundary Condition:-

If the field is propogating (travelling) from one medium to other medium we need to find which field components are same and which field components are different. ✓

Any field Component can be represented as Vector Sum of tangential and normal component i.e $\bar{E}_1 = \bar{E}_{1t} + \bar{E}_{1n}$.

where $\bar{E}_{1t}$ = tangential Component of $\bar{E}$ in medium ① Which is always parallel to interface (or) boundary.

$\bar{E}_{1n}$ = Normal component which is ⊥r to interface or Boundary



Apply closed line integral $\oint_l \bar{E}\cdot\bar{dl}=0$ to the rectangular oop of above figure

$$\oint_l \bar{E}\cdot\bar{dl}=0$$

$$0 = \Delta w\,\bar{E}_{1t} - \frac{\Delta h}{2}\cancel{\bar{E}_{1n}} - \frac{\Delta h}{2}\cancel{\bar{E}_{2n}} - \Delta w\,\bar{E}_{2t} + \frac{\Delta h}{2}\cancel{\bar{E}_{2n}} + \frac{\Delta h}{2}\cancel{\bar{E}_{1n}}$$

$$= \Delta w[\bar{E}_{1t} - \bar{E}_{2t}] = 0$$

$$\boxed{\bar{E}_{1t} = \bar{E}_{2t}}$$

That is the tangential Components of electric field intensities are continous (Same).

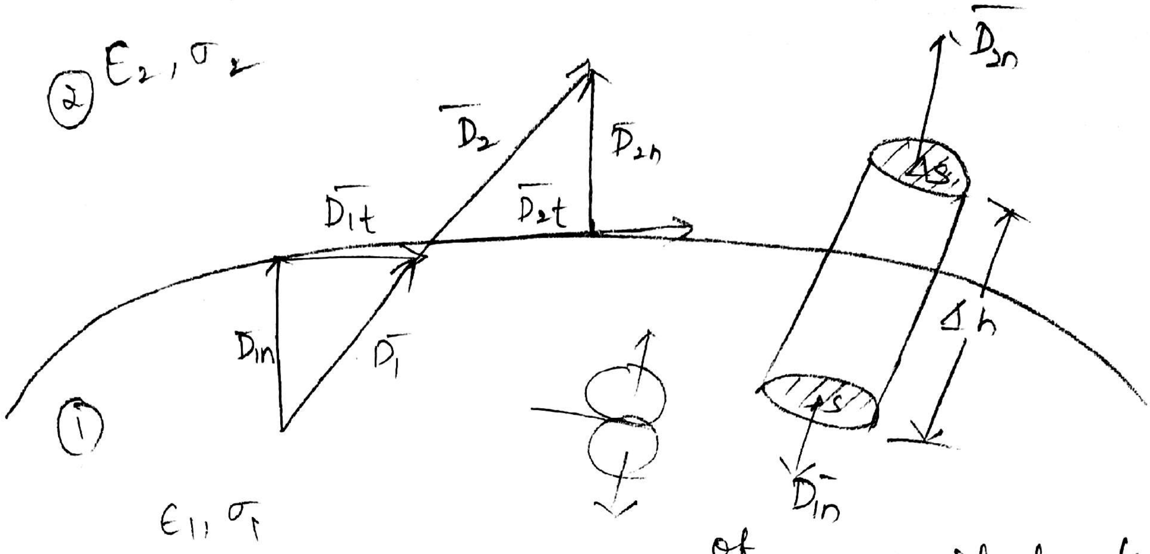
$$\bar{D} = \epsilon\bar{E}$$

$$\bar{E} = \frac{\bar{D}}{\epsilon}$$

$$\bar{E}_{1t} = \bar{E}_{2t}$$

$$\boxed{\frac{\bar{D}_{1t}}{\epsilon_1} = \frac{\bar{D}_{2t}}{\epsilon_2}}$$

The tangential Components of electric flux densities are discontinous.



As $\Delta h \to 0$ The Contribution ~~due~~ of the fields due to ~~Side~~ Sides will be zero.

Now Apply $\iint_S \bar{D}\cdot\bar{ds} = Q_{enclosed}$ to the cylindrical pill box of above figure.

$Q_{enc} = ?$

$\rho_s = \frac{Q}{S} = \frac{\Delta Q}{\Delta S}$

$Q_{enc} = \Delta Q = \rho_s \Delta_s$

$\iint_S \bar{ds} = \Delta s$

~~$= \Delta s \bar{D}_{2n} - \Delta S D_{1n}$~~

$= \Delta S \bar{D}_{2n} - \Delta S \bar{D}_{1n} = \rho_s \Delta S$

$\bar{D}_{2n} - \bar{D}_{1n} = \rho_s$

Let $\rho_s = 0$, $\bar{D}_{2n} = \bar{D}_{1n}$.

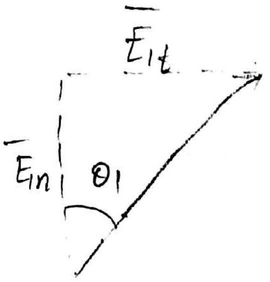
the normal components of electric flux density are continous.

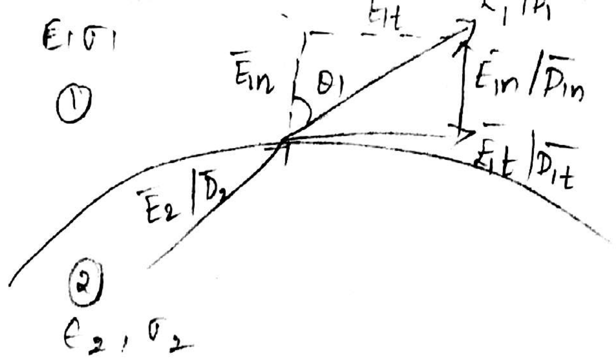
Law of Refraction

Boundary Conditions are used to find refraction of the Electric field across the interface.

Let $\bar{D}_1$ or $\bar{E}_1$ and $\bar{D}_2$ (or) $\bar{E}_2$ makes an angle θ_1, θ_2 w.r.t normal to the interface

$$\sin\theta_1 = \frac{\overline{E_{1t}}}{\overline{E_1}}$$





$$\overline{E_{1t}} = \overline{E_1} \sin\theta_1$$

Similarly $\overline{E_{2t}} = \overline{E_2} \sin\theta_2$

we Know that from boundary conditions $\overline{E_{1t}} = \overline{E_{2t}}$

$$\overline{E_1} \sin\theta_1 = \overline{E_2} \sin\theta_2 \longrightarrow ①$$

Similarly Apply $\cos\theta_1 = \frac{\overline{E_{1n}}}{\overline{E_1}}$

$$\overline{E_{1n}} = \overline{E_1} \cos\theta_1$$

Similary $\overline{E_{2n}} = \overline{E_2} \cos\theta_2$

we know that from boundary conditions

$$\overline{D_{1n}} = \overline{D_{2n}}$$

$$\overline{D} = \epsilon \overline{E}$$

Substitute $\overline{D}$ in above eqn

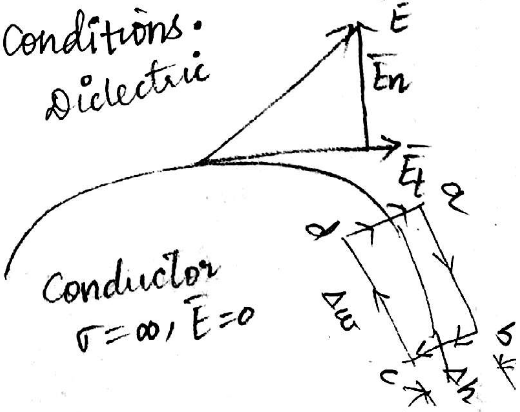
$$\epsilon_1 \overline{E_{1n}} = \epsilon_2 \overline{E_{2n}}$$

$$\epsilon_1 \overline{E_1} \cos\theta_1 = \epsilon_2 \overline{E_2} \cos\theta_2 \longrightarrow ②$$

$$\frac{①}{②} \Rightarrow \frac{1}{\epsilon_1} \tan\theta_1 = \frac{\tan\theta_2}{\epsilon_2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2}}$$

Case- ii) Dielectric - Conductor Boundary Conditions:



Conductor, $\sigma = \infty$

$$\overline{J} = \sigma \overline{E}$$

$$\overline{E} = \frac{\overline{J}}{\sigma}$$

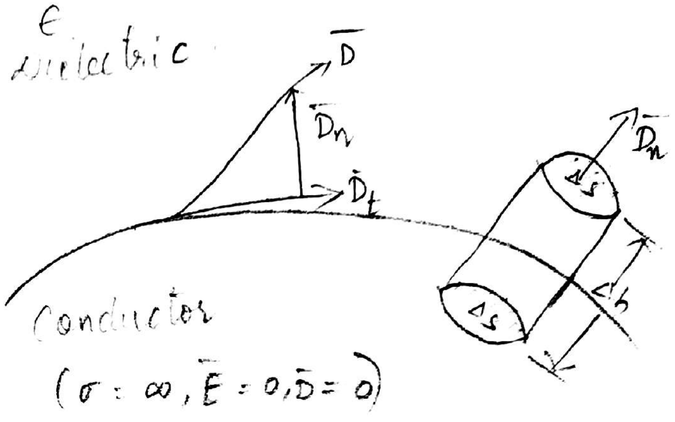
$$\overline{E} = \frac{\overline{J}}{\infty} = 0$$

Apply $\oint_l \bar{E} \cdot \bar{dl} = 0$

$$0 = \bar{E}_t \Delta w - \cancel{\frac{\Delta h}{2}} \bar{E}_n + 0 + 0 + 0 + \cancel{\frac{\Delta h}{2}} \bar{E}_n$$

$\bar{E}_t \Delta w = 0$

$\bar{E}_t = 0.$



Dielectric - conductor:-

Apply $\iint_S \bar{D} \cdot \bar{ds} = Q_{enc}$

$Q_{enc} = ?$

$\rho_s = \frac{Q}{S} = \frac{\Delta Q}{\Delta S}$

$Q_{enc} = \Delta Q = \rho_s \Delta S$

$\int dx = x$
$\int dl = l$
$\iint_S \bar{ds} = S = \Delta S$

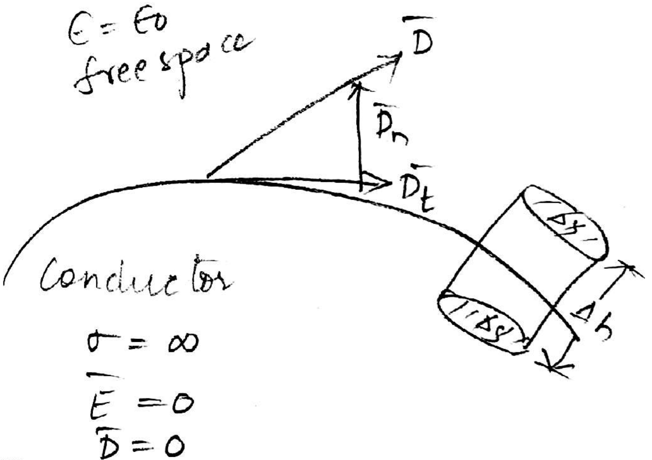
$\iint_S \bar{D} \cdot \bar{ds} = \Delta S \bar{D}_n$

$\iint_S \bar{D} \cdot \bar{ds} = Q_{enc}$

$\cancel{\Delta S} \bar{D}_n = \rho_s \cancel{\Delta S}$

$\rho_s = \bar{D}_n.$

If $\rho_s = 0$, $\bar{D}_n = 0.$



Conductor-free space:-

$\bar{D}_n = 0$, if $\rho_s = 0$

$\bar{E}_t = 0.$

Two extensive Homogeneous isotropic dielectrics meet on plane $Z = 0$, for $z > 0$ $\epsilon_{r_1} = 4$, and for $z < 0$ ϵ_{r_2} is 3 - a uniform electric field intensity $\bar{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ K/m exists for $Z \geq 0$ find 1) $\bar{E}_2$ for $z \leq 0$ Nil

the angles of $\bar{E}$ and $\bar{E}_2$ makes with the inter space

3) The Energy density (J/m3).

Nil

4 The energy density
~~with~~ a side of 1ltr matters centred at (3,4,-7) .

---

① Two extensive homogeneous isotropic Dielectrics meet on plane $z=0$ for $z>0$, $\epsilon_{r1}=4$ and for $z<0$ $\epsilon_{r2}=3$. A uniform electric field intensity $\bar{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ kV/m exist for $z \geq 0$. find

i) E_2 for $z \leq 0$ ii) The angles $\bar{E}_1$ and $\bar{E}_2$ makes with the interface.

iii) The energy density (J/m³) in Both Dielectrics iv) The energy with in a cube of side (2m) Centered at (3, 4, -5)

Note:
Based on the condition given in the question which is seperating the mediums that Component will represent normal component and the remaining two Components are called tangential Components.

$$\bar{E}_2 = \bar{E}_{2t} + \bar{E}_{2n}.$$

Across z-axis boundary z = normal, x, y = tangentia

$$\bar{E}_{1n} = 3\hat{a}_z$$
$$\bar{E}_{1t} = 5\hat{a}_x - 2\hat{a}_y$$

from Boundary condition, we know that

$$\bar{E}_{1t} = \bar{E}_{2t}$$
$$\bar{E}_{2t} = 5\hat{a}_x - 2\hat{a}_y$$

2nd Boundary cond, $\bar{D}_{2n} = \bar{D}_{1n}$ if $\rho_s = 0$

$$\bar{D} = \epsilon \bar{E}.$$

$$\epsilon_2 \overline{E}_{2n} = \epsilon_1 \overline{E}_{1n}$$

$$\overline{E}_{2n} = \frac{\epsilon_1}{\epsilon_2} \overline{E}_{1n}$$

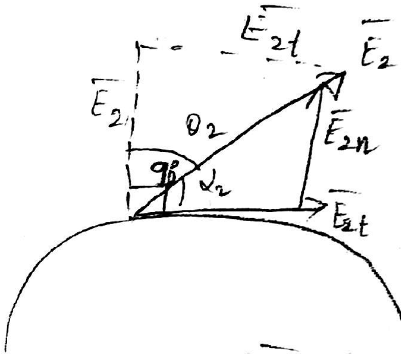
$$= \frac{\cancel{\epsilon_0} 4}{\cancel{\epsilon_0} 3} \overline{E}_{1n}$$

$$= \frac{4}{\cancel{3}} \times \cancel{3} \hat{a_z}$$

$$\overline{E}_{2n} = 4\hat{a_z}.$$

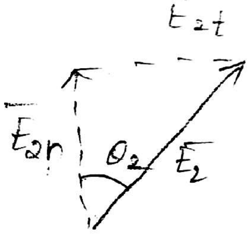
$$\overline{E}_2 = 5\hat{a_x} - 2\hat{a_y} + 4\hat{a_z} \text{ Kv/m}$$

ii) Let α_1, α_2 be the angles of $\overline{E}_1$ and $\overline{E}_2$ make with the interface while θ_1, θ_2 are the angles make with normal to the interface



$$\tan\theta_2 = \frac{|\overline{E}_{2t}|}{|\overline{E}_{2n}|}.$$

$$\overline{E}_2 = \underbrace{5\hat{a_x} - 2\hat{a_y}}_{\overline{E}_{2t}} + \underbrace{4\hat{a_z}}_{\overline{E}_{2n}}$$



$$\tan\theta_2 = \frac{\sqrt{25+4}}{\sqrt{16}}$$

$$\theta_2 = 53.4^\circ$$

$$\tan\theta_2 = \frac{\sqrt{29}}{4}$$

$$\alpha_2 = 90^\circ - \theta_2$$

$$= 90 - 53.4^\circ$$

$$\alpha_2 = 36.6^\circ$$

$$\tan\theta_1 = \frac{|\overline{E}_{1t}|}{|\overline{E}_{1n}|}$$

$$\overline{E}_1 = 5\hat{a_x} - 2\hat{a_y} + 3\hat{a_z}$$

$$\tan\theta_1 = \frac{\sqrt{25+4}}{\sqrt{9}} = \frac{\sqrt{29}}{3} = 60.9^\circ$$

$$\theta_1 = 60.9^\circ$$

$\alpha_1 = 90^\circ - \theta_1$

$= 90^\circ - 60.9^\circ$

$\alpha_1 = 21.1^\circ$

Verify the answer we can use the concept of law of refraction

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2}$$

$$\frac{\tan(60.9^\circ)}{\tan(53.4^\circ)} = \frac{4}{3}$$

$$\frac{1.8}{1.34} = \frac{4}{3}$$

$$1.33 = 1.33$$

iii) As we know that Energy density in medium ① is given as

$\overline{E_1} = 5\hat{a_x} - 2\hat{a_y} + 3\hat{a_z}$.

$\epsilon_1 = \epsilon_0 \epsilon_{r1}$

$$w_{E1} = \frac{1}{2}\epsilon_1 |\overline{E_1}|^2$$

$$= \frac{1}{2}(4)\left(\sqrt{25+4+9}\right)^2 \times \frac{10^{-9}}{36\pi}$$

$$= 6.7\times10^{-10}$$

$$6.7\times10^{+4}\times10^{-10} \text{ (or) } 6.7\times10^{-1}\times10^{-10}$$

$$w_{E1} = 0.61\,nJ/m^3$$

$\overline{E_2} = 5\hat{a_x} - 2\hat{a_y} + 4\hat{a_z}$

$|\overline{E_2}| = \sqrt{25+4+16}$

$= 3\sqrt{5}$.

$$w_{E_2} = \frac{1}{2}\epsilon_2 |\overline{E_2}|^2$$

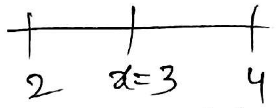
$$= \frac{1}{2}\times\frac{10^{-9}}{36\pi}\times3\times(3\sqrt{5})^2$$

$$w_{E_2} = 0.597\,nJ/m^3$$

At the center $(3,4,-5)$ of the cube of side 2m which lies in second region.

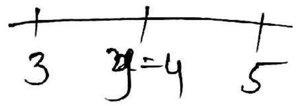
$$w_E = \frac{dW_E}{dv}$$

$$W_{E_2} = \int_v w_{E_2}\, dv.$$



$2 \le x \le 4$

$$= 0.597n \int_x \int_y \int_z dx\, dy\, dz$$



$$= 0.597\times10^{-9}\, (x)_2^4\, (y)_3^5\, (z)_{-6}^{-4}$$

$3 \le y \le 5$

$W_E = 4.776\times10^{-9}$. J

$-6 \le z \le -4$

2) The region $y<0$ contains a dielectric material for which $\epsilon_{r1} = 2$ and the region $y>0$ contains a dielectric material for which $\epsilon_{r2} = 4$. If $\bar{E}_1 = -3\hat{a}_x + 5\hat{a}_y + 7\hat{a}_z$ V/m find $\bar{E}_2$ and $\bar{D}_2$ in medium '2'.

we know that $\bar{E}_{1t} = \bar{E}_{2t}$.

$$\bar{E}_{1t} = -3\hat{a}_x + 7\hat{a}_z = \bar{E}_{2t}$$

$$\bar{D}_{2n} = \bar{D}_{1n} \quad \text{if } \rho_s = 0$$

$$\bar{D}_{2n} = \epsilon_2 \bar{E}_{2n}$$

In region 1 $z<0$ and in region 2 $z>0$, $\sigma_1 = \sigma_2 = 0$.

if $\bar{E}_1 = \hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$ V/m. find $\bar{E}_2$ and $\bar{D}_2$.

$\epsilon_{r_1} = \epsilon_{r_2} = 1$.

z - Norm

x, y - 1

A homogeneous dielectric medium whose $\epsilon_r = 2.5$ fills region $x<0$ while region 2 $x>0$ is free space. If $\bar{D}_1 = 12\hat{a}_x - 10\hat{a}_y + 4\hat{a}_z$ nc/m² find $\bar{D}_2$ and θ_2.

