

ANTENNAS & WAVE PROPAGATION

**B.TECH
(III YEAR – I SEM)**

Department of Electronics and Communication Engineering

**SVR ENGINEERING COLLEGE
NANDYAL.**



AYYALURU METTA, NANDYAL– 518 503 (A.P)

(Affiliated to JNTUA Anantapur, Approved by AICTE, New Delhi)

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR

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15A04501 ANTENNAS & WAVE PROPAGATION				

Course Objectives:

- Fundamentals of electromagnetic radiation: Maxwell's equations, potential functions, wave equation, retarded potential, short current element, near and far fields, Poynting's theorem.
- Design of antenna arrays: principle of pattern multiplication, broadside and end fire arrays, array synthesis, coupling effects and mutual impedance, parasitic elements, Yagi-Uda antenna.

Course Outcomes:

Upon successful completion of the course, students will be able to:

- Approximate parametric equations for the calculation in the farfield region.
- Write parametric integral expressions for a given current source.
- Calculate electromagnetic fields for a given vector potential.
- Discover pattern multiplication principle for array antennas.

UNIT - I

Antenna Basics & Dipole antennas: Introduction, Basic antenna parameters- patterns, Beam Area, Radiation Intensity, Beam Efficiency, Directivity-Gain-Resolution, Antenna Apertures, Effective height, Fields from oscillating dipole, Field Zones, Shape-Impedance considerations, Polarization – Linear, Elliptical, & Circular polarizations, Antenna temperature, Antenna impedance, Front-to-back ratio, Antenna theorems, Radiation – Basic Maxwell's equations, Retarded potential-Helmholtz Theorem, Radiation from Small Electric Dipole, Quarter wave Monopole and Half wave Dipole – Current Distributions, Field Components, Radiated power, Radiation Resistance, Beam width, Natural current distributions, far fields and patterns of Thin Linear Center-fed Antennas of different lengths, Illustrative problems.

UNIT- II

VHF, UHF and Microwave Antennas - I: Loop Antennas - Introduction, Small Loop, Comparison of far fields of small loop and short dipole, Radiation Resistances and Directives of small and large loops (Qualitative Treatment), Arrays with Parasitic Elements - Yagi - Uda Arrays, Folded Dipoles & their characteristics. Helical Antennas-

Helical Geometry, Helix modes, Practical Design considerations for Monofilar Helical Antenna in Axial and Normal Modes. Horn Antennas- Types, Fermat's Principle, Optimum Horns, Design considerations of Pyramidal Horns, Illustrative Problems.

UNIT - III

VHF, UHF and Microwave Antennas - II: Micro strip Antennas- Introduction, features, advantages and limitations, Rectangular patch antennas- Geometry and parameters, characteristics of Micro strip antennas, Impact of different parameters on characteristics, reflector antennas - Introduction, Flat sheet and corner reflectors, parabola reflectors- geometry, pattern characteristics, Feed Methods, Reflector Types - Related Features, Lens Antennas - Geometry of Non-metallic Dielectric Lenses, Zoning , Tolerances, Applications, Illustrative Problems.

UNIT- IV

Antenna Arrays: Point sources - Definition, Patterns, arrays of 2 Isotropic sources- Different cases, Principle of Pattern Multiplication, Uniform Linear Arrays – Broadside Arrays, Endfire Arrays, EFA with Increased Directivity, Derivation of their characteristics and comparison, BSA with Non-uniform Amplitude Distributions - General considerations and Binomial Arrays, Illustrative problems.

Antenna Measurements: Introduction, Concepts- Reciprocity, Near and Far Fields, Coordination system, sources of errors, Patterns to be Measured, Pattern Measurement Arrangement, Directivity Measurement , Gain Measurements (by comparison, Absolute and 3-Antenna Methods).

UNIT – V

Wave Propagation: Introduction, Definitions, Characterizations and general classifications, different modes of wave propagation, Ray/Mode concepts, Ground wave propagation (Qualitative treatment) - Introduction, Plane earth reflections, Space and surface waves, wave tilt, curved earth reflections, Space wave propagation - Introduction, field strength variation with distance and height, effect of earth's curvature, absorption, Super refraction, M-curves and duct propagation, scattering phenomena, tropospheric propagation, fading and path loss calculations, Sky wave propagation - Introduction, structure of Ionosphere, refraction and reflection of sky waves by ionosphere, Ray path, Critical frequency, MUF, LUF, OF, Virtual height and Skip distance, Relation between MUF and Skip distance, Multi-HOP propagation, Energy loss in ionosphere, Summary of Wave Characteristics in different frequency ranges, Illustrative problems.