$\epsilon_{r1} = 2.5, \epsilon_{r2} = 1.$

WKT $\bar{E}_{1t} = \bar{E}_{2t}$.

$$\bar{D} = \epsilon \bar{E}$$

$$\bar{E} = \frac{\bar{D}}{\epsilon}$$

$$\frac{\bar{D}_{1t}}{\epsilon_1} = \frac{\bar{D}_{2t}}{\epsilon_2}$$

$$\bar{D}_{2t} = \frac{\epsilon_2}{\epsilon_1} \times \bar{D}_{1t}$$

$$= \frac{\cancel{\epsilon_0}\epsilon_{r2}}{\cancel{\epsilon_0}\epsilon_{r1}} \times [-10\hat{a}_y + 4\hat{a}_z]$$

$$= \frac{1}{2.5} \times [-10\hat{a}_y + 4\hat{a}_z]$$

$$= 0.4[-10\hat{a}_y + 4\hat{a}_z]$$

$$= -4\hat{a}_y + 1.6\hat{a}_z.$$

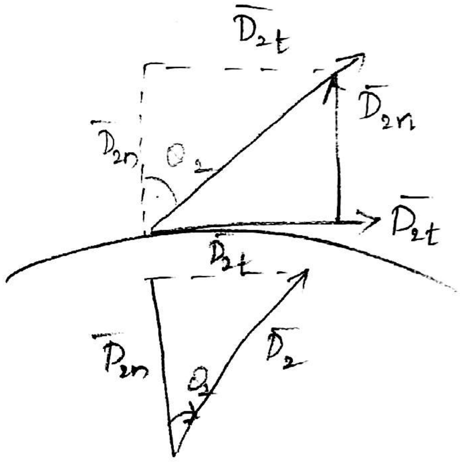
$$\bar{D}_2 = \bar{D}_{2t} + \bar{D}_{2n}$$

$$\bar{D}_{2n} = \bar{D}_{1n} = 12\hat{a}_x.$$

$$\bar{D}_2 = 12\hat{a}_x - 4\hat{a}_y + 1.6\hat{a}_z \text{ nc/m}^2.$$

$$\tan\theta_2 = \frac{|\bar{D}_{2t}|}{|\bar{D}_{2n}|} = \frac{\sqrt{16 + (1.6)^2}}{\sqrt{144}}$$

$$\theta_2 = 19.75°$$



Magnetic Boundary Condition:-

The Guass law for magneto static fields is given as

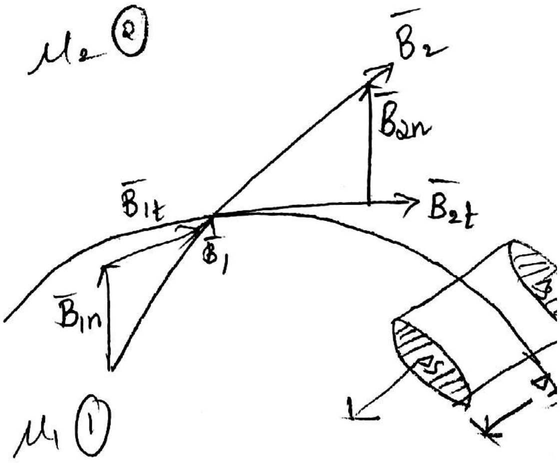
$$\oiint \bar{B} \cdot \bar{ds} = 0$$

As $\Delta h \to 0$

$$0 = \Delta S \bar{B}_{2n} - \Delta S \bar{B}_{1n}$$

$$0 = \Delta S[\bar{B}_{2n} - \bar{B}_{1n}]$$

$$\boxed{\bar{B}_{2n} = \bar{B}_{1n}}$$



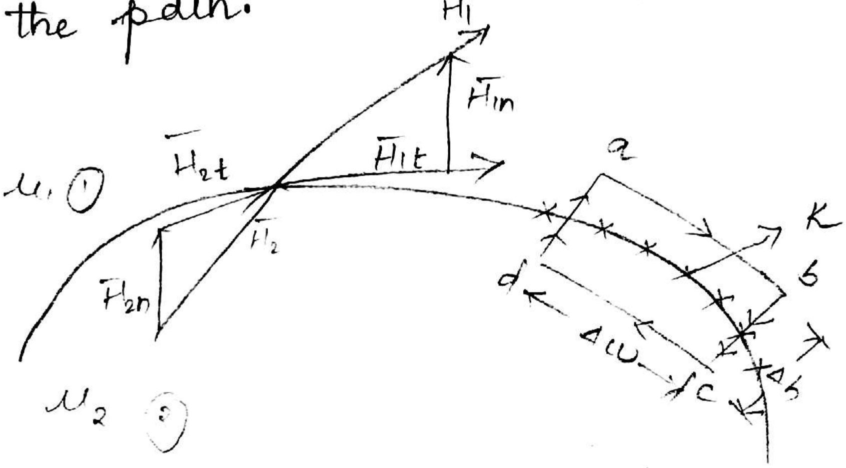
Thus the normal components of Magnetic flux densities are continous.

~~apply~~ $\bar{B}_{2n} = \bar{B}_{1n}$

WKT $\bar{B} = \mu \bar{H}$

$\boxed{\mu_2 \bar{H}_{2n} = \mu_1 \bar{H}_{1n}}$ → discontinous.

Apply Ampere's circuit law on the rectangular loop a, b, c, d, a where the surface current 'k' on the boundary is assume normal to the path.



Apply $\oint_l \bar{H}\cdot d\bar{l} = I_{enc}$

$I_{enc} = k\,\Delta w$

$$\oint_l \bar{H}\cdot d\bar{l} = \bar{H}_{1t}\,\Delta w - \frac{\Delta h}{2}\bar{H}_{1n} - \cancel{\frac{\Delta h}{2}\bar{H}_{2n}} - \Delta w\,\bar{H}_{2t}$$

$$+ \cancel{\frac{\Delta h}{2}\bar{H}_{2n}} + \cancel{\frac{\Delta h}{2}\bar{H}_{1n}}$$

$$\oint_l \bar{H}\cdot d\bar{l} = \bar{H}_{1t}\,\Delta w - \bar{H}_{2t}\,\Delta w$$

$$(\bar{H}_{1t} - \bar{H}_{2t})\,\Delta w = k\,\Delta w$$

$$\bar{H}_{1t} - \bar{H}_{2t} = k$$

In general the above eqn can be written as

$$[\bar{H}_1 - \bar{H}_2] \times \bar{a}_n = k.$$

If $k = 0$ then $\boxed{\bar{H}_{1t} = \bar{H}_{2t}}$ → continious

$\bar{B} = \mu \bar{H}$, $\bar{H} = \frac{\bar{B}}{\mu}$ $\boxed{\frac{\bar{B}_{1t}}{\mu_1} = \frac{\bar{B}_{2t}}{\mu_2}}$ → discontinous

EBC	MBC
1) $\bar{E}_{1t} = \bar{E}_{2t}$	1) $\bar{H}_{1t} = \bar{H}_{2t}$ if $K=0$
② $\bar{D}_{2n} = \bar{D}_{1n}$ if $\rho_s = 0$	2) $\bar{B}_{2n} = \bar{B}_{1n}$.

Two Homogenous and linear and isotropic medium have an interface at $x=0$ for $x<0$ describes a medium 1 and for $x>0$ describes medium 2. If $\mu_{r_1} = 2$, $\mu_{r_2} = 5$ and $\bar{H}_1 = 150\hat{a}_x - 400\hat{a}_y + 250\hat{a}_z$ ampere permeter find

i) Magnetic field intensity in medium ②

ii) Magnetic flux density in medium ①

iii) Magnetic fluxe densiry in medum ②

$$\text{Given } \bar{H}_1 = \underbrace{150\hat{a}_x}_{\bar{H}_{1n}} \underbrace{-400\hat{a}_y + 250\hat{a}_z}_{\bar{H}_{1t}} \text{ A/m}$$

1st BC $\Rightarrow \bar{H}_{1t} = \bar{H}_{2t}$ if $k=0$

$$\bar{H}_{2t} = -400\hat{a}_y + 250\hat{a}_z.$$

$$\bar{H}_2 = \bar{H}_{2t} + \bar{H}_{2n}.$$

2nd BC $\bar{B}_{2n} = \bar{B}_{1n}$.

$\bar{B} = \mu\bar{H}$ $\quad \mu_2 \bar{H}_{2n} = \mu_1 \bar{H}_{1n}$.

$$\bar{H}_{2n} = \frac{\mu_1}{\mu_2} \bar{H}_{1n}$$

$$= \frac{\cancel{\mu_0}\mu_{r_1}}{\cancel{\mu_0}\mu_{r_2}} 150\hat{a}_x$$

$$= \frac{2}{5} \overset{30}{\cancel{150}} \hat{a}_x$$

$$\bar{H}_{2n} = 60\hat{a}_x.$$

$$\bar{H} = \bar{H}_{2t} + \bar{H}_{2n} = -400\hat{a}_y + 250\hat{a}_z + 60\hat{a}_x \text{ A/m}$$

$$\overline{B_2} = \mu_2 \overline{H_2}$$

$$= \mu_{r_2} \mu_0 [\overline{H_2}) = 4\pi \times 10^{-7} \times 5 \times (-400\hat{a_y} + 250\hat{a}_z + 60\hat{a_x}) \, wb/m$$

$$\overline{B_1} = \mu_1 \overline{H_1}$$

$$= \mu_{r_1} \mu_0 [\overline{H_1}]$$

$$\overline{B_1} = 2 \times 4\pi \times 10^{-7} \times [150\hat{a_x} - 400\hat{a_y} + 250\hat{a}_z) \, wb/m^2$$

for $z < 0$ defines region 1 and for $z > 0$ defines region 2 region 1 is characterised by $\mu_{r_1} = 2$, $\mu_{r_2} = 4$. If magnetic field in ~~medium~~ region 1 is given by $\overline{H_1} = 4\hat{a_x}\ 4.5\hat{a_y} - 3\hat{a}_z$ A/m find magnetic field intensity in medium ~~H~~ 2 and also Calculate $|\overline{H_2}|$.

3) ~~Sol~~ The xy plane acts as the interface between two different mediums. medium 1 ($z<0$) is filled with a material whose $\mu_r = 6$. and medium 2 ($z>0$) is filled with a material whose $\mu_r = 4$ if the interface carries a current of $\frac{1}{\mu_0}\hat{a}_y$ mA/m and $\bar{B}_2 = 5\hat{a}_x + 8\hat{a}_z$ mwb/m². find $\bar{H}_1$ and $\bar{B}_1$

$$\bar{B}_2 = \underbrace{5\hat{a}_x}_{\bar{B}_{2t}} + \underbrace{8\hat{a}_z}_{\bar{B}_{2n}}$$

from 2nd Boundary condition $\bar{B}_{2n} = \bar{B}_{1n}$.

$$\bar{B}_{2n} = \bar{B}_{1n} = 8\hat{a}_z$$

$b = \mu H$.

$H = \frac{b}{\mu}$

$$\bar{B}_1 = \bar{B}_{1t} + \bar{B}_{1n}.$$

from $\bar{B}_{1t} = ?$

1st Boundary condition.

$$(\bar{H}_1 - \bar{H}_2) \times \hat{a}_n = K.$$

$$\left(\frac{[B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z]}{6\mu_0} - \frac{5\hat{a}_x + 8\hat{a}_z}{4\mu_0}\right) \times \hat{a}_z = \frac{1}{\mu_0}\hat{a}_y.$$

$\bar{B} = \mu\bar{H}$

$\bar{B}_2 = \mu_2\bar{H}_2$.

$$\bar{H}_2 = \frac{\bar{B}_2}{\mu_2} = \frac{5\hat{a}_x + 8\hat{a}_z}{\mu_0(4)}$$

Similarly $\bar{H}_1 = \frac{\bar{B}_1}{\mu_1}$.

$$\bar{B}_1 = \frac{B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z}{\mu_0 6}$$

$$\left(\frac{B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z}{6\mu_0} - \frac{5\hat{a}_x + 8\hat{a}_z}{4\mu_0}\right) \times \hat{a}_z = \frac{1}{\mu_0}\hat{a}_y$$

$$\Rightarrow \frac{-B_x\hat{a}_y + B_y\hat{a}_x + 0}{6} = \hat{a}_y + \frac{-5\hat{a}_y + 0}{4}.$$

$$\frac{-B_x\hat{a}_y + B_y\hat{a}_x}{6} = \hat{a}_y + \frac{-5\hat{a}_y}{4}.$$

compare on both sides

$\hat{a}_y$

$$\frac{-B_x}{6} = 1 - \frac{5}{4}$$

$\hat{a}_x$

$$\frac{B_y}{6} = 0$$

$$\frac{-B_x}{6} = \frac{-1}{4}$$

$$B_y = 0.$$

$$B_x = \frac{6}{4}$$

$$B_x = \frac{3}{2}$$

$$B_x = 1.5$$

$$= \underbrace{B_x \hat{a_x} + B_y}\, \hat{a_y} + \underbrace{B_z}_{n} \hat{a_z}.$$

$$= 1.5 \hat{a_x} + 0 + 8 \overset{z}{a_z} \quad m\,wb/m^2$$

$$\overline{B_1} = \mu \overline{H_1}$$

$$H_1 = \frac{\overline{B_1}}{\mu_1} = \frac{1.5 \hat{a_x} + 8 a \hat{z} \times w \cdot x\, wb/m^2}{\mu_0 \mu_r}$$

$$= \frac{1.5 \hat{a_x} + 8 \hat{a_z} \quad A/m}{4\pi \times 10^{-7} \times 6}$$

=

t parallel plate Capacitor with a ~~light~~ plate area of -cm² and plates seperation of three (3 mm) has a Voltage 50 sin(10³t) V is applied to its plate. calcul isplacement current by assuming $\varepsilon = 2\varepsilon_0$.

Given $\varepsilon = 2\varepsilon_0$

$$\frac{\partial \overline{D}}{\partial t} = ?$$

$$\overline{D} = \varepsilon \overline{E}.$$

$$= 2 \varepsilon_0 \overline{E}.$$

$$D =$$

$$\overline{E} = \frac{v}{d} \quad V/m$$

$$\overline{E} = \frac{50 \sin(10^3 t)}{3 \times 10^{-3}}$$

2/7/17

UNIT-IV

WAVE PROPAGATION

Wave is a means of transport of infoumation, the first appliation of maxwells eqn is related to wave peopagation.

Maxwells equations in time hormonic foem:-

We know that the time haemonic in electric field Component is given as

$$\bar{E}_s = \underset{\overset{\uparrow}{\text{Amplitude}}}{E_0}\ \underbrace{e^{j\omega t}}_{\rightarrow\ \text{phase}}$$

Consider first Maxwells eqn $\nabla \cdot \bar{D} = \rho_v$

$$\nabla \cdot \epsilon \bar{E}_s = \rho_v$$

$$\nabla \cdot \bar{E}_s = \frac{\rho_v}{\epsilon} \longrightarrow ①$$

Consider second Maxwells eqn $\nabla \cdot \bar{B} = 0$

$$\bar{B} = \mu \bar{H}$$

$$\bar{H}_s = H_0 e^{j\omega t}$$

$$\nabla \cdot \mu H_0 e^{j\omega t} = 0$$

$$\nabla \cdot \bar{H} = 0 \longrightarrow ②$$

Consider 3rd Maxwells eqn $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$

$$\bar{J} = \sigma \bar{E}_s = \sigma E_0 e^{j\omega t}$$

$$\bar{D} = \epsilon \bar{E}_s$$

$$\frac{\partial \bar{D}}{\partial t} = \frac{\partial}{\partial t} \left(\epsilon \left[E_0 e^{j\omega t} \right] \right)$$

In phasor form, $\frac{\partial}{\partial t} = j\omega$

$$\frac{\partial \bar{D}}{\partial t} = j\omega \epsilon \left[E_0 e^{j\omega t} \right]$$

Substitute $\bar{J}$, $\frac{\partial \bar{D}}{\partial t}$ values in

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla\times\bar{H} = \sigma E_0 e^{j\omega t} + j\omega\epsilon\ E_0 e^{j\omega t}$$

$$\nabla\times\bar{H} = E_0\ e^{j\omega t}\{\sigma + j\omega\epsilon\}$$

$$\nabla\times\bar{H} = \bar{E}_s\ [\sigma + j\omega\epsilon] \longrightarrow ③$$

Consider 4th Maxwells eqn $\nabla\times\bar{E} = \frac{-\partial\bar{B}}{\partial t}$

$$\nabla\times\bar{E}_s = -j\omega\mu\bar{H}_s$$

$$\nabla\cdot\bar{E}_s = \frac{\rho_v}{\epsilon}$$

$$\nabla\cdot\bar{H} = 0$$

$$\nabla\times\bar{H}_s = \bar{E}_s\ [\sigma + j\omega\epsilon]$$

$$\nabla\times\bar{E}_s = -j\omega\mu\bar{H}_s$$

Types of mediums:-

1. loss less dielectric (or) perfect dielectric:-
Conductivity $\sigma = 0$ $(\sigma << \omega\epsilon)$, $\epsilon = \epsilon_0\epsilon_r$, $\mu = \mu_0\mu_r$

2. lossy dielectric medium:- (partially conducting medium)
Conductivity $\sigma \neq 0$,
$\mu = \mu_0\mu_r$, $\epsilon = \epsilon_0\epsilon_r$

3. Free space medium (or) air medium:-
Conductivity $\sigma = 0$, $\mu = \mu_0$, $\epsilon = \epsilon_0$

4. perfect conductor medium:-
conductivity $\sigma = \infty$ $[\sigma >> \omega\epsilon]$
$\mu = \mu_0\mu_r$
$\epsilon = \epsilon_0$

Wave propagation in lossy dielectrics:-

Lossy dielectric is also called as partially conducting medium i.e $\sigma \neq 0$.

In time harmonic form the first Maxwells eqn is given as $\nabla \cdot \bar{E}_s = \frac{\rho_v}{\epsilon}$

Assume there is no change in medium (or) the medium is charge-free i.e $\rho_v = 0$

$$\nabla \cdot \bar{E}_s = 0 \longrightarrow ①$$

$$\nabla \cdot \bar{H}_s = 0 \longrightarrow ②$$

$$\nabla \times \bar{H}_s = \bar{E}_s\,[\sigma + j\omega\epsilon] \longrightarrow ③$$

$$\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s \longrightarrow ④$$

Consider eqn ④ $\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s$

for the above eqn apply the vector identity $\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla$

Apply curl to eqn ④

$$\nabla \times (\nabla \times \bar{E}_s) = -j\omega\mu\, \nabla \times \bar{H}_s$$

$$\nabla(\nabla \cdot \bar{E}_s) - \nabla^2 \bar{E}_s = -j\omega\mu\, \nabla \times \bar{H}_s \qquad (\nabla \cdot \bar{E}_s = 0)$$

from eqn ① $-\nabla^2 \bar{E}_s = -j\omega\mu\, \nabla \times \bar{H}_s$

$$\nabla^2 \bar{E}_s = j\omega\mu\,[\sigma + j\omega\epsilon]\,\bar{E}_s \quad [\text{from eqn ③}]$$

$$\frac{\partial^2}{\partial x^2}\bar{E}_s + \frac{\partial^2}{\partial y^2}\bar{E}_s + \frac{\partial^2}{\partial z^2}\bar{E}_s = j\omega\mu\,[\sigma + j\omega\epsilon]\,\bar{E}$$

Assume the wave is travelling in z-direction having only z-component

$$\frac{\partial^2}{\partial z^2}\bar{E}_s = j\omega\mu\,[\sigma + j\omega\epsilon]\,\bar{E}_s$$

$$\frac{\partial^2 \bar{E}_s}{\partial z^2} = \gamma^2 \bar{E}_s \quad \text{where } \gamma^2 = j\omega\mu\,[\sigma + j\omega\epsilon]$$

= propagation constant of a wave

Now the above eqn can be written as

$$\frac{\partial^2 \bar{E}_s}{\partial z^2} - \gamma^2 \bar{E}_s = 0$$

$$\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0$$

This eqn is called Helm holtz eqn (or) wave eqn for electric field.

The above eqn is in the form of 2nd order differential eqn

$$[D^2 - \gamma^2)\,\overline{E}_s = 0$$

Auxilary Eqn, $m^2 - \gamma^2 = 0$

$$m = \pm\gamma$$

$\therefore$ The roots are real and unequal

$\therefore$ The general Solution is $\overline{E_s} = E_0 e^{-\gamma z} + E_0' e^{\gamma z}$

The term $E_0 e^{-\gamma z}$ is called as wave travelling in positive direction.

The term $E_0' e^{\gamma z}$ is called as wave travelling in negative z direction

Acc to our assumption E_0' is zero

$$\therefore \overline{E}_s = \overline{E}_0 e^{-\gamma z}$$

The above eqn can also be written as

$$\overline{E}(\underset{\text{space}}{z}, \underset{\text{time}}{t}) = E_0 e^{-\gamma z} e^{j\omega t} \hat{a_x}$$

$$= Re\left\{E_0 e^{-\alpha z} e^{-j\beta x} e^{j\omega t}\right\} \hat{a_x}$$

$$= Re\left\{E_0 e^{-\alpha z} e^{j(\omega t - \beta z)}\right\} \hat{a_x}$$

$$= Re\left\{E_0 e^{-\alpha z} \cos(\omega t - \beta z) + j \sin(\omega t - \beta z)\right\} \hat{a_x}$$

$\overline{E}(z,t) =$ ~~Re~~ $\left\{\underbrace{E_0 e^{-\alpha z}}_{\text{amp}} \underbrace{\cos(\omega t - \beta z)}_{\text{phase}}\right\} \hat{a_x}$

We know that $\gamma = \alpha + j\beta$

$$|\gamma| = \sqrt{\alpha^2 + \beta^2}$$

$$|\gamma|^2 = \alpha^2 + \beta^2 \longrightarrow ⑤$$

from the previous part $\gamma^2 = j\omega\mu[\sigma + j\omega\epsilon]$

$$|\gamma^2| = \sqrt{(\omega\mu)^2}\left[\sqrt{\sigma^2 + \omega^2\epsilon^2}\right]$$

$$|\gamma^2| = \omega\mu\left[\sqrt{\sigma^2+\omega^2\epsilon^2}\right]$$

$$|\gamma^2| = \omega\mu\sqrt{\omega^2\epsilon^2\left[1+\frac{\sigma^2}{\omega^2\epsilon^2}\right]}$$

$$= \omega\mu . \omega\epsilon\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}$$

$$|\gamma^2| = \omega^2\mu\epsilon\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} \longrightarrow$$ ⑥

⑤ = ⑥ $\Rightarrow$ $\alpha^2+\beta^2 = \omega^2\mu\epsilon\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} \longrightarrow$ ⑦

we know that $\gamma = \alpha+j\beta$

Square on both sides.

$$\gamma^2 = \alpha^2-\beta^2+2j\alpha\beta$$

$$Re\{\gamma^2\} = \alpha^2-\beta^2$$

But $\gamma^2 = j\omega\mu[\sigma+j\omega\epsilon]$

$$\alpha^2-\beta^2 = Re\{\gamma^2\} = -\omega^2\mu\epsilon \longrightarrow$$ ⑧

⑦ + ⑧ $\Rightarrow$ $\alpha^2+\beta^2 = \omega^2\mu\epsilon\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}$

$$\alpha^2-\beta^2 = -\omega^2\mu\epsilon$$

$+ \quad + \qquad +$

$$2\alpha^2 = \omega^2\mu\epsilon\left[\sqrt{1+\frac{\sigma^2}{\epsilon^2\omega^2}}-1\right]$$

$$\alpha^2 = \frac{1}{2}\omega^2\mu\epsilon\left[\sqrt{1+\frac{\sigma^2}{\epsilon^2\omega^2}}-1\right]$$

$$\alpha = \sqrt{\frac{\omega^2\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\epsilon^2\omega^2}}-1\right]}$$

Sub the value of α^2 in eqn ⑧

$$\alpha^2-\beta^2 = -\omega^2\mu\epsilon$$

$$\beta^2 = \alpha^2+\omega^2\mu\epsilon = \frac{1}{2}\omega^2\mu\epsilon\left[\sqrt{1+\frac{\sigma^2}{\epsilon^2\omega^2}}-1\right]+\omega^2\mu\epsilon$$

$$\beta^2 = \omega^2\mu\epsilon\left[\frac{1}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}-1\right] + \frac{2\omega^2\mu\epsilon}{2}$$

$$\beta^2 = \frac{\omega^2\mu\epsilon}{2}\left[\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}-1\right]+2\right]$$

$$\beta^2 = \frac{\omega^2\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}+1\right]$$

$$\beta = \sqrt{\frac{\omega^2\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}+1\right]}$$

from third Maxwell's eqn $\nabla\times\bar{H}_s = \bar{E}_s\left[\sigma+j\omega\epsilon\right]$

Apply Curl product on both sides to the above eqn.

$$\nabla\times(\nabla\times\bar{H}_s) = [\sigma+j\omega\epsilon]\,\nabla\times\bar{E}_s$$

W.K.T from Vector identity $\nabla\times\nabla\times\bar{A} = \nabla(\nabla\cdot A)-\nabla^2\bar{A}.$

then $\nabla\times(\nabla\times\bar{H}_s) =$

$$\nabla(\nabla\cdot\bar{H}_s)-\nabla^2\bar{H}_s = [\sigma+j\omega\epsilon]\,\nabla\times\bar{E}_s.$$

$$\nabla(\nabla\cdot\bar{H}_s)-\nabla^2\bar{H}_s = [\sigma+j\omega\epsilon][-j\omega\mu\bar{H}_s] \rightarrow ④$$

$$\cancel{-}\nabla^2\bar{H}_s = +j\omega\mu\bar{H}_s\,[\sigma+j\omega\epsilon]$$

$$+\nabla^2\bar{H}_s = \bar{H}_s\gamma^2$$

$$\nabla^2\bar{H}_s-\gamma^2\bar{H}_s = 0$$

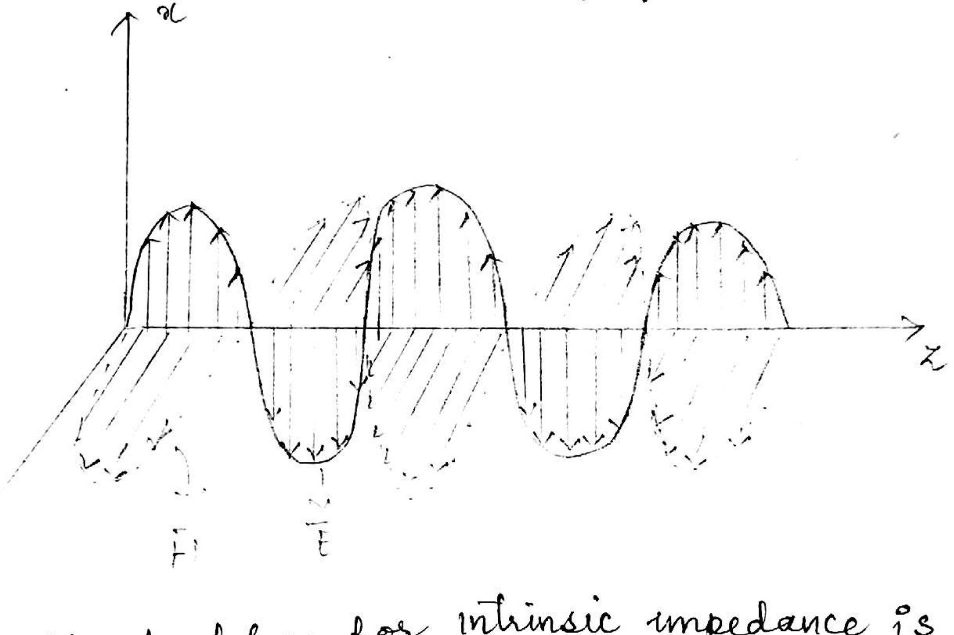
This is called as helmholtz Eqn (or) wave Eqn in magnetic field.

If we apply the same Analysis then we can get finally

$$\bar{H}(z,t) = H_0 e^{-\alpha z}\cos(\omega t-\beta z)\,\hat{a}_y.$$

Define Uniformplane wave propagation?

At each instant of time both electric fields and magnetic fields are ⊥lr to each other and these field Components are perpendicular to wave propagation This is Called as uniform plain wave propagation.



The standard eqn for intrinsic impedance is represented as η: η = ratio of $\frac{E}{H}$. $\eta = \frac{E}{H}\,\Omega$

$\eta = 377\Omega$ for free space
$= 120\pi$.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon\left[1+\frac{\sigma}{j\omega\epsilon}\right.}}$$

$$= \sqrt{\frac{\mu/\epsilon}{\left[(1+\frac{\sigma}{j\omega\epsilon}\right]}}$$

In general $\eta = \underset{\text{amp}}{|\eta|}\, e^{i\underset{\text{phase}}{\theta}\eta}$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\sqrt{\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}}} \qquad \theta_\eta = \frac{1}{\tan^{-1}\left[\frac{\frac{-\sigma}{\omega\epsilon}}{1}\right]}$$

$$= -\tan^{-1}\left[\frac{-\sigma}{\omega\epsilon}\right]$$

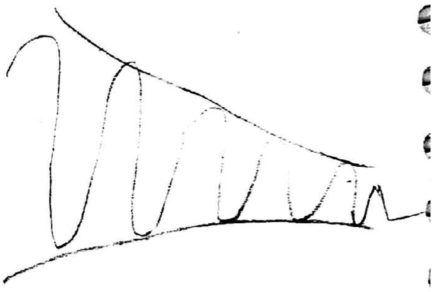
$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}, \quad \theta_\eta = \tan^{-1}\left[\frac{\sigma}{\omega\epsilon}\right]$$

$$\tan(2\theta_\eta) = \frac{\sigma}{\omega\epsilon}$$

WKT $\eta = \frac{E}{H} = \frac{\bar{E}(z,t)}{\bar{H}(z,t)}$

$$\bar{H}(z,t) = \frac{\bar{E}(z,t)}{\eta}$$

$$\bar{H}(z,t) = \frac{1}{\eta} E_0 e^{-\alpha z} e^{j(\omega t-\beta z)} \hat{a_y}$$



Where α = attenuation constant

1 Neper = $20\log_{10}^{e}$ = 8.686 dB

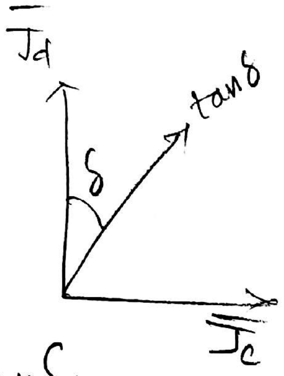
from ③ $\nabla \times \bar{H}_s = \bar{E}_s[\sigma + j\omega\epsilon]$

$$= \underset{\substack{\downarrow \\ \bar{J}_c}}{\sigma\bar{E}_s} + \underset{\substack{\downarrow \\ \bar{J}_d}}{j\omega\epsilon\bar{E}_s}$$

$\bar{J}_c$ ↓ Conduction current density

$\bar{J}_d$ displacement current density

Divide $\bar{J}_c$ and $\bar{J}_d$ i.e $\frac{|\bar{J}_c|}{|\bar{J}_d|} = \frac{\sigma\bar{E}_s}{\omega\epsilon\bar{E}_s}$



$$\frac{|\bar{J}_c|}{|\bar{J}_d|} = \frac{\sigma}{\omega\epsilon} = \underset{\substack{\downarrow \\ \text{loss tangent}}}{\tan\delta}$$

A medium is loss less if $\tan\delta$ is very small and the medium is acting as a good conductor if $\tan\delta$ is very hig Characteristic behaviour of a wave not only depends on σ, μ, ϵ

but also depends on frequency. A medium which behave as a good conductor at low frequency may behave as dielectric at high frequency

from eqn ③ $\Rightarrow \nabla \times H_s = E_s\,[\sigma + j\omega\epsilon]$

$$= E_s\, j\omega\epsilon\left[1 + \frac{\sigma}{j\omega\epsilon}\right]$$

Where $\epsilon_c = \epsilon\left[1 + \frac{\sigma}{j\omega\epsilon}\right]$

$$= \epsilon + \frac{\sigma}{j\omega}$$

Let $\epsilon_c = \underset{\epsilon'}{\underset{\downarrow}{\epsilon}} - j\,\underset{\epsilon''}{\underset{\Downarrow}{\frac{\sigma}{\omega}}}$

$$\tan\delta = \frac{\sigma}{\omega\epsilon} = \frac{\epsilon''}{\epsilon'} = \frac{\epsilon''}{\epsilon}.$$

Plane waves in lossless dielectric medium :-

$\sigma = 0,\ \mu = \mu_0\mu_r,\ \epsilon = \epsilon_0\epsilon_r,\ \sigma << \omega\epsilon$

WKT from previous concept

$$\alpha = \sqrt{\frac{\omega^2\mu\epsilon}{2}\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}} - 1}$$

$$\alpha = 0.$$

$$\beta = \sqrt{\frac{\omega^2\mu\epsilon}{2}\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}} + 1} \Rightarrow \sqrt{\frac{\sqrt{\omega^2\mu\epsilon}}{\sqrt{2}}\sqrt{2}}$$

$$\beta = \sqrt{\frac{\omega^2\mu\epsilon}{2}\cdot 2} = \omega\sqrt{\mu\epsilon}$$

$\tan\delta = \frac{\sigma}{\omega\epsilon}$

$\tan\delta = 0.$

$$\eta = \frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} = \frac{\sqrt{\mu/\epsilon}}{[1+0]^{1/4}}$$

$$= \sqrt{\frac{\mu}{\epsilon}}\ \angle 0^\circ$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}}$$

$$v_p = \frac{1}{\sqrt{\mu\epsilon}}$$

**

$$\boxed{\alpha = 0\,\text{Np},\ \beta = \omega\sqrt{\mu\epsilon}\ \text{rad},\ \eta = \sqrt{\frac{\mu}{\epsilon}}\,\Omega,\ v_p = \frac{1}{\sqrt{\mu\epsilon}}\ \text{m/sec}}$$

plane waves in free space:-

$$\sigma = 0,\ \mu = \mu_0,\ \epsilon = \epsilon_0$$

$$\alpha = \sqrt{\frac{\omega^2\mu\epsilon}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}-1}$$

$$\alpha = 0$$

$$\beta = \sqrt{\frac{\omega^2\mu\epsilon}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}+1}$$

$$\beta = \sqrt{\frac{\omega^2\mu\epsilon}{2}\cdot 2}$$

$$= \omega\sqrt{\mu\epsilon}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0}$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8\ \text{m/sec}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\angle 0^\circ \Rightarrow 377\,\Omega\angle 0^\circ$$

**

$$\eta = 377\,\Omega\ \text{(or)}\ 120\pi\,\Omega\ \angle 0^\circ$$

$$\boxed{\alpha = 0\,\text{NP},\ \beta = \omega\sqrt{\mu_0\epsilon_0}\ \text{rad},\ \eta = \sqrt{\frac{\mu_0}{\epsilon_0}}\,\Omega;\ v_p = \frac{1}{\sqrt{\mu_0\epsilon_0}}\ \text{m/sec}}$$

A lossy dielectric has intrinsic impedance of $200\angle 30^\circ\,\Omega$ at a particular freq 'ω'. if at that freq the plane wave propagating through a dielectric has magnitude field component $\bar{H} = 10e^{-\alpha x}\cos(\omega t - \frac{1}{2}x)\hat{a_y}$ A/m find $\bar{E}$, α, skin depth and wave polarization.

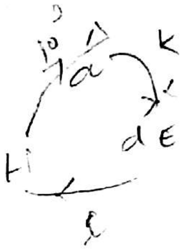
Polarization:-

Orientation of the path followed by electric field $\bar{E}$ w.r.t time is called polarization.

for free space $\overline{H} = H_0 e^{-\alpha x} \cos(\omega t - \beta x)\, \hat{a_y}$.

By comparing $\beta = \frac{1}{2}$, $H_0 = 10$

$$\hat{a_K} = \hat{a_E} \times \hat{a_H} \quad [\text{std eqn}]$$



$$= \hat{a_K} \times \hat{a_H} = -\hat{a_E}$$

$$\hat{a_x} \times \hat{a_y} = -\hat{a_z}.$$

$$\theta_\eta = 30^\circ$$

WKT $\tan 2\theta_\eta = \frac{\sigma}{[\omega\epsilon]}$

$$\frac{\sigma}{\omega\epsilon} = \tan 60^\circ$$

$$\frac{\sigma}{\omega\epsilon} = \sqrt{3}.$$

WKT $\alpha = \sqrt{\frac{\omega^2\mu\epsilon}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} - 1}$

$$\beta = \sqrt{\frac{\omega^2\mu\epsilon}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} + 1}.$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} - 1}}{\sqrt{\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} + 1}} = \frac{\sqrt{\sqrt{1+3} - 1}}{\sqrt{\sqrt{1+3} + 1}}$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{2-1}}{\sqrt{2+1}} = \frac{1}{\sqrt{3}}.$$

$$\alpha = \beta \cdot \frac{1}{\sqrt{3}} = \frac{1}{2\sqrt{3}} \text{ Np}$$

$$\overline{E} = E_0 e^{-\alpha x} \cos\left(\omega t - \frac{1}{2}x\right) \cancel{\hat{a_y}} \left(-\hat{a_z}\right) \text{ V/m}$$

$\eta = \frac{E_0}{H_0} \Rightarrow \overline{E} = 200\, e^{-\frac{1}{2\sqrt{3}}x} \cos\left(\omega t - \frac{1}{2}x\right) \left(-\hat{a_z}\right) \text{ V/m}$

$E_0 = \eta H_0$

$= 200 \times 10 = 2000$

Skin depth is represented as δ

$$\delta = \frac{1}{\alpha}$$

$$= \frac{1}{\frac{1}{2\sqrt{3}}}$$

$$\boxed{\delta = 2\sqrt{3}}$$

A plane wave travelling through a medium with $\epsilon_r = 8, \mu_r = 2$ & has electric field $\bar{E} = 0.5\, e^{-z/3} \sin(10^8 t - \beta z)\, \hat{a_x}$. find β, loss tangent, wave impedance, wave velocity v_p and $\bar{H}$.

$$\bar{E} = 0.5\, e^{-z/3} \sin(10^8 t - \beta z)\, \hat{a_x}$$

$$E_0 = 0.5, \quad \alpha = +1/3, \quad \beta = ? \quad \omega = 10^8$$

$$\hat{a_K} = \hat{a_E} \times \hat{a_H}$$

$$\hat{a_K} \times \hat{a_E} = \hat{a_H}.$$

$$\begin{tikzcd} & a_K \arrow[dr] & \\ a_H \arrow[ur] & & a_E \arrow[ll] \end{tikzcd}$$

$$\hat{a_z} \times \hat{a_x} = \hat{a_y}\ (H).$$

Since α is present so it represents lossy medium

W.K.T $\alpha = \sqrt{\frac{\omega^2 \mu \epsilon}{2} \sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} \cdot -1}$

$\frac{1}{3} \neq$ Let $\sqrt{1 + \frac{\sigma^2}{\omega^2 \epsilon^2}} = x$

$$\alpha = \sqrt{\frac{\omega^2 \mu \epsilon}{2} (cx) - 1)}$$

$$\frac{1}{3} = \sqrt{\frac{(10^8)^2\, 4\pi \times 10^{-7} \times 8.85 \times 10^{-12} (2 \times 8)}{2} (x-1)}$$

$$\left(\frac{1}{3}\right)^2 = \frac{10^{16} \times 4\pi \times 10^{-7} \times 8.85 \times 10^{-12} \times 16}{2} (x-1)$$

$$x - 1 = 0.124$$

$$x = 1.124$$

$$x^2 = 1 + \frac{\sigma^2}{\omega^2 \epsilon^2} = 1.26$$

$$\frac{\sigma^2}{\omega^2\epsilon^2} = 1.26 - 1$$

$$= 0.26$$

$$\frac{\sigma}{\omega\epsilon} = \sqrt{0.26} = 0.5.$$

W.K.T $$\beta = \sqrt{\frac{\omega^2\mu\epsilon}{2}\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} + 1}$$

$$= \sqrt{\frac{10^{16} \times 4\pi \times 10^{-7} \times 8.85 \times 10^{-12} (2\times 8)}{2}\sqrt{(.25+1)} + 1}$$

$$\beta = 1.378 \text{ rad}$$

Loss tangent $\tan\delta = \frac{\sigma}{\omega\epsilon} = 0.5$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} = \frac{\sqrt{\frac{4\pi\times10^{-7}\times 2}{8.85\times10^{-12}\times 8}}}{\left[1+(0.5)^2\right]^{1/4}} = 178\,\Omega$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon}$$

$$\tan 2\theta_\eta = 0.5 \Rightarrow \theta_\eta = \frac{\tan^{-1}\left(\frac{1}{2}\right)}{2}$$

$$= 13.28$$

$$v_p = \frac{\omega}{\beta} = \frac{10^8}{1.378} = 72.5\times10^6 \text{ m/sec}$$

$$\overline{H} = H_o e^{-z/3}\sin(10^8 t - \beta z)\,\hat{a_y}$$

$$\eta = \frac{E_o}{H_o} \Rightarrow H_o = \frac{E_o}{\eta} = \frac{0.5}{178} = 2.8\times10^{-3}$$

$$\overline{H} = 2.8\times10^{-3}\times e^{-z/3}\sin(10^8 t - 1.378 z)\,\hat{a_y} \text{ A/m}$$

plane waves in Good Conductor :-

$\sigma >> \omega\epsilon \Rightarrow \sigma = \infty,\ \mu = \mu_o\mu_r,\ \epsilon = \epsilon_o.$

W.K.T $$\alpha = \sqrt{\frac{\omega^2\mu\epsilon}{2}\sqrt{1+\frac{\sigma^2}{\omega\epsilon^2}} - 1}$$

$$\alpha = \sqrt{\frac{\omega^2\mu\epsilon}{2}\sqrt{\left(\frac{\sigma}{\omega\epsilon}\right)^2 \cancel{-1}}}$$

$$= \sqrt{\frac{\omega^2\mu\epsilon}{2}\left[\frac{\sigma}{\omega\epsilon}\right] \cancel{-1}}$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$= \sqrt{\frac{\cancel{2}\pi f\mu\sigma}{\cancel{2}}}$$

$$\alpha = \beta = \sqrt{\pi f\mu\sigma}$$

$$\frac{\sigma}{\omega\epsilon} >> 1$$

$$\sqrt{\frac{(\omega\epsilon)^2+\sigma^2}{\omega\epsilon} - \omega\epsilon}$$

$$\frac{\sigma - \omega\epsilon}{\omega^2}$$

$$v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\sqrt{\pi f\mu\sigma}}$$

$$v_p = 2\sqrt{\frac{\pi f}{\mu\sigma}}$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{2\pi f\mu\sigma}{2}}} = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \sqrt{\frac{2\omega}{\mu\sigma}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

Neglect $\omega\epsilon$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\sqrt{j}$$

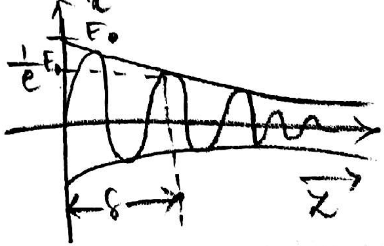
$$\sqrt{j} = 0 + \sqrt{j} \; ; |\sqrt{j}| = 1$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{1}{0}\right) = \frac{\pi}{2}$$

$$\eta = \underbrace{\sqrt{\frac{\omega\mu}{\sigma}}}_{\text{Mag}} \; \underbrace{\pi/2}_{\text{phase}}$$

Skin depth :-

The distance 'δ' through in Which the Amplitude of Signal decreases to a factor of 1/e is called as Skindepth(or) Penetration depth of the medium.



i.e $E_0 e^{-\alpha z} = E_0 e^{-1}$

$$\alpha z = 1$$

Replace $z = \delta$

$$\alpha \delta = 1$$

$$\boxed{\delta = \frac{1}{\alpha}}.$$

The surface resistance or Skin resistance $R_s(\Omega) = \frac{1}{\sigma\delta}$

$$R_s = \frac{1}{\sigma \cdot \frac{1}{\alpha}} = \frac{\alpha}{\sigma}$$

$$R_s = \frac{\sqrt{\pi . f \mu \sigma}}{\sigma} = \sqrt{\frac{\pi f \mu}{\sigma}}\ \Omega$$

we know that $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$

neglect $\omega\epsilon$ $\eta = \sqrt{\frac{\omega\mu}{\sigma}} \cdot \sqrt{j}$

from the Complex Variable j can be written as $j = e^{j\pi/2}$

$$\sqrt{j} = e^{j\pi/4}$$

Therefore $\eta = \sqrt{\frac{\omega\mu}{\sigma}} \cdot e^{j\pi/4}$

$$= \sqrt{\frac{\omega\mu}{\sigma}} \left(\cos \pi/4 + j \sin \pi/4\right)$$

$$= \sqrt{\frac{\omega\mu}{\sigma}} \left(\frac{1}{\sqrt{2}} + j \cdot \frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{\frac{2\pi f \mu}{\sigma}} \cdot \left[\frac{1 + j}{\sqrt{2}}\right]$$

$$= \sqrt{\frac{\pi f \mu}{\sigma} \times \frac{\sigma}{\sigma}} \left[1 + j\right] \qquad \left(\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}\right.$$