TEXT BOOKS:

1. John D. Kraus and Ronald J. Marhefka and Ahmad S.Khan, "Antennas and wave propagation," TMH, New Delhi, 4th Ed., (special Indian Edition), 2010.
2. E.C. Jordan and K.G. Balmain, "Electromagnetic Waves and Radiating Systems," PHI, 2ndEdn, 2000.

REFERENCES:

1. C.A. Balanis, "Antenna Theory- Analysis and Design," John Wiley & Sons, 2ndEdn., 2001.
2. K.D. Prasad, SatyaPrakashan, "Antennas and Wave Propagation," Tech. India Publications, New Delhi, 2001.

(19A04502) ANTENNAS AND WAVE PROPAGATION

Course Objectives:

- To introduce radiation mechanisms and basic characteristics of antennas.
- To derive mathematical expressions and their application for complete design of antennas.
- To demonstrate various modes of EM wave propagation.
- To explain measurement of antenna parameters
- To introduce design concepts of various types of antennas including micro strip antenna.

UNIT- I

Antenna Characteristics: Radiation mechanism and current distribution, radiation pattern, directivity, gain, Input impedance, polarization, bandwidth, HPBW. Reciprocity, equivalence of radiation and receive patterns, equivalence of impedances, effective aperture, vector effective length, antenna temperature, Friis transmission formula, problem solving.

Learning Outcomes:

At the end of this unit, the student will be able to

- Understand radiation mechanism and basic antenna characteristics. (L1)
- Compute radiation intensity, gain and directivity of antennas. (L2)

UNIT- II

Wire and Antenna Arrays: Wire and antenna arrays: Radiation resistance and directivity and other characteristics of short dipole, monopole, half-wave dipole, small loop antenna.

Linear array and pattern multiplication, two-element array, uniform array, binomial array, broadside and end-fire arrays.

Rhombic antennas, Yagi-Uda array, Turnstile Antenna, Helical antenna - axial and normal modes, log-periodic Array, spiral antenna.

Learning Outcomes:

At the end of this unit, the student will be able to

- Derive expressions for radiation resistance, directivity of wire antennas. (L3)
- Obtain radiation pattern of various array antennas using pattern multiplication. (L3)
- Compare radiation pattern and other antenna parameters of broadside and endfire array antennas. (L5)
- To know the design aspects of antenna arrays. (L4)

UNIT- III

Aperture Antennas and Lens Antennas: Aperture Antennas and Lens Antennas: Slot antenna, pyramidal and conical horn antennas, reflector Antenna: flat plate, corner and parabolic reflectors - common curved reflector shapes, Feed mechanisms.

Lens Antennas - Introduction, Geometry of Non-metallic Dielectric Lenses, Zoning, Tolerances, Applications.

Learning Outcomes:

At the end of this unit, the student will be able to

- Understand basic principles of aperture and lens antennas. (L1)
- Design aperture and lens antennas. (L4)

UNIT- IV

Micro-Strip Antennas And Antenna Measurements: Micro-strip Antennas and Antenna Measurements: Basic characteristics, feeding methods, methods of analysis - Design of Rectangular and Circular Patch Antennas, Introduction to Smart Antennas - Concept of adaptive beam forming, Measurement of Antenna Parameters, basic setup, radiation pattern measurement, gain, directivity.

Learning Outcomes:

At the end of this unit, the student will be able to

- Describe feeding methods for micro-strip antennas. (L2)
- Apply the concepts to measure antenna parameters. (L2)
- Design rectangular and circular patch antennas for given specifications. (L4)

UNIT- V

Wave Propagation - I: Introduction, Definitions, Categorizations and General Classifications, Different Modes of Wave Propagation, Ray/Mode Concepts, Ground Wave Propagation (Quantitative Treatment) - Introduction, Plane Earth Reflections, Space and Surface Waves, Wave Tilt, Curved Earth Reflections, Space Wave Propagation - Introduction, Field Strength

Variation with Distance and Height, Effect of Earth's Curvature, Absorption, Super retraction, M- Curves and Duct Propagation, Scattering Phenomena, Tropospheric Propagation.