* **

$$\boxed{\eta = \frac{1}{\delta\sigma}(1 + j)\ \Omega}$$

Problem:

In a loss less medium for which $\eta = 60\pi$, $\mu_r = 1$, ~~$H = 01$~~

$\bar{H} = -0.1 \cos(\omega t - z)\ \hat{a_x} + 0.5 \sin(\omega t - z)\ \hat{a_y}$ A/m. find $\epsilon_r, \omega, \bar{E}$

$$\bar{H} = \bar{H}_1 + \bar{H}_2$$

Here $\bar{H}_1 = -0.1 \cos(\omega t - z)$, $\bar{H}_2 = 0.5 \sin(\omega t - z)$

Since it is a $\alpha = 0$, $\sigma = 0$.
loss less medium

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$60\pi = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

$$60\pi = \sqrt{\frac{4\pi\times10^{-7}\times1}{8.854\times10^{-12}\times\epsilon_r}}.$$

$$(60\pi)^2 = \frac{4\pi\times10^{-7}}{8.85\times10^{-12}\times\epsilon_r}$$

$$\epsilon_r = \frac{4\pi\times10^{-7}}{(60\pi)^2\times8.85\times10^{-12}}$$

$$\epsilon_r = 3.99 \approx 4.$$

from the question $\beta=1$.

$$\beta = \omega\sqrt{\mu\epsilon}$$

$$\omega = \frac{\beta}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{4\pi\times10^{-7}\times1\times8.85\times10^{-12}\times4}}$$

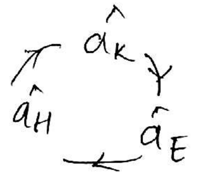
$$\omega = 149.8\times10^{6}\ \text{rad}$$

$$\bar{E} = \bar{E}_1 + \bar{E}_2$$

$\bar{E}_1 = ?$ we know that $\hat{a}_K = \hat{a}_E \times \hat{a}_H$

$$\hat{a}_K \times \hat{a}_H = -\hat{a}_E.$$

$$\hat{a}_z \times \hat{a}_x = -\hat{a}_y$$



$$\eta = \frac{E_{01}}{H_{01}}$$

$$E_{01} = \eta H_{01} = 60\pi(-0.1) = -6\pi$$

$$\therefore\ \bar{E}_1 = -6\pi\cos(\omega t - z)(-\hat{a}_y)\ \text{v/m}$$

$E_2 = ?$ we know that $\hat{a}_K = \hat{a}_E \times \hat{a}_H$

$$\hat{a}_K \times \hat{a}_H = -\hat{a}_E$$

$$\hat{a}_z \times \hat{a}_y = -(-\hat{a}_x)$$

$$\hat{a}_z \times \hat{a}_y = \hat{a}_x$$

$$E_{02} = \eta H_{02} = 60\pi\times0.5 = 30\pi$$

$$\bar{E}_2 = +30\pi \sin(\omega t - \beta z)\hat{a}_x \text{ V/m}$$

$$\bar{E} = \bar{E}_1 + \bar{E}_2$$

$$= -6\pi \cos(\omega t - \beta z)(-\hat{a_y}) + 30\pi \sin(\omega t - z)\,\hat{a}_x \text{ V/m}$$

A uniform plane wave propagating in a medium has $\bar{E} = 2e^{-\alpha z}\sin(10^8 t - \beta z)\,\hat{a_y}$ V/m. If the medium is characterised by $\epsilon_r = 1, \mu_r = 20, \sigma = 3$ S/m. find $\alpha, \beta, \bar{H}$.

$$\frac{\sigma}{\omega\epsilon} = \frac{3}{10^8 \times 8.854\times10^{-12}\times 1}$$

$$\frac{\sigma}{\omega\epsilon} = 3.38\times10^{3} >> 1.$$

∴ The medium is acting as a good conductor.

we know that for a good conductor

we know that $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{\pi f\mu\sigma}{2}}$

$$\alpha = \beta = \sqrt{\frac{10^8\times 4\pi\times10^{-7}\times 20\times 3}{2}} = 61.39 \text{ rad}$$

$$\eta = \sqrt{\frac{\mu\omega}{\sigma}} = \sqrt{\frac{4\pi\times10^{-7}\times 20\times10^{8}}{3}} = 28.94\ \Omega$$

$$\tan(2\theta_\eta) = \frac{\sigma}{\omega\epsilon} = 3.38\times10^{3}$$

$$2\theta_\eta = \tan^{-1}(3.38\times10^{3})$$

$$\theta_\eta = \frac{89.98}{2}$$

$$\theta_\eta = 44.99^\circ$$

$$\theta_\eta \approx 45^\circ$$

$$\bar{H} = H_0 e^{-\alpha z}\sin(10^8 t - \beta z)\,\hat{a}_H.$$

$$\hat{a}_K = \hat{a}_E \times \hat{a}_H$$

$$\hat{a}_K \times \hat{a}_E = \hat{a}_H$$

$$\hat{a}_z \times \hat{a_y} = -\hat{a}_x.$$

$\bar{H} = ?$ $\quad \eta = \frac{E_0}{H_0}$, $H_0 = \frac{E_0}{\eta} = \frac{2}{28.94} = 0.069$

$$\bar{H} = 0.069 e^{-\alpha z} \sin(10^{8}t - \beta z)(-\hat{a}_x) \text{ A/m}$$

Power pointing Vector Theorem:-

Energy can be transported (or) transform or transferred from (transmitter) one point to another point ~~such~~ by means of electromagnetic waves. The rate of such change transportation can be done by using Maxwell's eqn.

We know that $\nabla \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t}$

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \longrightarrow ①$$

Similarly $\nabla \times \bar{H} = \sigma \bar{E} + \epsilon \dfrac{\partial \bar{E}}{\partial t} \longrightarrow ②$

~~from~~ for eqn ② perform dot product with $\bar{E}$.

$$\bar{E} \cdot (\nabla \times \bar{H}) = \bar{E} \cdot \sigma \bar{E} + \bar{E} \cdot \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$\bar{E} \cdot (\nabla \times \bar{H}) = \sigma \bar{E}^2 + \bar{E} \cdot \epsilon \frac{\partial \bar{E}}{\partial t} \longrightarrow ③$$

But for any vector field $\bar{A}$ and $\bar{B}$.

$$\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot (\nabla \times \bar{A}) - \bar{A} \cdot (\nabla \times \bar{B}) \longrightarrow ④$$

Now Apply the above Vector identity by assuming that $\bar{A} = \bar{H}$, $\bar{B} = \bar{E}$.

$$④ \Rightarrow \nabla \cdot (\bar{H} \times \bar{E}) = \bar{E} \cdot (\nabla \times \bar{H}) - \bar{H} \cdot (\nabla \times \bar{E}) \longrightarrow ⑤$$

~~Compare ③ and ⑤~~

$$\bar{E} \cdot (\nabla \times \bar{H}) = \nabla \cdot (\bar{H} \times \bar{E}) + \bar{H} (\nabla \times \bar{E}) \longrightarrow ⑥$$

Compare ③ and ⑥

$$\nabla \cdot (\bar{H} \times \bar{E}) + \bar{H} \cdot (\nabla \times \bar{E}) = \sigma \bar{E}^2 + \bar{E} \cdot \epsilon \frac{\partial \bar{E}}{\partial t} \longrightarrow ⑦$$

from eqn ① $\nabla \times \bar{E} = -\mu \dfrac{\partial \bar{H}}{\partial t}$

$$\bar{H} \cdot (\nabla \times \bar{E}) = \bar{H} \cdot \left(-\mu \frac{\partial \bar{H}}{\partial t}\right) = -\mu \frac{\partial \bar{H}^2}{\partial t}$$

Substitute the above value in eqn ⑦

$$⑦ \Rightarrow \nabla\cdot(\bar{H}\times\bar{E}) - \mu\frac{\partial\bar{H}^2}{\partial t} = \sigma\bar{E}^2 + \frac{\partial\bar{E}^2}{\partial t}\cdot\epsilon$$

$$\nabla\cdot(\bar{H}\times\bar{E}) = \sigma\bar{E}^2 + \epsilon\frac{\partial\bar{E}^2}{\partial t} + \mu\frac{\partial\bar{H}^2}{\partial t}$$

$$\mu = \mu/2, \quad \epsilon = \epsilon/2$$

$$\nabla\cdot(\bar{H}\times\bar{E}) = \sigma\bar{E}^2 + \frac{\epsilon}{2}\frac{\partial\bar{E}^2}{\partial t} + \frac{\mu}{2}\frac{\partial\bar{H}^2}{\partial t}$$

$$-\frac{\mu}{2}\frac{\partial\bar{H}^2}{\partial t} = \sigma\bar{E}^2 + \frac{\epsilon}{2}\frac{\partial\bar{E}^2}{\partial t} - \nabla\cdot(\bar{H}\times\bar{E})$$

$$-\frac{\mu}{2}\frac{\partial\bar{H}^2}{\partial t} = \sigma\bar{E}^2 + \frac{\epsilon}{2}\frac{\partial\bar{E}^2}{\partial t} + \nabla\cdot(\bar{E}\times\bar{H}).$$

Multiply with '−'

By rearranging the above terms and by taking Volume integral on both sides.

$$-\int_v \nabla\cdot(\bar{E}\times\bar{H})\,dv = \int_v\left(\sigma\bar{E}^2 + \frac{\epsilon}{2}\frac{\partial\bar{E}^2}{\partial t}\right)dv + \int_v \frac{\mu}{2}\frac{\partial\bar{H}^2}{\partial t}dv$$

Multiply with negative sign on both sides.

$$\int_v \nabla\cdot(\bar{E}\times\bar{H})\,dv = -\int_v\left(\sigma\bar{E}^2 + \frac{\epsilon}{2}\frac{\partial\bar{E}^2}{\partial t}\right)dv - \int_v \frac{\mu}{2}\frac{\partial\bar{H}^2}{\partial t}dv.$$

Apply divergence theorem to L.H.S

$$\underbrace{\oiint_S(\bar{E}\times\bar{H})\cdot\bar{ds}}_{\substack{\text{total power}\\ \text{leaving the}\\ \text{given surface}}} = \underbrace{\frac{-\partial}{\partial t}\left[\int_v \frac{1}{2}\epsilon\bar{E}^2 + \frac{1}{2}\mu\bar{H}^2\right]dv}_{\substack{\text{rate of decreement}\\ \text{in energy stored in}\\ \text{electric and Magnetic field}}} - \underbrace{\int_v \sigma\bar{E}^2 dv}_{\substack{\text{ohmic /}\\ \text{conductor}\\ \text{losses.}}}$$

$$P_{total} = \bar{E}\times\bar{H}.$$

Pointing theorem states that net power flowing out of a given surface is equal to rate of decrease in energy with in the volume minus Conduction loss.

we know that $\bar{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a_x}$

$$\bar{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z \underset{-\theta_\eta}{}) \hat{a_y}$$

Therefore $$p(z,t) = \bar{E} \times \bar{H} = \bar{E} \cdot \frac{\bar{E}}{\eta} = \frac{E_0^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta x - \theta_\eta)\hat{a_z}.$$

we know that $\cos A \cos B - \frac{~~1~~}{~~[illegible]~~} = 2[\cos[A-B) + \cos(A+B)]$

~~$P(z,t) = \frac{E_0^2}{\eta} e^{-2\alpha z} \_ \cos(\theta_\eta) + \cos(2\omega t - 2\beta z - \theta_\eta)\hat{a_z}$~~

$$P(z,t) = \frac{E_0^2}{2\eta} e^{-2\alpha z} \cos(\theta_\eta) + \cos(2\omega t - 2\beta z + \theta_\eta)\hat{a_z} \longrightarrow ⑧$$

$$P_{avg} = \frac{1}{T}\int_0^T p(z,t)\, dt$$

$$P_{avg} = \frac{1}{T}\left\{\frac{E_0^2}{2\eta} e^{-2\alpha z} \cos(\theta_\eta)\int_0^T dt\ \hat{a_z}\right.$$

$$= \frac{1}{T}\left(\frac{E_0^2}{2\eta} e^{-2\alpha z} \cos(\theta_\eta)(T)\right)\hat{a_z}$$

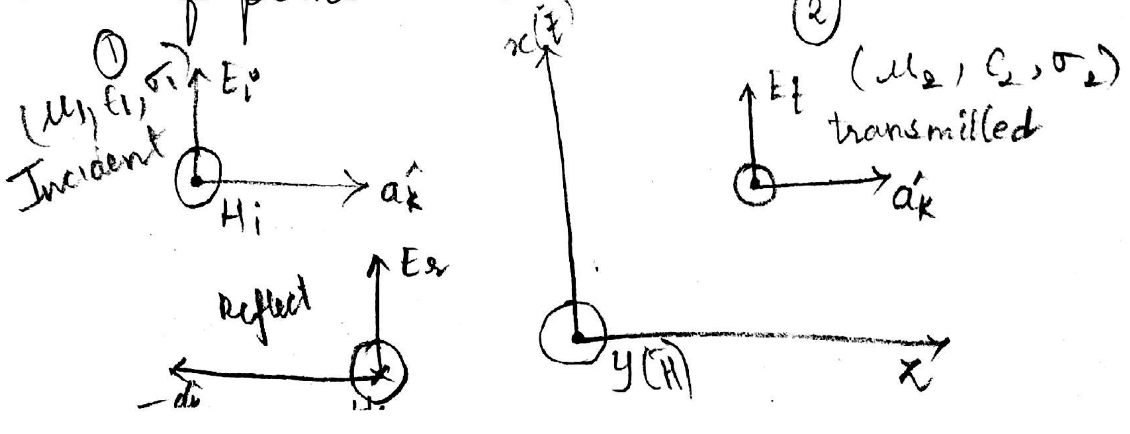
$$= \frac{E_0^2}{2\eta} e^{-2\alpha z} \cos(\theta_\eta)\ \hat{a_z}\ \text{m watt}/m^2$$

The ~~refore~~ total average power crossing a given surface area S is given by total average power

$$\text{S}\quad P(avg)_{total} = \int_S P_{avg}\, ds$$

Imp

*Reflection of plane wave at normal incidents.



Consider $E_i = E_{i0}\, e^{-\gamma_1 z}\, \hat{a_x}$

$H_i = H_{i0}\, e^{-\gamma_1 z}\, \hat{a_y}$

$E_t = E_{t0}\, e^{-\gamma_2 z}\, \hat{a_x}$

$H_t = H_{t0}\, e^{-\gamma_2 z}\, \hat{a_y}$

$E_r = E_{r0}\, e^{\gamma_1 z}\, \hat{a_x}$

$H_r = H_{r0}\, e^{\gamma_1 z}\, (-\hat{a_y})$

Now we need to apply the boundary conditions of dielectric-dielectric medium.

$\bar{E}_{1t} = \bar{E}_{2t} \rightarrow$ tangential

$\bar{E}_{i0} + \bar{E}_{r0} = \bar{E}_{t0} \xrightarrow{①}$ transmitted wave

medium ① = med ②

$\bar{H}_{1t} = \bar{H}_{2t}$ (MBC)

$H_{i0} - H_{r0} = H_{t0} \rightarrow ②$

$$\frac{①}{\eta_1} \Rightarrow \frac{E_{i0}}{\eta_1} + \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_1} \rightarrow ③$$

we know that $\eta = \frac{E}{H} \Rightarrow H = \frac{E}{\eta}$

Sub the above value in ②

$$② \Rightarrow \frac{E_{i0}}{\eta_1} - \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2} \rightarrow ④$$

Add· ③ + ④

$$2\frac{E_{i0}}{\eta_1} = \frac{E_{t0}}{\eta_1} + \frac{E_{t0}}{\eta_2}$$

$$= \cancel{E_{t0}\left[\frac{E_{t02} - E_{t0}}{\eta_1 \eta_2}\right]}$$

$$= E_{t0}\left[\frac{\eta_2 + \eta_1}{\eta_1 \eta_2}\right]$$

$$\frac{E_{i0}}{E_{t0}} = \frac{\eta_2+\eta_1}{2\eta_2}$$

the ratio of $\frac{E_{t0}}{E_{i0}}$ is called as transmission coefficient as τ.

$$\boxed{\frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_1+\eta_2}} = \tau.$$

Divide ① with E_{i0}

$$\frac{E_{i0}}{E_{i0}} + \frac{E_{r0}}{E_{i0}} = \frac{E_{t0}}{E_{i0}} \qquad \Gamma = \frac{E_{r0}}{E_{i0}}$$

$$**\boxed{1+\Gamma=\tau}$$

$$\Gamma = \tau - 1.$$

$$= \frac{2\eta_2}{\eta_1+\eta_2} - 1 \Rightarrow \frac{2\eta_2-\eta_1-\eta_2}{\eta_1+\eta_2}$$

$$(\text{reflection coefficient})\ \Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$$

Conclusions:-

① $1+\Gamma=\tau$

2) $0 \le \Gamma \le 1$

3) $1 \le S \le \infty$

In a non-magnetic medium $E = 4\sin(2\pi\times10^7 t - 0.8x)\,\hat{a}_z$ V/m find i) ϵ_r, η ii) time average power carried by the wave iii) Total power crossing 100 sq cm of a plane $(2x+y)=5$.

for a non-magnetic medium $\mu_r = 1$.

Since in the Question $\alpha=0$. So the medium is a loss less medium

$\alpha = 0, \; \beta = \omega\sqrt{\mu\epsilon}$

$$0.8 = 2\pi \times 10^{7} \sqrt{4\pi\times10^{-7}\times1 \times 8.85\times10^{-12} \times \epsilon_r}$$

$$\frac{(0.8)^2}{(2\pi\times10^{7})^2 \times 4\pi\times10^{-7}\times1\times8.85\times10^{-12}} = \epsilon_r$$

$$\epsilon_r = 14.57$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \sqrt{\frac{4\pi\times10^{-7}}{8.85\times10^{-12}\times14.57}}$$

$$\eta = 98.69\angle 0^{\circ}\,\Omega$$

In general $P_{avg} = \frac{E_0^2}{2\eta}\cos\theta_\eta \, \hat{a}_x$

$$= \frac{E_0^2}{2\eta} \, 1 \, \hat{a}_x$$

$$= \frac{(4)^2}{2(98.69)} \hat{a}_x$$

$$P_{avg} = 81.06 \, \hat{a}_x \text{ mwatt/m}^2$$

Total power crossing $= \int_S P_{avg} \, \overline{ds} = P_{avg}(S)\,\hat{a}_n \cdot$

$$= 81.06\,\text{m}\, \hat{a}_x \cdot (100\times10^{-4}) \left(\frac{2\hat{a}_x + \hat{a}_y}{5}\right)$$

Given $2x + y = 5$.

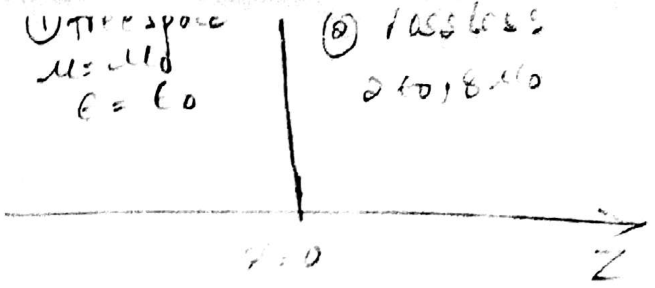
$$\hat{a}_n = \frac{\bar{a}}{|\bar{a}|} = \frac{2\hat{a}_x + 1\hat{a}_y}{\sqrt{4+1}}$$

$$= \frac{2\hat{a}_x + \hat{a}_y}{\sqrt{5}}$$

$= 725 \,\mu$ watts

∗∗ In a free space $z \le 0$, a plane wave with $H_i = 10\cos(10^{8}t - \beta x)\,\hat{a}_x$ mA/m is incident normally on a loss less medium. ($\epsilon = 2\epsilon_0, \mu = 8\mu_0$ in region $z \ge 0$. Deteemine the reflected wave H_r, E_r and the transmitted wave E_t, H_t.

for a free space



phase velocity $v_p = \frac{\omega}{\beta_1}$

$$= \frac{\omega}{\omega\sqrt{\mu_0 \epsilon_0}}$$

$$v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times10^8 \, m|sec$$

$$\beta_1 = \frac{\omega}{v_p} = \frac{10^8}{3\times10^8} = 0.33 \text{ rad}|m.$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega \text{ (or) } 120\pi\,\Omega$$

for a loss less medium

$$\beta_2 = \omega\sqrt{\mu_2 \epsilon_2}$$

$$= 10^8\sqrt{8\mu_0 \times 2 \times \varepsilon_0}$$

$$= 10^8 \times 4 \cdot \times \sqrt{\cancel{8}\mu_0 \times \varepsilon_0} = 10^8 \times 4 \cdot$$

$$\beta_2 = \frac{4\times\omega}{v_p}$$

$$= 4\beta_1 = \frac{4}{3} \text{ rad}/m$$

$$\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{8\mu_0}{2\varepsilon_0}} = 2\eta_1$$

$$= 2(120\pi)$$

$$\eta_2 = 240\pi\,\Omega$$

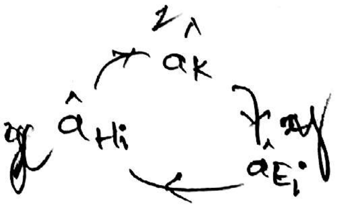
Given That $H_i = 10\cos(10^8 t - \beta z)\,\hat{a}_x \times 10^{-3}$

$$E_i = E_{i0}\cos(10^8 t - \beta z)\,\hat{a}_{Ei} \times 10^{-3}$$

$$\hat{a}_K = \hat{a}_{Ei} \times \hat{a}_{Hi}$$

$$\hat{a}_K \times \hat{a}_{Hi} = -\hat{a}_{Ei}$$

$$\hat{a}_z \times \hat{a}_x = -\hat{a}_y = \hat{a}_{Ei}$$



$$\eta_1 = \frac{E_{i0}}{H_{i0}} \Rightarrow E_{i0} = \eta_1 H_{i0} = 120\pi \times 10$$

$$E_{i0} = 1200\pi$$

$$\bar{E}_i = 1200\pi \cos(10^8 t - \beta z)(-\hat{a}_y)\ mV/m.$$

Now we know that reflection coefficient

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{240\pi - 120\pi}{240\pi + 120\pi} = \frac{1\cancel{20}\cancel{\pi}}{\frac{3\cancel{60}\cancel{\pi}}{3}} = \frac{1}{3}$$

$$\frac{E_{ro}}{E_{io}} = \frac{1}{3}$$

$$E_{ro} = \frac{1}{3} E_{io}$$

$$= \frac{1}{3} \times 1200\pi$$

$$\boxed{E_{ro} = 400\pi}$$

$$\therefore E_r = 400\pi \cos(10^8 t - \beta z)\, \hat{a}_{Er}$$

$$= 400\pi \cos(10^8 t - 0.33z)\, \hat{a}_{Er}\ mV/m.$$

$$\hat{a}_k = \hat{a}_{Er} \times \hat{a}_{Hr} \qquad \hat{a}_{Hi} = \hat{a}_x$$

$$\hat{a}_k \times \hat{a}_{Hr} = -\hat{a}_{Er} \qquad \hat{a}_{Hr} = -\hat{a}_x$$

$$\hat{a}_z \times -\hat{a}_x = -(-\hat{a}_y)$$

$$= \hat{a}_y = \hat{a}_{Er}$$

Therefore $E_r = 400\pi \cos(10^8 t - 0.3z)\, \hat{a}_y\ mV/m.$

$$H_r = H_{ro} \cos(10^8 t - 0.3z)(\hat{a}_x)\ m\cancel{A}m.$$

$$\eta_1 = \frac{E_{ro}}{H_{ro}} \Rightarrow H_{ro} = \frac{E_{ro}}{\eta_1}$$

~~$H_{ro} =$ [illegible]~~

$$H_{ro} = \frac{40\cancel{0}\cancel{\pi}}{12\cancel{0}\cancel{\pi}} = \frac{10}{3} = 3.3$$

$$H_r = \left[3.33 \times \cos(10^8 t) - 0.3z\right](-\hat{a}_x)\ mA/m.$$

$$\tau = 1 + \Gamma = \frac{2\eta_2}{\eta_1 + \eta_2} = 1 + \frac{1}{3} = \frac{4}{3}.$$

$$\tau = \frac{E_{to}}{E_{io}} = \frac{4}{3}$$

$$E_{to} = \frac{4}{3} E_{io} = \frac{4}{3} 1200\pi = 1600\pi$$

$$E_t = E_{to} \cos(10^8 t - \beta_2 z)\, \hat{a}_{Et} \text{ mv/m}$$

Since $\hat{a}_{Ei} = -\hat{a}_y$, $\hat{a}_{Et} = -\hat{a}_y$
$\hat{a}_{Hi} = \hat{a}_x$, $\hat{a}_{Ht} = \hat{a}_x$

reflected waves are completely opposite to incident waves Whereas transmitted waves are similar to incident waves.

$$E_t = 1600\pi \cos(10^8 t - 1.3z)\, (-\hat{a}_y) \text{ mv/m}$$

$$\eta_2 = \frac{E_{to}}{H_{to}} \Rightarrow H_{to} = \frac{E_{to}}{\eta_2} = \frac{1600\pi}{240\pi} = \frac{20}{3}$$

$$H_t = \frac{20}{3} \cos\left(10^8 t - \frac{4}{3} z\right) \hat{a}_x \text{ mA/m.}$$

Reflection of a plane wave at oblique incident:

The uniform plane wave takes the general form as

$$E(r,t) = E_0 \cos(\omega t - Kr)$$

$E(r,t) = E_0\, e^{-\alpha z}$
$\cos(\omega t - \beta z)$
Here $\alpha = 0$

$r \to z$
$\beta \to K$

$$= E_0 \cos(Kr - \omega t)$$

$$E(r,t) = E_0 \operatorname{Re}\left\{e^{j(Kr - \omega t)}\right\}$$

K is propagation vector

Where r = position vector.
i.e $\Rightarrow \bar{r} = x\hat{a}_x + y\hat{a}_y + a_z \hat{a}_z$.

$$K = K_x \hat{a}_x + K_y \hat{a}_y + K_z \hat{a}_z$$

and also K indicate the direction of

The Mag of K is related to ω according to dispersion relation as

$$K^2 = K_x^2 + K_y^2 + K_z^2 = \omega^2 \mu \epsilon$$

for a lossless medium $\alpha = 0$, β = constant $= K$.

Now the Maxwell's eqn for a Source free region is reduces ~~that~~ at. $\rho_v = 0$, $\sigma = 0$ $\nabla = K$

$$\frac{\partial}{\partial t} = -\omega$$

$\nabla \cdot D = 0 \Rightarrow \epsilon k \cdot \bar{E} = 0 \longrightarrow ①$

$\nabla \cdot \bar{B} = 0 \Rightarrow \mu k \cdot \bar{H} = 0 \longrightarrow ②$

$$\nabla \times \bar{H} = 0 + \frac{\partial D}{\partial t} \Rightarrow K \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t} \longrightarrow ③$$

$$= -\omega \epsilon \bar{E} \longrightarrow ③$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow K \times \bar{E} = \omega \mu \bar{H} \longrightarrow ④$$

The above eqns mention thats

i) $\bar{E}$, k, $\bar{H}$ are mutually orthogonal

ii) $\bar{E}$ and $\bar{H}$ are lies in same plane

iii) $K \cdot r$ = Scalar

iv) $K \cdot r = xK_x + yK_y + zK_z$ = constant

from eqn ④ Mag field the corresponding to electric field is given by $\bar{K} \times \bar{E} = \omega \mu \bar{H}$

$$H = \frac{1}{\omega\mu} \bar{K} \times \bar{E} \longrightarrow ⑤$$

Until we have express $\bar{E}$ and $\bar{H}$ are in general form now we can discuss the Concept of Oblique incident of uniform plane wave at a plein boundary.

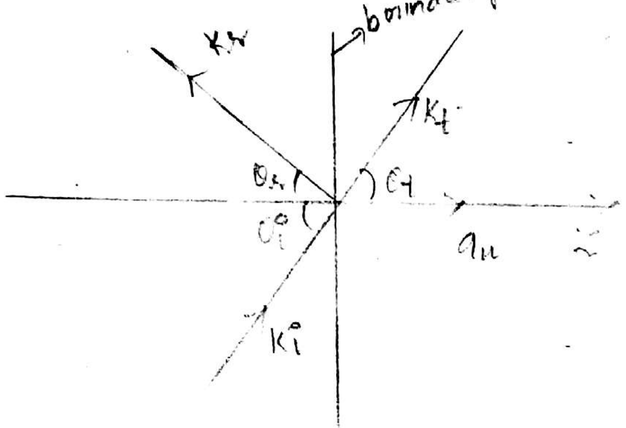
The plain defined by the propagation due to boundary by K and unit normal ~~p~~Vector $\hat{a}_n$ is called plain of incident.

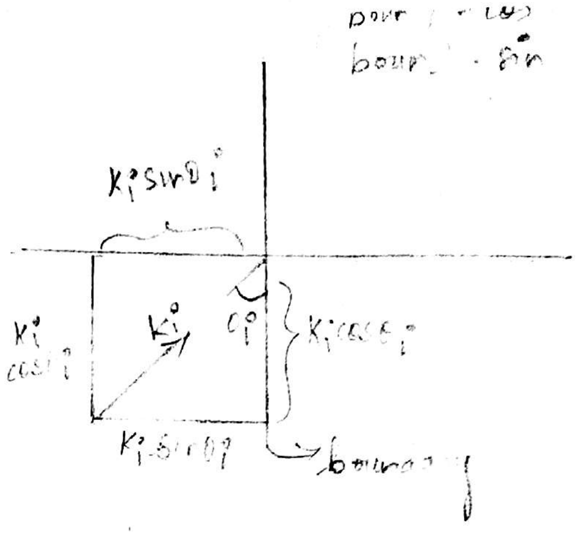
Let $E_i = E_{i0} \cos(xK_{ix} + yK_{iy} + zK_{iz} - \omega_i t) \longrightarrow ⑥$

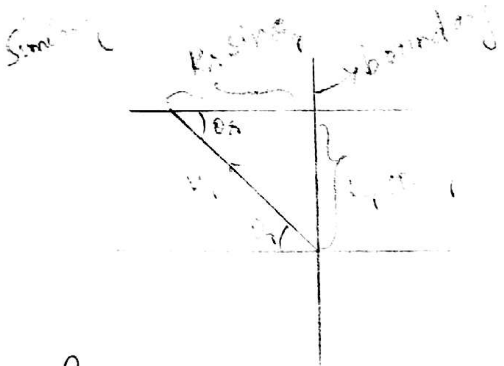
Similarly $E_r = E_{r0} \cos(xK_{rx} + yK_{ry} + zK_{rz} - \omega_r t) \rightarrow ⑦$

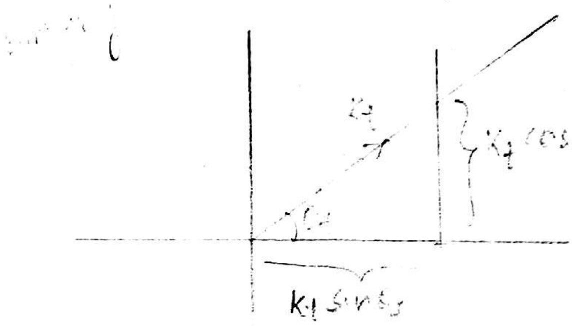
$E_t = E_{t0} \cos(xK_{tx} + yK_{ty} + zK_{tz} - \omega_t t) \rightarrow ⑧$

θ_i = angle of incidence









for loss less medium $k_t=\beta_t$, $k_i=\beta_i$, $k_r=\beta_r$

Types of polarizations :-

The Orientation of the path followed by the electric field with respect to time is called as polarization. polarization are classefied into three types

1) Linear — vertical $\bar{E}_y$
   — Horizontal $\bar{E}_x$

2) Circular polarization — Left hand cp $\bar{E}$ (CW)
   — right hand cp $\bar{E}$ (ACW)

Axial ratio = 1. $E_x^2+E_y^2=r^2$

$|E_x| = |E_y|$, E_x & E_y = 90°

3) ~~Electrical~~ Elliptical pola — Left hand Ep cw
   — Right hand Ep Acw

magnitude of $|E_x| \neq |E_y|$

Axial ratio = $\frac{\text{Major axis}}{\text{Minor axis}} > 1$.

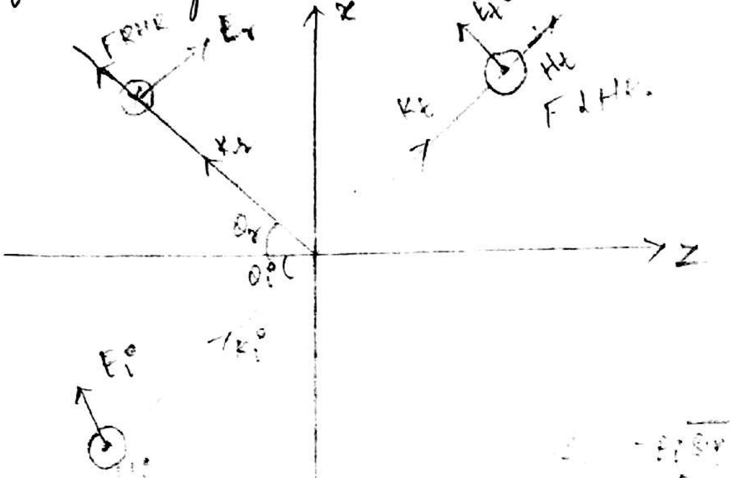
Any polarization may be considered as a linear ~~po~~combination of 1) electric field perpendicular to plane of incident (perpendicular polarization)

2) Electric field is parallel to plane of incident (or) parallel polarization.

adj - cos
opp - sin

Parallel polarization:-

Here Electric field component is parallel to the plane of incident (the plane of incident is xz plane) (E is lying in both x and z axis). Here magnetic field component is perpendicular to plane of incidence means magnetic field component lies only in Y-axis.



$$E_{is} = (E_{i0} \cos\theta_i \hat{a}_x - \sin\theta_i \hat{a}_z)$$
$$e^{-j\beta_1 (x \sin\theta_i + z\cos\theta_i)}$$

$$E_{is} = E_{i0}(\cos\theta_i \hat{a}_x - \sin\theta_i \hat{a}_z) e^{-j\beta_1 (x \sin\theta_i + z\cos\theta_i)} \rightarrow ①$$

$$H_{is} = \frac{E_{i0}}{\eta_1} e^{-j\beta_1 (x \sin\theta_i + z\cos\theta_i)} \hat{a}_y \longrightarrow ②$$

Similarly $E_{rs} = E_{r0}(\cos\theta_r \hat{a}_x + \sin\theta_r \hat{a}_z) e^{-j\beta_1 (x\sin\theta_r - z\cos\theta_r)} \longrightarrow ③$

$$H_{rs} = \frac{-E_{r0}}{\eta_1} e^{-j\beta_1 (x\sin\theta_r - z\cos\theta_r)} \hat{a}_y \longrightarrow ④$$

$$E_{ts} = E_{t0}(\cos\theta_t \hat{a}_x - \sin\theta_t \hat{a}_z) e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)} \longrightarrow ⑤$$

$$H_{ts} = \frac{E_{t0}}{\eta_2} e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)} \hat{a}_y \longrightarrow ⑥ \quad (\theta_i = \theta_r)$$

To maintain angle of reflection = angle of incidence the tangential component of E and H must be continous across the boundary so that we may get as

$$(E_{in} + E_{r0}) \cos\theta_i = E_{t0} \cos\theta_t \longrightarrow ⑦$$

Scanned by CamScanner

$$H_{io} - H_{ro} = H_{to}$$

But H can be written as

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2} \longrightarrow ⑧$$

$$⑦ + ⑧ \rightarrow (E_{io} + E_{ro}) = E_{to} \frac{\cos\theta_t}{\cos\theta_i}$$

$$E_{io} - E_{ro} = E_{to}\frac{\eta_1}{\eta_2}$$

$$+ \quad +$$

$$2E_{io} = E_{to}\left[\frac{\cos\theta_t}{\cos\theta_i} + \frac{\eta_1}{\eta_2}\right] \longrightarrow ⑨$$

perform $⑦ - ⑧ \Rightarrow 2E_{ro} = E_{to}\left[\frac{\cos\theta_t}{\cos\theta_i} - \frac{\eta_1}{\eta_2}\right] \longrightarrow ⑩$

Now reflection coefficient

$$\Gamma_{uel} = \frac{E_{ro}}{E_{io}} = \frac{\left[\frac{\cos\theta_t}{\cos\theta_i} - \frac{\eta_1}{\eta_2}\right]}{\left[\frac{\cos\theta_t}{\cos\theta_i} + \frac{\eta_1}{\eta_2}\right]}$$

$$= \frac{\frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\cos\theta_i\eta_2}}{\frac{\cos\theta_t\eta_2 + \eta_1\cos\theta_i}{\cos\theta_i\eta_2}}$$

$$\Gamma_{uel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

Transmission coefficient $\tau_{uel} = 1 + \Gamma_{uel}$

$$= 1 + \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$= \frac{\eta_2\cos\theta_t + \cancel{\eta_1\cos\theta_i} + \eta_2\cos\theta_t - \cancel{\eta_1\cos\theta_i}}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$= \frac{2\eta_2\cos\theta_t}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

Brewster angle:

In parallel polarization it is possible to obtain no reflection at a particular angle of incidence. This angle is called Brewster angle.

Reflection coefficient $\Gamma_{\parallel} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$

Refflection = 0.

⇒ Make numerator to zero

$$\eta_2 \cos\theta_t - \eta_1 \cos\theta_i = 0$$

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_i$$

If $\theta_i = \theta_{B\parallel}$ then

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_{B\parallel}$$

$$\sin^2\theta_i = \frac{\epsilon_2}{\epsilon_1+\epsilon_2}, \quad \cos^2\theta_i = \frac{\epsilon_1}{\epsilon_1+\epsilon_2}.$$

$$\tan^2\theta_i = \frac{\epsilon_2}{\epsilon_1}$$

$$\tan\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \qquad \theta_i = \theta_{B\parallel}$$

$$\theta_{B\parallel} = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

Critical angle $\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$

Unit-V

Smith chart:

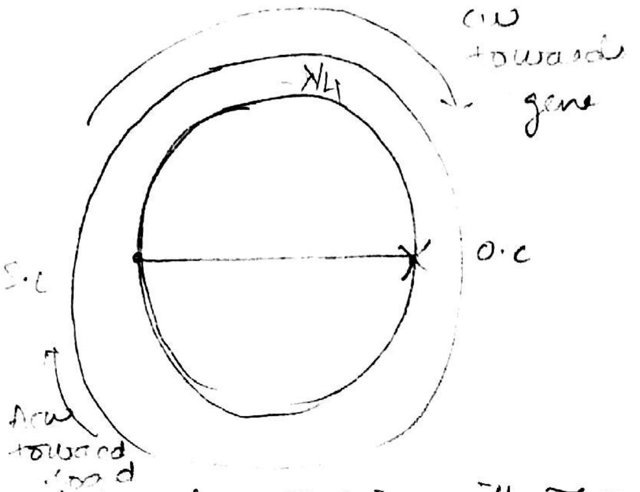
A Smith chart consist of two types of circles and those two circles are orthogonal to each other

1) Resistance circle
2) Reactance circle

Reactance circles may be inductive or capacitive.

above axis - inductive
below axis - capacitive.

$\frac{\lambda}{4} + \frac{\lambda}{4} = 360°$ $\quad \frac{\lambda}{2} = 360°$ $\quad 1\lambda = 720°$



A loss less Tx line with $Z_0 = 50\Omega$ of 30m long and operates at 2MHz the line is terminated with a load impedance of $Z_L = 60+j40\Omega$. If V(velocity) = 0.6 C on the line find

i) reflection coefficient

2) VSWR

3) input impedance

$Z_0 = 50\Omega$, $V = 0.6c$, $Z_L = 60+j40\Omega$

Verifying.

$$K = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60+j40-50}{60+j40+50}$$

$$K = \frac{10+j40}{110+j40} = \frac{41.23\angle 75.96}{117.0\angle 19.9}$$

$$K = \frac{(10+j40)(110-j40)}{(110+j40)(110-j40)}$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$l = 30m$

$= 0.197 + 0.29i$

$\beta = \frac{2\pi}{\lambda}$ $\lambda = \frac{c}{f} = \frac{0.6c}{f}$

$$S = \frac{1+|K|}{1-|K|}$$

$= 0.35 \angle 55.9$

$= 0.35 \angle 56°$

$$\lambda = \frac{0.6 \times 3 \times 10^8}{2 \times 10^6}$$

$S = 2.07$

Now we need to apply the Smith chart procedure.

1) To calculate K, we need to find normalised impedance

i) Normalised impedance $z_l = \frac{Z_L}{Z_0}$

$$= \frac{60+j40}{50} = 1.2+j0.8$$

Locate normalised impedance on the Smith chart at a point 'p' where resistance (R) = 1.2 and reactance X = 0.8

To get reflection coefficient K or γ extend op line to meet the inner most circle at point Q and measure op and OQ.

$$K = \frac{OP}{OQ} = \frac{2.8}{8} = 0.35$$

~~VSWR~~.

Take op as a radius and O as a center draw the Voltage standing wave ratio circle which meets the axis of the Smith chart at point 'S'.