Wave Propagation - II: Sky Wave Propagation - Introduction, Structure of Ionosphere, Refraction and Reflection of Sky Waves by Ionosphere, Ray Path, Critical Frequency, MUF, LUF, OF, Virtual Height and Skip Distance, Relation between MUF and skip Distance, Multi-hop Propagation, illustrative problems.

Learning Outcomes:

At the end of this unit, the student will be able to

- Understand effects of earth's magnetic field on wave propagation (L1)
- Apply the concepts to solve problems related to wave propagation(L2)
- Analyze tropospheric propagation and derive the expression for received field strength (L3)
- Identify layers in ionosphere and their ionization densities (L1)

Course Outcomes:

- Understand various antenna parameters, principle of operation of various antennas viz. wired, aperture, micro strip antennas.
- Discuss various EM wave propagation methods in ionosphere and troposphere
- Analyze mathematical aspects of wave propagation, Derive expressions related to radiation mechanisms for antennas
- Design various antennas namely array, micro strip, horn, lens and aperture antennas, etc., for a given application.
- Compare performance of various antennas.

TEXT BOOKS:

1. John D. Kraus, Ronald J. Marhefka, Ahmad S. Khan, "Antennas and Wave Propagation", 4thEdition, TMH, 2010.
2. Jordan, E.C. and Balmain. K. G., "Electromagnetic Waves and Radiating Systems", Prentice-Hall Publications.

REFERENCES:

1. Constantine A. Balanis, "Antenna Theory-Analysis and Design", Wiley Publication, 2016.
2. K.D. Prasad, "Antenna & Wave Propagation", Satya Prakash Publications, 2009.
3. Matthew N.O.Sadiku, "Principle of Electromagnetics", 4th edition, Oxford (International), 2012.

Unit – 1

Antenna Basics: Introduction, basic Antenna parameters, patterns, beam area, radiation intensity, beam efficiency, directivity and gain, antenna apertures, effective height, bandwidth, radiation efficiency, antenna temperature and antenna field zones.

Introduction:-

It is a source or radiator of EM waves, or a sensor of EM waves. It is a transition device or transducer between a guided wave and a free space wave or vice versa. It is an electrical conductor or system of conductors that radiates EM energy into or collects EM energy from free space. It is an impedance matching device, coupling EM waves between Transmission line and free space or vice versa.

Some Antenna Types

- Wire Antennas- dipoles, loops and Helical
- Aperture Antennas-Horns and reflectors
- Array Antennas-Yagi, Log periodic
- Patch Antennas- Microstrips, PIFAs

Principle- Under time varying conditions, Maxwell's equations predict the radiation of EM energy from current source (or accelerated charge). This happens at all frequencies, but is insignificant as long as the size of the source region is not comparable to the wavelength. While transmission lines are designed to minimize this radiation loss, radiation into free space becomes main purpose in case of Antennas. For steady state harmonic variation, usually we focus on time changing current. For transients or pulses, we focus on accelerated charge. The radiation is perpendicular to the acceleration. The radiated power is proportional to the square of