Calculate $\angle PCS = 56°$.

ii) VSWR can be calculated as by finding the distance between O and S

$$S = 2.8.$$

iii) Input impedance:-

To find input impedance Zin, first we need to express length in terms of Wavelength i.e $l \rightarrow \lambda$.

$$l = 30m, \ \lambda = \frac{v}{f} = \frac{0.6c}{f} = \frac{0.6 \times 3 \times 10^8}{2 \times 10^6}$$

$$1\lambda = 90$$

$$\begin{matrix} 1\lambda = 90m \\ ? = 30m \end{matrix} \Rightarrow \frac{30\lambda}{90} = \frac{\lambda}{3} = \frac{720}{3} = 240°.$$

The length of the line corresponds to an angular momentum of 240° i.e we need to move towards generator (clockwise) from point 'P' to point 'G' on VSWR circle

$$z_{in} = 0.5 + j0.035.$$

$$Z_{in} = z_{in} Z_0$$

$$= (0.5 + j0.035)50$$

$$= 2.5 + 1.75j.$$

2) A load of $100 + j150\,\Omega$ is connected to a $75\,\Omega$ loss less lines find i) reflection coefficient, ii) VSWR, iii) load admittance(Y_L),

iv) input impedance at 0.4λ from the load.

v) V_{max} and V_{min} location w.r.t load if the line is

0.6λ. fixed length ~~of~~

vi) Input impedance at generator

i) Given $100 + j150\,\Omega$

To calculate K, first of all we need to calculate normalised load impedance

$$z_L = \frac{Z_L}{Z_0} = \frac{100 + j150}{75}$$

$$z_L = 1.33 + 2j$$

we locate this value at a point 'P' on the smith chart

$$K = \frac{OP}{OQ} = \frac{5.2}{8} = 0.65$$

$$\angle POS = 40^\circ$$

To obtain Y_L extend PO line to POP' line where P' is a point which will intersect VSWR circle

$y_l = r - jX_c$

$y_l = 0.22 - j0.36$

$Y_L = y_l Y_0$

$Y_L = \frac{y_l}{Z_0} = \frac{0.22 - j0.36}{75} = 2.93 - j4.8 \text{ ms}$

iv) The 0.4λ corresponds to an angular moment of (0.4×720°) = 288° on the constant VSWR circle. from P we move 288° towards generator (clockwise) on the 'S' circle. To reach 'R'.

At R calculate $z_{in} = 0.32 + j0.64$.

$Z_{in} = z_{in} Z_0$

$= 75(0.32 + j0.64)$

$Z_{in} = 24 + j48\ \Omega$

The 0.6λ corresponds to an angular moment of $0.6 \times 720 = 432° = 360° + 72° = 1$ revolution +72°

Thus we start from point 'p' (load end), and we move along the S circle with an angle of 432° and reaching the generator at point 'G'. To reach point 'G' we can have 2 Vmax and one Vmin location. The first Vmax is located at

$P \text{ to } S = 40° \quad 1\lambda = 720°$

$? = 40°$

$\frac{1\lambda\, 40°}{720} =$

0.055λ

2nd Vmax is located at

$= 0.055\lambda + \frac{\lambda}{2} = 0.555\lambda$

V_{min} is located at

$$= 0.055\lambda + \frac{\lambda}{4}$$

$$= 0.3055\lambda.$$

5) At G_1,

$$z_{in} = 1.8 - j2.2$$

$$Z_{in} = z_{in} Z_o$$

$$= (1.8 - j2.2)\,75$$

$$= 135 - 165j$$

3) An Antenna with an impedance of $\overset{Z_L}{40+j30\ \Omega}$ is to be matched to a $\overset{Z_0}{100\Omega}$ lossless lines with a shorted stub. find i) the required stub admittance. (ii) The distance between the stub and the antenna (iii) stub length ~~(iv) Than~~

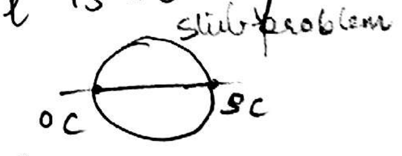
Normalised load impedance $z_l = \frac{Z_L}{Z_o} = \frac{40+j30}{100}$

$$= 0.4 + j0.3.$$

Locate z_l on the Smith chart and from this draw the VSWR circle so that y_l (is said to be normalised load impedance) can be located opposite to z_L.

Note: i) Always stub will be connected in parallel to the load impedance so in the stub problem we will prefer normalised load admittance as a reference point. (p').

ii) for a stub problem Analysis since y_l is the reference point



At y_l find resistance and reactance value

$$y_l = 1.6 - j1.2.$$

2) locate points A and B where the VSWR circle intersects ~~Gpa~~ · g=1 circle. at point A, $y_s = +j1.04$ and at point B, $y_s = \pm j1.04$. Thus the required stub admittance is

$$Y_s = Y_0 y_s = \frac{\pm j1.04}{100}$$

$$Y_s = \pm j10.4 \, m℧$$

→ feom figure we determine the distance between the load y_l and the stub

$l_B = ?$

$1\lambda = 720^\circ$

$? = 23^\circ$

$$\frac{23\lambda}{720} = 0.032\lambda$$

$l_A = ?$

$1\lambda = 720^\circ$

$? = 262^\circ$

$$\frac{262 \times 1\lambda}{720} = 0.36\lambda$$

→ Locate points A' and B' corresponding to stub admittance now find stub length (distance feom short circuit to A' and distance feom S.c to B.

To find stub length S.c is acting as a reference point

$d_A = ?$

sc to A' = ? angle

$1\lambda = 720^\circ$

$? = 87^\circ$

$$\frac{87 * 1\lambda}{720^\circ}$$

$d_A = 0.122\lambda$

$d_\beta = ?$

$1\lambda = 720^\circ$

$? = 270^\circ$

$$\frac{1\lambda \times 270^\circ}{720^\circ}$$

$d_\beta = 0.3778\lambda$

3rd Imp Ques :

1) Electromagnetic Boundary Condition

need of going for B.C.
Electric Boundary con
electric

2) problems on Maxewell's Eqn

3) problems on Boundary condition

4) Maxewell's eqn in all forms.

5) Inconsistency of ampere's law.

5th Unit Imp Quest

1. Basic Transmission line
2. Input impedance in lossy, lossless, S.C and O.C
3. VSWR
4. Reflection coeff K
5. Applications of Tramsion line.
6. peoblems on Transmission line.
7. Smith + Smith problems.

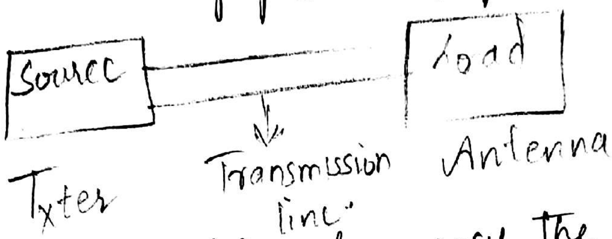
4th unit Imp Quest :

1. plane wave in lossy medium (or) Derive Holtmoltz's wave Eqn.
2. plane weave in loss less
3. plane weave in good conductor $\eta_2 = \frac{1}{\delta \sigma}(1+j)$
4. plane weave in free space (problem) (2nd)
5. Noemal incidence
6. power poynting Theorem
7. parallel polarization
8. perpendicular polarization
9. peoblems.

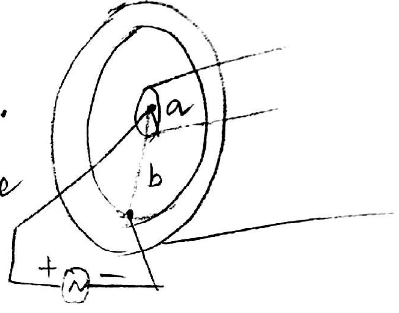
Unit - 5.

Transmission Lines.

Transmission of power is done by guided structure. This structure guides the propogation of power from source to load.



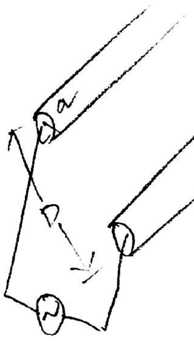
Based on the operating frequency the transmission lines are classefied as



1) co-axial cable transmission line.
It is used in television purpose
the frequency is
(400-700) MHZ

characteistic ~~output~~ impedance $= Z_0 = \frac{138}{\sqrt{\epsilon_r}} \log \frac{b}{a}$ Ω

(100-200HZ)
LAN



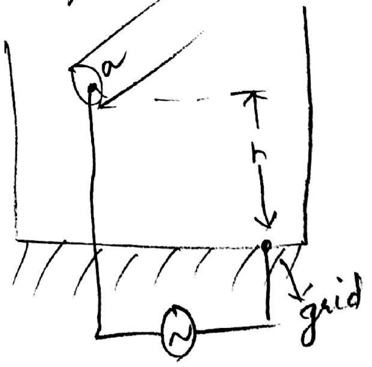
2) parallel wire conductor
100-200HZ
used for LAN

$$Z_0 = \frac{276}{\sqrt{\epsilon_r}} \log \frac{D}{a}$$

D = distance b/w two cond
a = radius of conductor

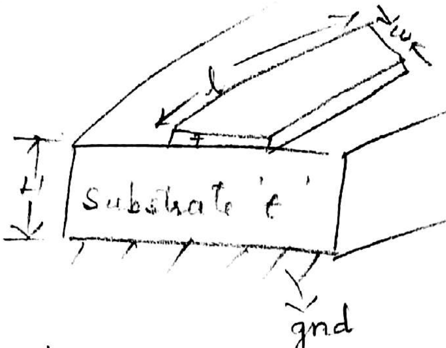
3) a conductor placed above the grnd plane at low frequency

$$Z_0 = \frac{138}{\sqrt{\epsilon_r}} \log \left[\frac{2h}{a}\right]$$



4) Microstrip transmission lines.
It is used at B>1GHZ (micro wave frequency)

$$Z_0 = \frac{377}{\sqrt{\epsilon_r}\left[\frac{W}{H}+2\right]}$$

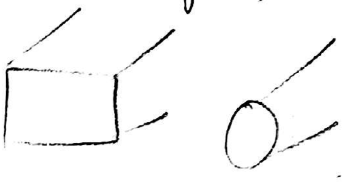


5) Wave guide transmission line:-

It is used at >1 GHz (Microwave freq)

⇒ Microwave ovens (2.4 GHz)

⇒ It is also called High pass filter



Note:-

At low frequency we can apply KVL's and KCL's where as at high frequency we cannot apply KVL's and KCL's

Transmission line parameters:-

Transmission line parameters are classified into two types

1) primary constants (Distributed Comp = R, L, G, C)

2) Secondary constants (γ, Z_0)

R → Ω/m, L → H/m, G → ℧/m, C → F/m

γ → Propagation constant (no units)

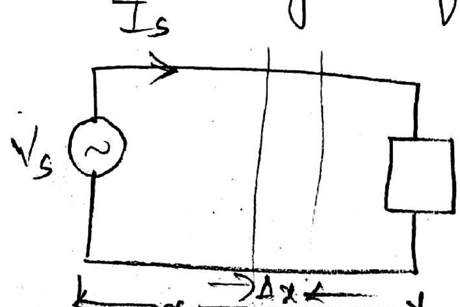
Z_0 → Characteristic impedance units are Ω.

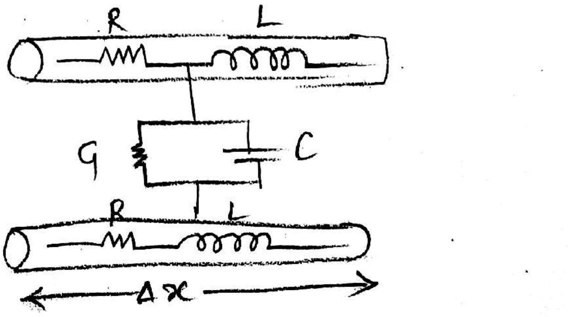
It can be written as $\gamma = \alpha + j\beta$

where α = attenuation constant Np/m (neper)

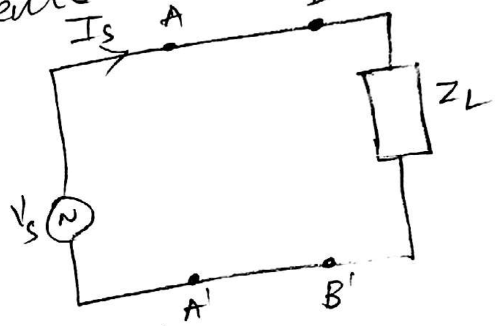
β = phase constant (rad/m)

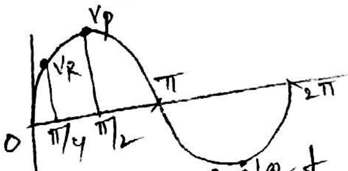
Internal diagram of transmission line:-





elements which are present internally
ents. Here R, L, G, C are called as distributed elements
these elements are inbuilt. we cannot remove them
reas in electrical circuits we can have R, L, G, C are called
lumped elements which can be connected or removed. The
ped elements donot introduce any line delays or phase
lays whereas the distributed elements will introduce
ne delays or phase delays. (lagging)





let at AA' the amplitude of
Signal is at V_p but at BB'
the amplitude of Signal is at
V_R i.e the amplitude at BB' lags
by amount of 45° (π/4) radians.
Similarly let at AA' the amplitude of Signal is at V_Q.
At BB' the amplitude of Signal is at V_p. i.e ~~at~~ the
amplitude at BB' lags by an amount of π radians.

Importance of α, β, Z_0 :-

A Signal can be divided into 2 parts

1) forward Signal (or) Incident Signal

It is moving from Source to load end

2) Backward Signal (or) Reflected Signal

It is moving from load end toward Source

The forward Signal eqn is $V^+e^{-\gamma x}$

$$\gamma = \alpha + j\beta$$

$$V^+e^{-(\alpha+j\beta)x} = \underbrace{V^+e^{-\alpha x}}_{amp} \cdot \underbrace{e^{-j\beta x}}_{phase}$$

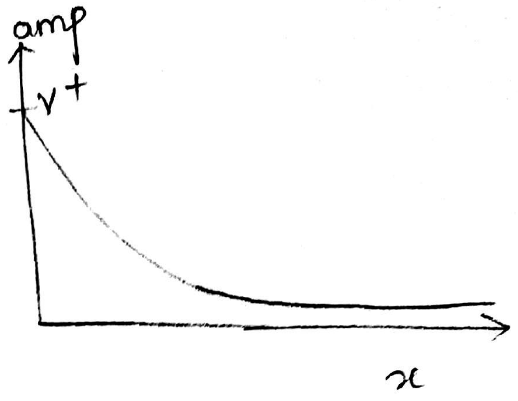
x = Signal is moving in +ve x axis

i) at $x=0$

forward Signal $= V^{+}e^{-\alpha x}$

$= V^{+}e^{0}$

$= V^{+}$



ii) at $x=\infty, V^{+}e^{-\infty}=0$.

attenuation constant is a parameter that shows how the amplitude reduction takes place when it travels from the transmission line.

⇒ Attenuation constant will specifies the characteristics lengths that show how the wave travels on the transmission line upto which distance.

β-phase constant $\beta = \frac{2\pi}{\lambda} = \frac{rad}{m}$

WKT $\gamma = \alpha + j\beta$

But from internal diagram $Z = R + j\omega L$

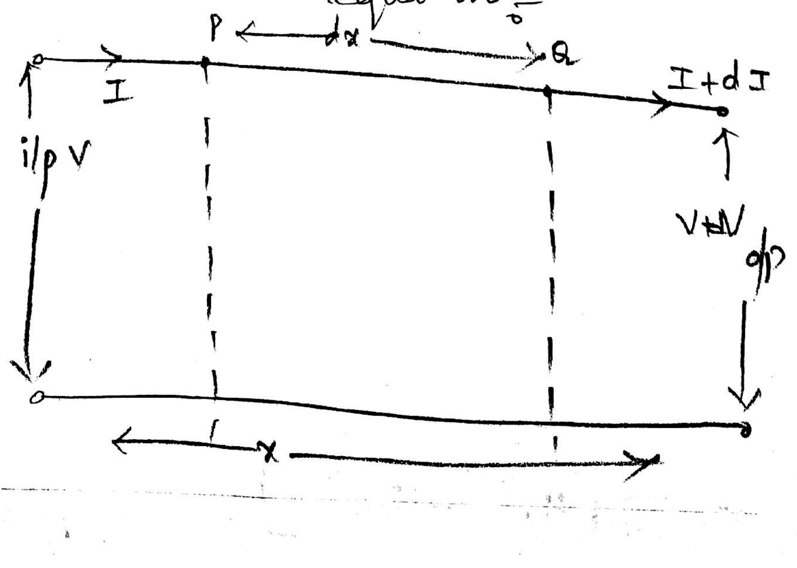
Similary $Y = G + j\omega C$

$$\gamma = \sqrt{ZY}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \ \Omega$$

Basic transmission line Equation:



ff Eqn ② W.r.t 'x' on both Sides.

$$-\frac{d^2I}{dx^2} = \frac{dv}{dx}\cdot(G+j\omega c)$$

Substitute the Value of $\frac{dv}{dx}$ from ① eqn in above eqn.

en above eqn can be written as.

$$-\frac{d^2I}{dx^2} = -I(R+j\omega L)[G+j\omega c]$$

$$\frac{d^2I}{dx^2} = I\gamma^2 \text{ where } \gamma = \sqrt{(G+j\omega c)(R+j\omega L)}$$

$$\frac{d^2I}{dx^2} - I\gamma^2 = 0$$

It is representing Second order D.E.

The auxilary polynomial of above d.E is

$$m^2 - \gamma^2 = 0$$

$$m^2 = \gamma^2$$

$$m = \pm\gamma$$

The roots are real and unequal

General Solution $I = Ce^{-\gamma x} + De^{\gamma x} \longrightarrow ④$

n the above ③, ④ $Ae^{-\gamma x}$, $Ce^{-\gamma x}$ represents The Signal which moving from Source to load. Similarly $Be^{+\gamma x}$, $De^{\gamma x}$ will presents the signal which is moving from load end to ource.

At $x = 0m$ in eqn ③, $V = V_s$, $I = I_s$.

Sub the above intitial Condition in eqn 3 & 4.

$$③ \Rightarrow V_s = Ae^0 + Be^0$$

$$V_s = A + B.$$

$$④ \Rightarrow I_s = C + D$$

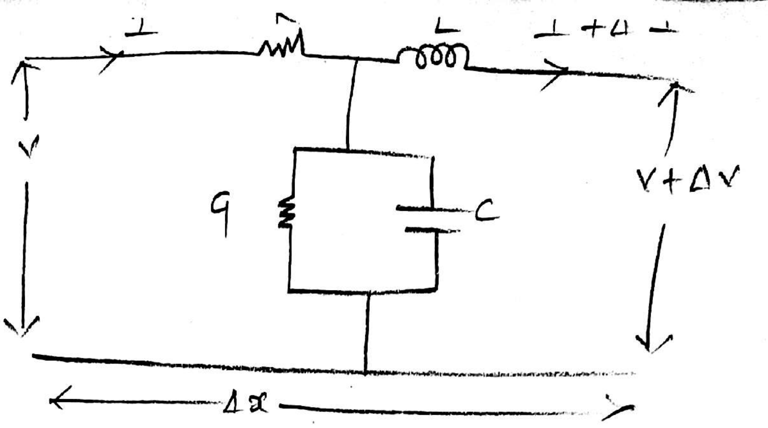
$$V = Ae^{-\gamma x} + Be^{\gamma x}.$$

differentiate above eqn w.r.t 'x' on b.s.

$$\frac{dV}{dx} = A\cdot e^{-\gamma x}(-\gamma) + Be^{\gamma x}(\gamma)$$

$$\frac{dV}{dx} = \gamma\left[Be^{\gamma x} - Ae^{-\gamma x}\right]$$

Scanned by CamScanner



The potential differeence between input and output is given as

$$V-(V+dV) = I\,[R+j\omega L]\,dx$$

$$\frac{-dV}{dx} = I\,[R+j\omega L] \longrightarrow ①$$

Similarly the current flowing through shunt admittance is given as

$$I-[I+dI] = V\,[G+j\omega C]\,dx$$

$$\frac{-dI}{dx} = V\,[G+j\omega C] \longrightarrow ②$$

$$\left[\because I = \frac{V}{R} = \frac{V}{Z} = VY\right]$$

Diff ① w.r.t 'x' on both sides

$$\frac{-d^2V}{dx^2} = \frac{dI}{dx}\,[R+j\omega L]$$

Substitute the value of $\frac{dI}{dx}$ from Eqn ② in the above Eqn

$$\frac{-d^2V}{dx^2} = -V\,[G+j\omega C]\,[R+j\omega L]$$

$$\frac{d^2V}{dx^2} = V\gamma^2 \quad \text{where } \gamma = \cancel{\sqrt{\frac{R+j\omega L}{G+j\omega C}}} \quad \gamma = \sqrt{(R+ju}$$

$$(G+ju$$

$$\frac{d^2V}{dx^2} - V\gamma^2 = 0$$

$$(D^2-\gamma^2)\,V = 0$$

It is representing 2^{nd} order defferential Eqn

The auxilary Eqn of above diff Eqn is

$$m^2-\gamma^2 = 0$$

$$m^2 = \gamma^2$$

$$m = \pm\gamma$$

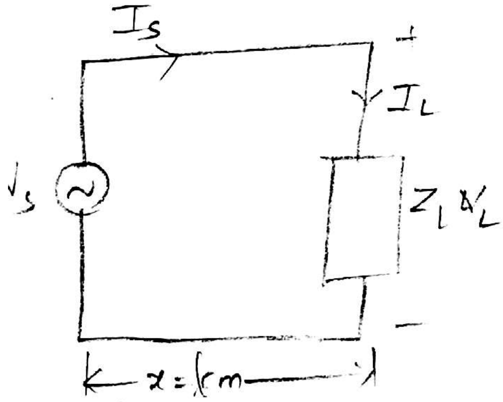
The roots are real and unequal, the general solution is

Input impedance of transmission line terminated with any load impedance (Z_L):-

we know that from basic transmission line eqn

$$V = V_s \cosh\gamma x - I_s Z_0 \sinh\gamma x \longrightarrow$$ ⑦

$$I = I_s \cosh\gamma x - \frac{V_s}{Z_0} \sinh\gamma x \longrightarrow$$ ⑧



At $x = l$ meters, $V = V_L$, $I = I_L$

Substitute above eqns in Eqn ⑦, ⑧

$$V_L = V_s \cosh\gamma l - I_s Z_0 \sinh\gamma l \longrightarrow$$ ①

$$I_L = I_s \cosh\gamma l - \frac{V_s}{Z_0} \sinh\gamma l \longrightarrow$$ ②

$$V_L = I_L Z_L \longrightarrow$$ ③

Substitute Eqn ①, ② in Eqn ③

Eqn ③ $\Rightarrow V_s \cosh\gamma l - I_s Z_0 \sinh\gamma l$

$$= \left[I_s \cosh\gamma l - \frac{V_s}{Z_0} \sinh\gamma l\right] Z_L$$

$$V_s \left[\cosh\gamma l + \frac{Z_L}{Z_0} \sinh\gamma l\right] = I_s \left[Z_L \cosh\gamma l + Z_0 \sinh\gamma l\right]$$

$$Z_{in} = \frac{V_s}{I_s} = \frac{Z_L \cosh\gamma l + Z_0 \sinh\gamma l}{\cosh\gamma l + \frac{Z_L}{Z_0} \sinh\gamma l}$$

$$Z_{in} = \frac{Z_0\left[Z_L \cosh\gamma l + Z_0 \sinh\gamma l\right]}{\left[Z_0 \cosh\gamma l + Z_L \sinh\gamma l\right]}$$

$$= \frac{Z_0 \cosh\gamma l \left[Z_L + Z_0 \frac{\sinh\gamma l}{\cosh\gamma l}\right]}{\cosh\gamma l \left[Z_0 + Z_L \frac{\sinh\gamma l}{\cosh\gamma l}\right]}$$

$$= Z_0 \frac{\left[Z_L + Z_0 \tanh \gamma l\right]}{Z_0 + Z_L \tanh \gamma l} \; //$$

// Input impedance of a transmission line with loss less :-

The condition for loss less is given as $\alpha = 0$ and $R=0, G=0$

We know that $\gamma = \underbrace{\alpha}_{\text{real}} + \underbrace{j\beta}_{\text{imag}} = \sqrt{(\underbrace{R}_{0}+j\omega L)(\underbrace{G}_{0}+j\omega C)}$ Since real part = 0

$$\gamma = j\beta = \sqrt{(j\omega L)(j\omega C)}$$

$$j\beta = j\omega\sqrt{LC}$$

$$\beta = \omega\sqrt{LC} \text{ rad/m}$$

We know that input impedance of a transmission line if it is terminated with any load impedance is given a

$$Z_{in} = Z_0 \cdot \frac{Z_L \cosh\gamma l + Z_0 \sinh\gamma l}{Z_0 \cosh\gamma l + Z_L \sinh\gamma l}$$

for a lossless transmission line $\alpha = 0, \gamma = j\beta$

$$Z_{in} = Z_0 \cdot \frac{Z_L \cosh j\beta l + Z_0 \sinh j\beta l}{Z_0 \cosh j\beta l + Z_L \sinh j\beta l}$$

we know that $\cosh\theta = \frac{e^{\theta}+e^{-\theta}}{2}$

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$\cosh j\beta l = \frac{e^{j\beta l}+e^{-j\beta l}}{2} = \cos\beta l.$$

$$\boxed{\cosh j\beta l = \cos\beta l.}$$

Similarly w.k.T $\sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$

$$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}.$$

$$\boxed{\sinh j\beta l = j\sin\beta l.}$$

$$Z_{in} = Z_o \cdot \frac{Z_L\cos\beta l + jZ_o\sin\beta l}{Z_o\cos\beta l + j\cdot Z_L\sin\beta l}\ \Omega$$

$$= Z_o\ \frac{\cancel{\cos\beta l}\left[Z_L + jZ_o\frac{\sin\beta l}{\cos\beta l}\right]}{\cancel{\cos\beta l}\left[Z_o + jZ_L\frac{\sin\beta l}{\cos\beta l}\right]}$$

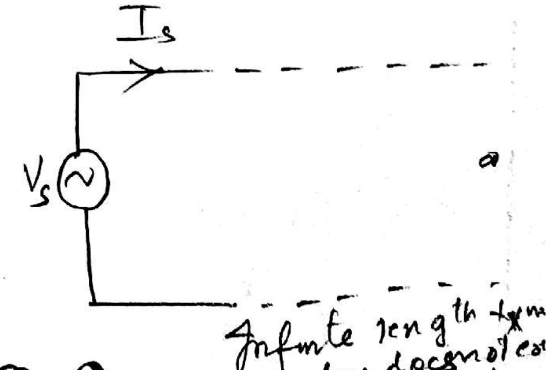
$$Z_{in} = Z_o \cdot \frac{[Z_L + jZ_o\tan\beta l]}{[Z_o + jZ_L\tan\beta l]}\ \Omega$$

Input impedance of infinite transmission line :-

we know that from the concept of basic transmission line eqn.

$$V = Ae^{-\gamma x} + Be^{\gamma x} \longrightarrow ①$$

$$I = Ce^{-\gamma x} + De^{\gamma x} \longrightarrow ②$$



At $x = \infty\,m$, $V = 0$, $I = 0$.

Substitute the above cond in ① & ②

$$0 = Ae^{-\infty} + Be^{\infty} \Rightarrow 0 = 0\cdot A + B \Rightarrow \boxed{B = 0}$$

Scanned by CamScanner

$② \rightarrow 0 = 0 + D$

$$\boxed{D=0}$$

Sub $B=0$ & $D=0$ in ① & ②

$$V = Ae^{-\gamma x} \longrightarrow ③$$

$$I = Ce^{-\gamma x} \longrightarrow ④$$

we know that $A+B=V_s$, $A-B=I_s Z_0$

$$\begin{array}{ll} A+B=V_s & C+D=I_s \\ A-B=I_s Z_0 & C-D=V_s/Z_0 \\ \hline 2A=V_s+I_s Z_0 & 2C=I_s+\frac{V_s}{Z_0} \end{array}$$

$$A = \frac{V_s + I_s Z_0}{2} \qquad C = \frac{1}{2}\left[I_s + \frac{V_s}{Z_0}\right]$$

At $x=0$ m, $I=I_s$, $V=V_s$

Sub the above condition in eqn ③ & ④

$$V_s = Ae^0 \qquad I_s = Ce^0$$

$$\boxed{V_s = A} \qquad \boxed{I_s = C}$$

w.k.T $Z_{in} = \frac{V_s}{I_s} = \frac{A}{C} = \dfrac{\frac{V_s+I_sZ_0}{2}}{\dfrac{I_s+\frac{V_s}{Z_0}}{2}}$

$$\Rightarrow \frac{V_s+I_sZ_0}{I_s+\frac{V_s}{Z_0}} \Rightarrow \frac{V_s+\cancel{I_s}Z_0}{\frac{Z_0\cancel{I_s}+V_s}{Z_0}} = Z_0$$

$$Z_{in} = Z_0$$

Thus for an infinite transmission line input impedance is equal to characteristic impedance of a transmission line.

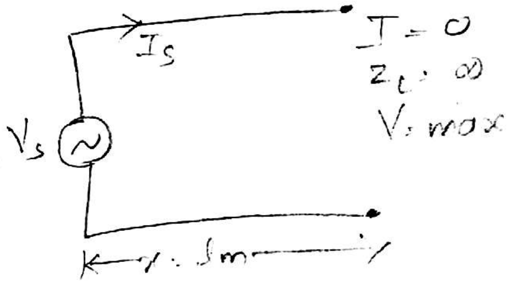
Input impedance of open Circuit & Short Circuit lines:-

We know that from basic transmission lines.

$$V = V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x \rightarrow ①$$

$$I = I_s \cosh \gamma x - \frac{V_s}{Z_0} \sinh \gamma x \rightarrow ②$$

(i) open circuit:-



At $x = lm$, $I = 0$ substitute the above condition in eqn ②

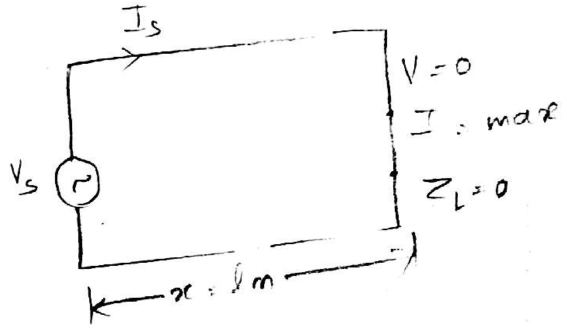
$$0 = I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l$$

$$I_s \cosh \gamma l = \frac{V_s}{Z_0} \sinh \gamma l$$

$$\frac{V_s}{I_s} = Z_0 \frac{\cosh \gamma l}{\sinh \gamma l}$$

$$\boxed{Z_{in(o.c)} = Z_0 \coth \gamma l}$$

ii) short Circuit:



At $x = lm$, $V = 0$

Substitute the above condition in Eq ①

$$0 = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l$$

$$V_s \cosh \gamma l = I_s Z_0 \sinh \gamma l$$

$$\frac{V_s}{I_s} = Z_0 \frac{\sinh \gamma l}{\cosh \gamma l} = Z_0 \cdot \tanh \gamma l$$

$$Z_{in(s.c)} = Z_0 \tanh \gamma l$$

Input impedance of loss less transmission line for open & short circuit

for loss less transmission lines $\gamma = j\beta$ $[\because \alpha = 0]$

$$Z_{o.c} = Z_0 \coth \gamma l$$

$$= Z_0 \frac{\cosh \gamma l}{\sinh \gamma l}$$

$$\gamma = j\beta \Rightarrow Z_0 \frac{\cosh j\beta l}{\sinh j\beta l}$$

$$= \frac{Z_0 \cos\beta l}{j \sin\beta l}$$

$$= \frac{Z_0}{j} \cot\beta l$$

$j^2 = -1$

$j.j = -1$

$= j^2 = -1, \; j.j = -1, \; \frac{1}{j} = -j \qquad \frac{1}{j} = -j$

$$\boxed{Z_{oc} = -jZ_0 \cot\beta l \ \Omega}$$

βl = electrical length

$$Z_{sc} = Z_0 \tanh \gamma l$$

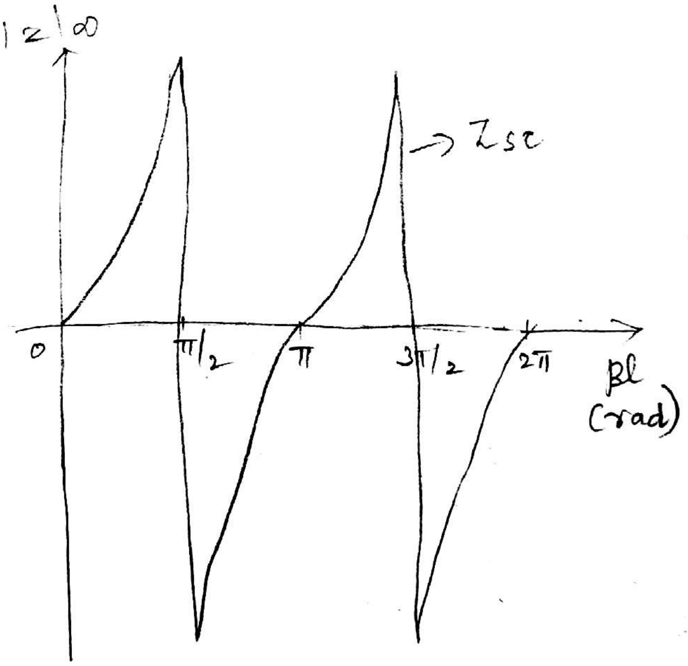
$$= Z_0 \frac{\sinh \gamma l}{\cosh \gamma l}$$

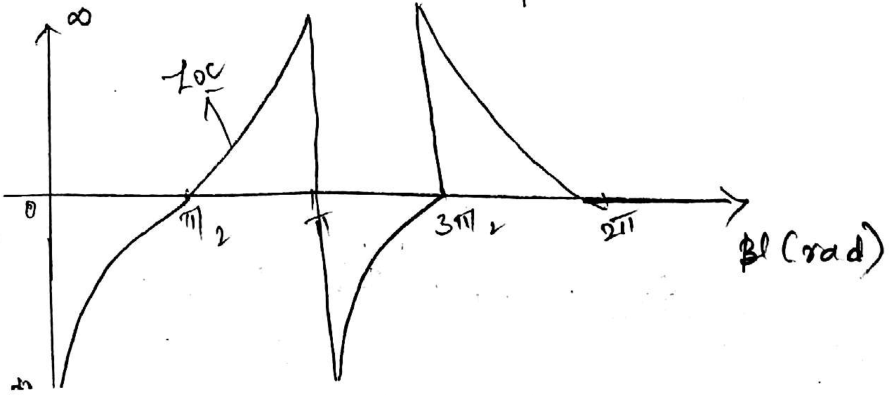
$$= Z_0 \frac{\sin j\beta l}{\cos j\beta l}$$

$$= Z_0 \, j \frac{\sin\beta l}{\cos\beta l}$$

$$Z_{sc} = Z_0 j \tan\beta l$$

$$\boxed{Z_{sc} = jZ_0 \tan\beta l}$$





Distortionless transmission lines:-

A signal normally consists of a band of frequencies, amplitudes of different frequency components will be attenuated differently in a lossy line as α' is frequency dependent. This results in distortion.

Distortionless transmission line is one in which attenuation constant α is independent of frequency while the phase shift constant β is linearly dependent on frequency. The condition for distortionless transmission line is

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

We know that $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$.

$$\gamma = \sqrt{R\left(1+\frac{j\omega L}{R}\right)G\left(1+\frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG\left[1+\frac{j\omega}{R/L}\right]\left[1+\frac{j\omega}{G/C}\right]}$$

$$= \sqrt{RG\left[1+\frac{j\omega}{R/L}\right]\left[1+\frac{j\omega}{R/L}\right]}$$

$$= \sqrt{RG}\left[1+\frac{j\omega}{R/L}\right]$$

$$= \sqrt{RG} + \frac{j\omega L\sqrt{RG}}{\sqrt{R}\sqrt{R}}$$

$$= \sqrt{RG} + j\omega L\sqrt{\frac{G}{R}}$$

$$\gamma = \alpha + j\beta \cdot \text{Np/m}$$

$$\alpha = \sqrt{RG}, \quad \beta = \omega L\sqrt{\frac{G}{R}} \cdot \text{rad/m}$$

$$\frac{R}{L} = \frac{G}{C}$$

$$\frac{G}{R} = \frac{C}{L}$$

phase Velocity $v_p = \frac{\omega}{\beta}$.

$$v_p = \frac{\omega}{\omega L\sqrt{G/R}} = \frac{1}{L\sqrt{\frac{G}{R}}} := \frac{1}{L\sqrt{\frac{C}{L}}}$$

$$= \frac{1}{\sqrt{\cdot L}\,\sqrt{L}\,\sqrt{\frac{C}{L}}}$$

$$\boxed{v_p = \frac{1}{\sqrt{LC}} \; m/s.}$$

We know that $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$$= \sqrt{\frac{R\left[1+\frac{j\omega L}{R}\right]}{G\left[1+\frac{j\omega C}{G}\right]}}$$

$$= \sqrt{\frac{R\left[1+\frac{j\omega}{R/L}\right]}{G\left[1+\frac{j\omega}{G/C}\right]}}$$

$$= \sqrt{\frac{R}{G}\,\frac{\left[1+j\omega/R/L\right]}{\left[1+j\omega/R/L\right]}}$$

$$\boxed{Z_0 = \sqrt{\frac{R}{G}} \; \Omega}$$

Loss less transmission line:-

The condition for Loss less transmission line is given as

$\alpha = 0, \; R = 0, \; G = 0$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\gamma = j\beta = \sqrt{(j\omega L)(j\omega C)}$$

$$j\beta = j\omega\sqrt{LC}$$

$\alpha = 0 Np/m; \quad \beta = \omega\sqrt{LC}$ rad/m

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} \; m/sec$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega L}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \; \Omega$$

Distortion less	Loss less
1) $\alpha = \sqrt{RG}$	$\alpha = 0$
2) $\beta = \omega L\sqrt{\frac{G}{R}}$	$\beta = \omega\sqrt{LC}$
3) $v_p = \frac{1}{\sqrt{LC}}$	$v_p = \frac{1}{\sqrt{LC}}$
4) $Z_0 = \sqrt{\frac{R}{G}}$	$Z_0 = \sqrt{\frac{L}{C}}$
$\frac{R}{L} = \frac{G}{C}$	$\alpha = 0, R = 0, G = 0$

Reflection coefficient:-

Reflection coefficient is represented as 'K' or 'Γ'

We know that from the basic transmission line eqn

$$V = Ae^{-\gamma x} + Be^{\gamma x} \longrightarrow ①$$

$$I = Ce^{-\gamma x} + De^{\gamma x} \longrightarrow ②$$

This Eqn (or) The other forms of above two eqn are represented as

$$V = V^{+}e^{-\gamma x} + V^{-}e^{\gamma x}$$

$$I = I^{+}e^{-\gamma x} + I^{-}e^{\gamma x}$$

Reflection coefficient is defined as ratio of reflected (signal) amplitude to incident (signal) amplitude

Case (i) K interms of Voltage:-

$$K = \frac{V_{reflected}}{V_{incident}} = \frac{Be^{\gamma x}}{Ae^{-\gamma x}} = \frac{B}{A}e^{2\gamma x}$$

W.K.T

$$A + B = V_s$$
$$A - B = I_s Z_0$$
$$2A = V_s + I_s Z_0$$
$$A = \frac{1}{2}\left[V_s + I_s Z_0\right]$$

$$A + B = V_s$$
$$A - B = I_s Z_0$$
$$2B = V_s - I_s Z_0$$
$$B = \frac{1}{2}\left[V_s - I_s Z_0\right]$$

$$K = \frac{V_s - I_s Z_0}{V_s + I_s Z_0} e^{2\gamma x}$$

$$= \frac{\cancel{I_s}\left[\frac{V_s}{I_s} - Z_0\right]}{\cancel{I_s}\left[\frac{V_s}{I_s} + Z_0\right]} e^{2\gamma x}$$

$$K = \frac{\left(\frac{V_s}{I_s} - Z_0\right)}{\left(\frac{V_s}{I_s} + Z_0\right)} e^{2\gamma x}$$

At $x = 'l' m$ $V = V_L, I = I_L$..

① $\Rightarrow$ $V_L = A e^{-\gamma l} + B e^{\gamma l}$

$$I_L = C e^{-\gamma l} + D e^{\gamma l}$$

$$K = \frac{V_{refl}}{V_{inc}} = \frac{B}{A}\frac{e^{\gamma l}}{e^{-\gamma l}}$$

$$K = \frac{B}{A} e^{2\gamma l}$$

$$= \frac{\left(\frac{V_L}{I_L} - Z_0\right)}{\left(\frac{V_L}{I_L} + Z_0\right)} e^{2\gamma l}$$

$$= \frac{(Z_L - Z_0)}{(Z_L + Z_0)} e^{2\gamma l}$$

Amplitude

phase

$$\boxed{|K| = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

If $Z_L = Z_0$.

$$|K| = \frac{0}{2Z_0} = 0.$$

When load impedance = characteristic impedance then magnitude of $K = 0$ it means only ~~reflected~~ incident Signals are present and there are no reflected Signal present

Tx line . Such transmission line is called as Matched

Smooth line.

K in terms of Current :- $I = C e^{-\gamma x} + D e^{\gamma x}$

$$K = \frac{D e^{\gamma x}}{C e^{-\gamma x}} = \frac{D}{C} e^{2\gamma x}$$

from the general transmission line equation

$$\begin{aligned} C + D &= I_s \\ C - D &= \frac{V_s}{Z_0} \\ + \quad + \quad & \quad + \\ \hline 2C &= I_s + \frac{V_s}{Z_0} \end{aligned}$$

$$C = \frac{1}{2}\left[I_s + \frac{V_s}{Z_0}\right]$$

$$\begin{aligned} C + D &= I_s \\ C - D &= \frac{V_s}{Z_0} \\ - \quad - \quad & \quad - \\ \hline 2D &= I_s - \frac{V_s}{Z_0} \end{aligned}$$

$$D = \frac{1}{2}\left[I_s - \frac{V_s}{Z_0}\right]$$

$$k = \frac{I_s - \frac{V_s}{Z_0}}{I_s + \frac{V_s}{Z_0}} e^{2\gamma x}$$

$$K = \left(\frac{I_s Z_0 - V_s}{I_s Z_0 + V_s}\right) e^{2\gamma x}$$

$$= \frac{I_s\left[Z_0 - \frac{V_s}{I_s}\right]}{I_s\left[Z_0 + \frac{V_s}{I_s}\right]} e^{2\gamma x}$$

$$= \frac{Z_0 - \frac{V_s}{I_s}}{Z_0 + \frac{V_s}{I_s}} e^{2\gamma x}$$

At $x = l$ m, $V = V_L$, $I = I_L$

$$K = \frac{D}{C} e^{2\gamma l}$$

$$= \frac{Z_0 - \frac{V_L}{I_L}}{Z_0 + \frac{V_L}{I}} e^{2\gamma l}$$

$$K = \underbrace{\frac{Z_0 - Z_L}{Z_0 + Z_L}}_{\text{Amp}} \; \underbrace{e^{2\gamma l}}_{\text{phase}}$$

Voltage Standing Wave Ratio (VSWR):-

We Know that from the basic transmission line Eqn

$$V = A e^{-\gamma x} + B e^{\gamma x} \longrightarrow ①$$

$$I = C e^{-\gamma x} + D e^{\gamma x} \longrightarrow ②$$

The other forms of the above eqn can be written as

$$V = V^+ e^{-\gamma x} + V^- e^{\gamma x} \longrightarrow ③$$

$$I = I^+ e^{-\gamma x} + I^- e^{\gamma x} \longrightarrow ④$$

Here $Z_0 = \frac{V^+}{I^+} = \frac{\text{Amplitude of Voltage V in forward direction}}{\text{Amplitude of Current I in forward direction}}$

Similarly the Backward Signal may consist of characteristic Impedance of $-Z_0$ Since in Backward direction it represents negative direction it means the power is recieved by generator but not supplied.

$$I^+ = \frac{V^+}{Z_0} \quad \text{similarly} \quad I^- = \frac{V^-}{-Z_0}$$

Substitute the above Value in eqn ③ and ④

$$④ \Rightarrow I = \frac{V^+}{Z_0} e^{-\gamma x} + \frac{V^-}{-Z_0} e^{\gamma x}$$

$$I = \frac{V^+}{Z_0} e^{-\gamma x} - \frac{V^-}{Z_0} e^{\gamma x}$$

Normally we have two special locations on the transmission line i) generator ii) load impedance.

We define Origin either at generator or at load impedance ow let us define origin at load

o location which is moving towards generator. So we can

place $x=-l$. Substitute $x=-l$ in Eqn ③, ④.

$$③ \Rightarrow V = V^+ e^{\gamma l} + V^- e^{-\gamma l} \longrightarrow ⑤$$

$$④ \Rightarrow I = \cancel{V^+} \frac{V^+}{Z_0} e^{\gamma l} - \frac{V^-}{+Z_0} e^{-\gamma l} \longrightarrow ⑥$$

let us assume loss less transmission lines, $\alpha=0$, $R=0$, $G=0$

$$\gamma = j\beta$$

Sub γ in ⑤ and ⑥

$$⑤ \Rightarrow V(l) = V^+ e^{j\beta l} + V^- e^{-j\beta l} \longrightarrow ⑦$$

$$⑥ \Rightarrow I(l) = \frac{V^+}{Z_0} e^{j\beta l} - \frac{V^-}{Z_0} e^{-j\beta l} \longrightarrow ⑧$$

$$\Gamma_l = \frac{\text{reflected}}{\text{Incident}} = \frac{V^-}{V^+} = K = \frac{Z_L - Z_0}{Z_L + Z_0}$$

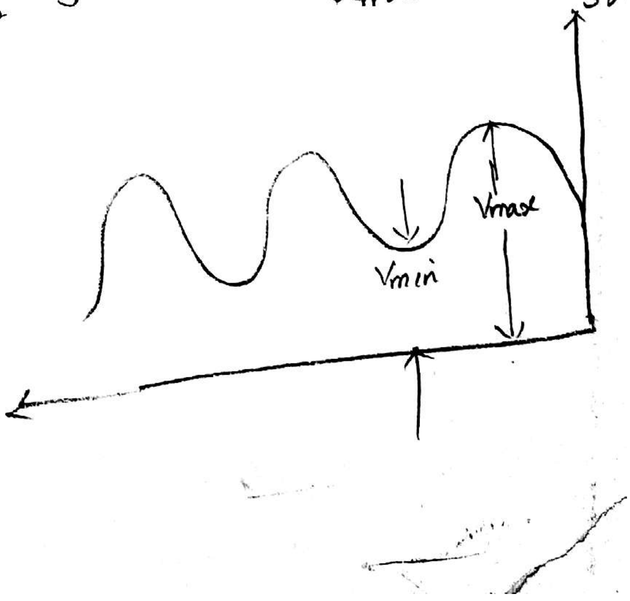
$$⑦ \Rightarrow V(l) = \underbrace{V^+ e^{j\beta l}}_{\text{Forward wave Signal}} \left\{ 1 + \underbrace{\frac{V^-}{V^+} e^{-2j\beta l}}_{\text{Backward wave Signal}} \right\}$$

Standing Wave is defined as sum of forward wave Signal and Backward Wave Signal.

Voltage Standing Wave ratio is defined as the ratio of Vmax to Vmin and it is represented as 'S' $S = \frac{V_{max}}{V_{min}}$

$$S = \frac{V_{inc} + V_{ref}}{V_{inc} - V_{ref}}$$

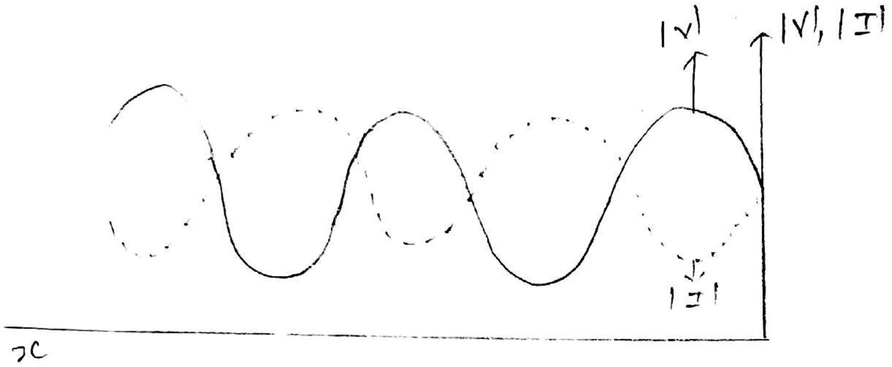
$$= \frac{V_{inc}\left[1 + \frac{V_{ref}}{V_{inc}}\right]}{V_{inc}\left[1 - \frac{V_{ref}}{V_{inc}}\right]}$$



$$\boxed{S = \frac{1+|K|}{1-|K|}}$$

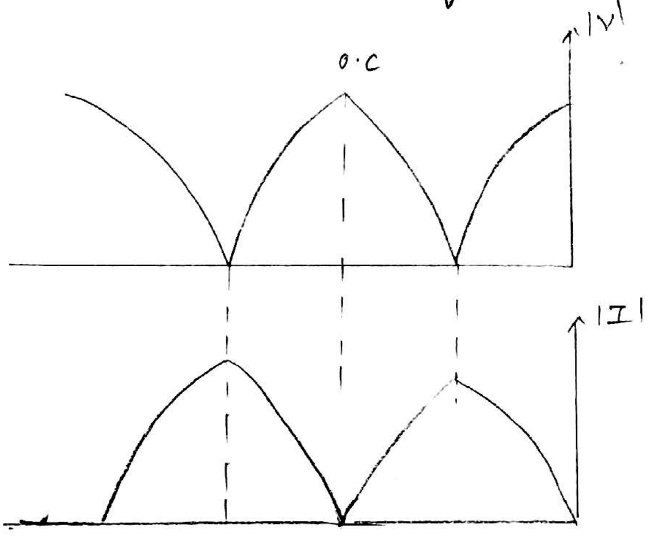
$$|-K| = \frac{Z_L - Z_0}{Z_L + Z_0} \quad , \quad S = \frac{1+|K|}{1-|K|}$$

On the transmission line the maximum Voltage and maximum Current ~~at~~ may not appear at Same line i.e standing wave Signal Voltage and current are shifted on the same transmission line.



for an open circuited transmission line $V_{min} = 0$

Case (ii):- K interms of Current

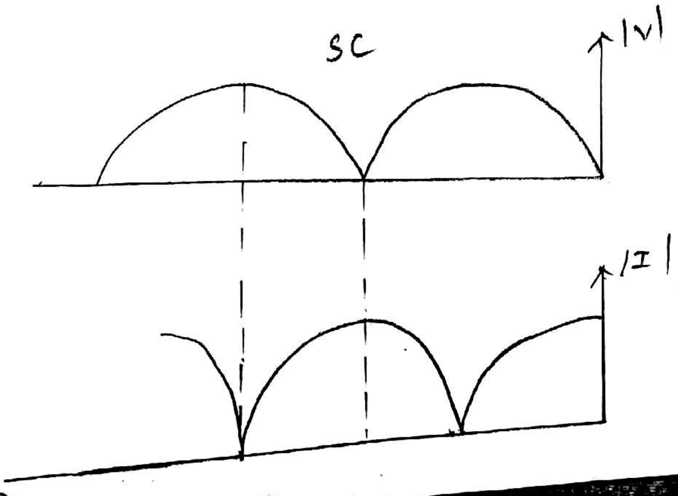


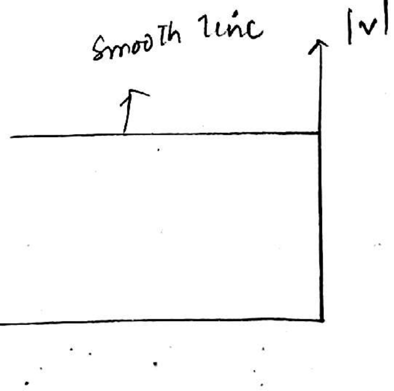
$0 \leq K \leq 1$

$1 \leq S \leq \infty$

$Z_1 = Z_0$

$S = 1 \Rightarrow V_{max} = V_{min}$



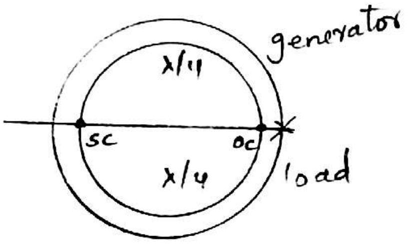


on the Smith chart clockwise indicates towards generator and anticlockwise indicates towards load.

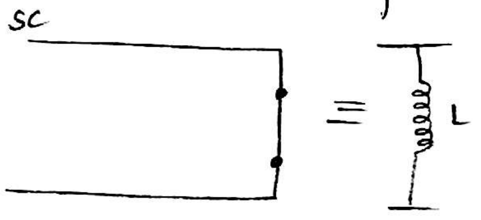
$$\frac{\lambda}{4} + \frac{\lambda}{4} = 360^\circ$$

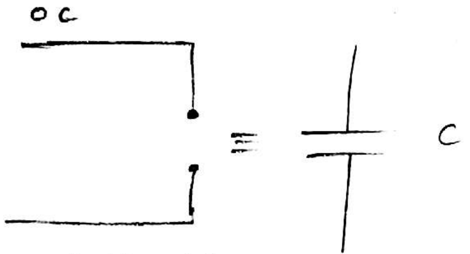
$$\frac{\lambda}{2} = 360^\circ$$

$$|\lambda| = 720^\circ$$

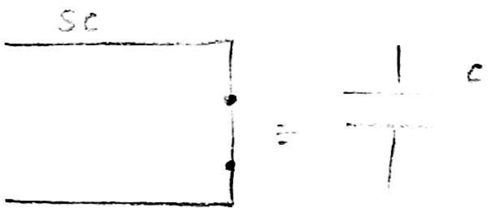


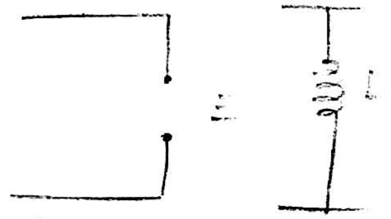
Depending on the length of the transmission line we can achieve inductance and capacitance





$0 < L < \lambda/4$ $\quad$ $0 < L < \lambda/4$





$\frac{\lambda}{4} < L_{sc} < \frac{\lambda}{2}$ $\quad$ $\frac{\lambda}{4} < L_{oc} < \frac{\lambda}{2}$

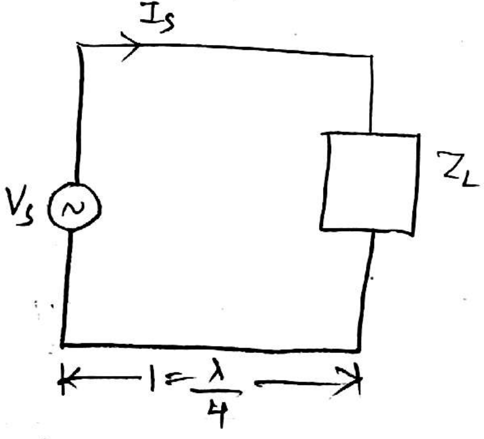
V.v.imp

Applications of transmission lines:

1. Impedance transformation:-

Input impedance of a transmission line depends on its line. Thus a short length transmission line is used as an impedance matching ling (or) impedance transformation.

Case-(i) $\frac{\lambda}{4}$ transmission line



let us consider the transmission line is a lossless

$$\alpha = 0, \gamma = j\beta$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$Bl = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} \qquad (\because \tan\frac{\pi}{2} = \infty)$$

$$Z_{in} = Z_0 \frac{\tan\beta l \left\{ \frac{Z_L}{\tan\beta l} + jZ_0 \right\}}{\tan\beta l \left\{ \frac{Z_0}{\tan\beta l} + jZ_L \right\}}$$

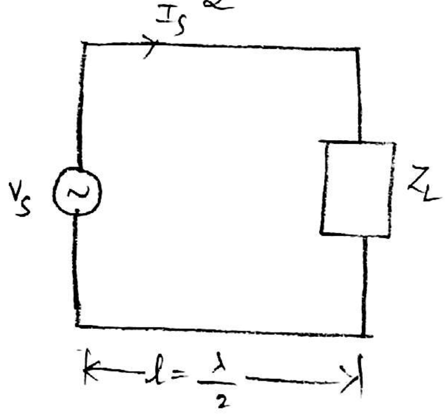
$$= \frac{Z_0 \{0 + jZ_0\}}{\{0 + jZ_L\}} = \frac{Z_0^2 j}{Z_L j}$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$$Z_0 = \sqrt{Z_{in} Z_L}$$

Thus a $\frac{\lambda}{4}$ line is considered as a quarter wave transformer that matches a load impedance of Z_L to a source impedance of Z_{in}

Case-ii) $\frac{\lambda}{2}$ transmission line :-



$$\alpha = 0, \quad \gamma = j\beta$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$$

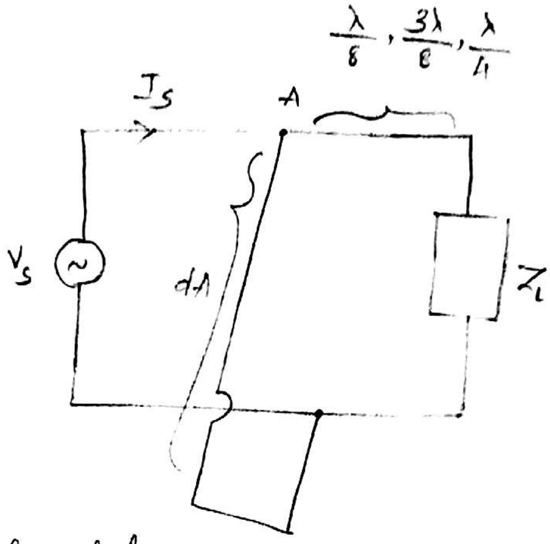
$$Z_{in} = Z_0 \frac{Z_L + 0}{Z_0 + 0}$$

$$\boxed{Z_{in} = Z_L}$$

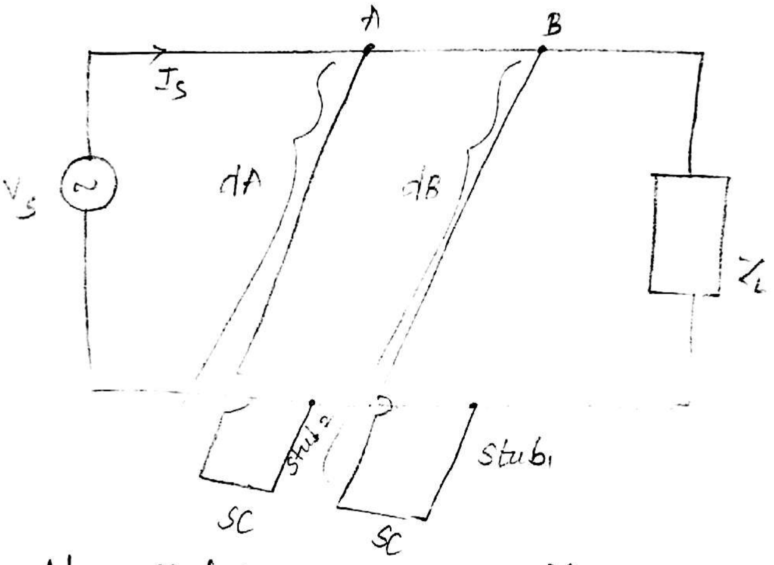
If the load and source cannot be made adjacent then a '$\frac{\lambda}{2}$' line may be connected at the load end point for accurate measurements.

Case-iii : single stub

⇒ single stub is used only for one particular frequency



Case-(iv) double stub:



→ Double stub is used for wide range of frequencies.

Power Calculations on the transmission line:

Here, we need to investigate

i) How much power is efficiently transfer to load

ii) How much amount of reflected power hits the generator with different phase and amplitude

At $l=0$ $V(l)=V(0) = V^+\{1+\Gamma_L\}$ } Fws

$I(l)=I(0) = I^+\{1-\Gamma_L\} = \frac{V^+}{Z_0}[1-\Gamma_L\}$

∴ power delivered to the load

Where $\frac{1}{2}$ is rms value

$$P = \frac{1}{2}VI$$
$$= \frac{1}{2}\frac{|V^+|^2}{Z_0}\{1-|\Gamma_L|^2\}$$

For a matched line $Z_L = Z_0$

$$\text{WKT } \Gamma = K = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$P = \frac{1}{2} \frac{|V^+|^2}{Z_0}$$

problems:-

1. A transmission line with air as dielectric has $Z_0 = 50\Omega$ and a phase constant of 3 rad/m at 10MHzs. find L, C of a transmission line.

Since in the given question the medium is air, it indicates loss less transmission line

$$\alpha = 0, R = 0, G = 0$$

$$Z_0 = \sqrt{\frac{L}{C}} \cancel{\rightarrow}$$

$$50 = \sqrt{\frac{L}{C}} \longrightarrow ①$$

$$\beta = w\sqrt{LC}$$

$$3 = 2\pi \times 10 \times 10^6 \sqrt{LC}$$

$$\sqrt{LC} = \frac{3}{2\pi \times 10 \times 10^6}$$

$$\sqrt{LC} = 47.74 \times 10^{-9} \longrightarrow ②$$

$$① \times ② = \sqrt{\frac{L}{C}} \times \sqrt{LC} = 50 \times 47.74 \times 10^{-9}$$

$$L = 2.387 \times 10^{-6} \text{ H/m}$$

$$\frac{①}{②} = \frac{\sqrt{\frac{L}{C}}}{\sqrt{Le}} = \frac{50}{47.74 \times 10^{-9}}$$

$$C = 10473 \text{ F/m}$$

2. A lossy cable has $R = 2.25\Omega/m$, $L = 1\mu H/m$ $C = 1PF/m$, $G = 0$ operates at $F = 0.5$ GigaHzs. find the attenuation constant of the line

WKT $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\gamma = \sqrt{(2.25 + j2\pi \times 0.5 \times 10^{9} \times 1 \times 10^{-6})(0 + j2\pi \times 0.5 \times 10^{9} \times 1 \times 10^{-12})}$$

$$\gamma = \sqrt{7.06 \times 10^{-3} j - 9.86}$$

$$\gamma = \sqrt{9.86 \angle 179.95}$$

$$= 3.14 \angle 179.95$$

$$= -3.13 + j2.74$$

$$= \alpha + j\beta$$

$$\alpha = -3.13.$$

3. for a transmission line which is terminated at with a normalised impedance (z_L), VSWR S = 2. find the magnitude of normalised impedance.

$S = Z$

$$z_L = \frac{Z_L}{Z_0}$$

$$S = \frac{1+K}{1-K}$$

$$\underline{Z} = \frac{1+K}{1-K}$$

$2 - 2K = 1 + K$

$3K = 1$

$K = \frac{1}{3}$

WKT $K = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

$$K = \frac{z_L - 1}{z_L + 1}$$

$$\frac{1}{3} = \frac{z_L - 1}{z_L + 1}$$

$z_L + 1 = 3z_L - 3$

$2z_L = 4$

$z_L = 2$

V.V. Imp

$$\boxed{1\,Np/m = \overbrace{8.686}^{20\log_{10}^{e}}\,dB/m}$$

V.V. Imp

4. A telephone wire of 20kms long has the following constants per loop km. R = 90Ω, C = 0.062 μF, L = 0.001H and leakage of 1.5×10^{-6} ℧ the line is terminated with its characteristic impedan

Lo and a potential difference of 2.1V having a frequency of 1000HZ is applied at the sending end (Source end). Calculate Zo, λ, phase Velocity (Vp)?

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{90+j(2\pi \times 1000) \times 0.001}{1.5 \times 10^{-6} + j(2\pi \times 1000)(0.062 \times 10^{-6})}}$$

$$= \sqrt{\frac{90+j6.283}{(1.5+j389.56)10^{-6}}}$$

$$= \sqrt{\frac{90.21\angle 3.99^\circ}{389.56 \times 10^{-6}\angle 89.76}}$$

$$= 481.24\angle -85.79^\circ \Omega$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(90+j2\pi \times 1000 \times 0.001)(1.5 \times 10^{-6} + j\, 2\pi \times 1000 \times 0.062 \times 10^{-6})}$$

$$\gamma = 0.187\angle 93.77^\circ = \underbrace{-0.0123}_{\alpha} + j\underbrace{0.187}_{\beta}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.187} = 33.6\,km$$

$$V_p = \frac{\omega}{\beta} = \frac{2\pi \times 1000}{0.187} = 33599.5\,km/s$$

5. A transmission line of 100Ω of a characteristic impedance is Connected to a load of 300Ω. Calculate reflection coeff, VSWR?

$$K = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300-100}{400} = \frac{200}{400} = \frac{1}{2}$$

$$S = \frac{1+K}{1-K} = \frac{1+0.5}{1-0.5} = \frac{1.5}{0.5} = \frac{3/2}{1/2} = 3.$$