$$I L \text{ or } Q V$$

Where

I = Time changing current in Amps/sec

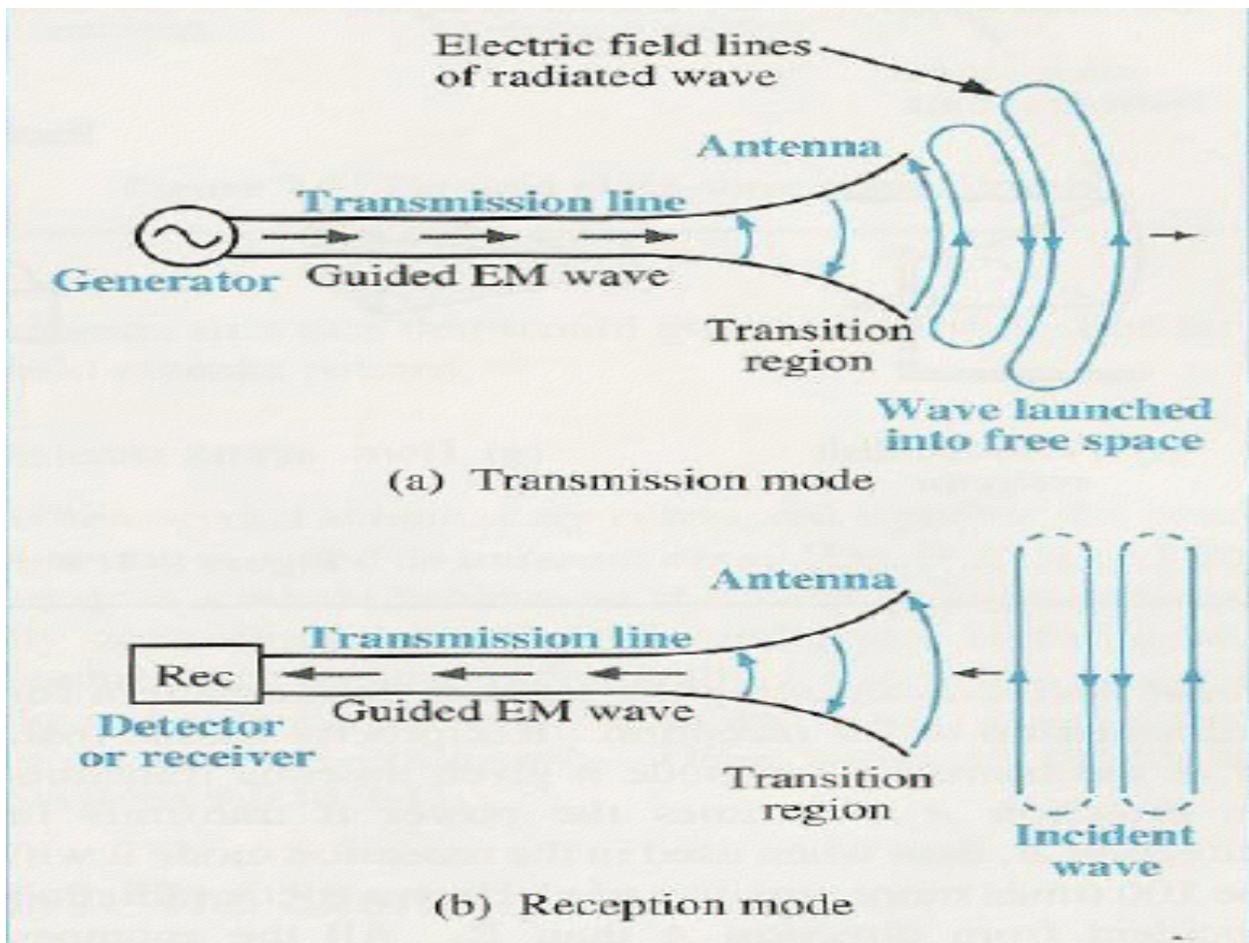
L = Length of the current element in meters

Q = Charge in Coulombs

$V =$ Time changing velocity

Transmission line opened out in a Tapered fashion as Antenna:

a) As Transmitting Antenna: –Here the Transmission Line is connected to source or generator at one end. Along the uniform part of the line energy is guided as Plane TEM wave with little loss. Spacing between line is a small fraction of λ . As the line is opened out and the separation b/n the two lines becomes comparable to λ , it acts like an antenna and launches a free space wave since currents on the transmission Line flow out on the antenna but fields associated with them keep on going. From the circuit point of view the antennas appear to the tr. lines As a resistance R_r , called Radiation resistance



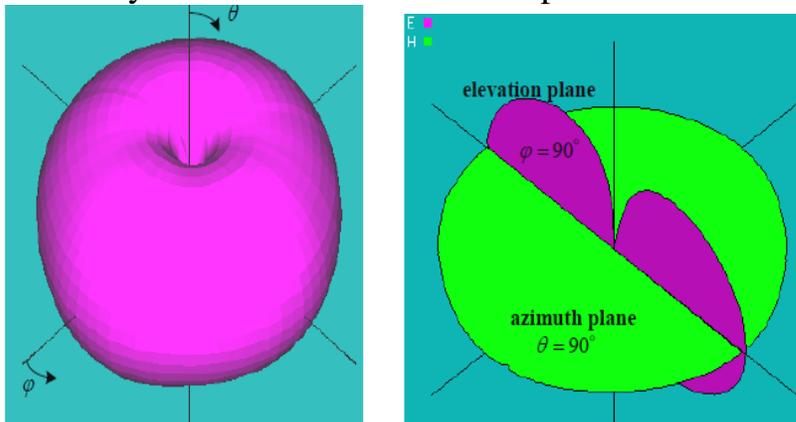
b) As Receiving Antenna –Active radiation by other Antenna or Passive radiation from distant objects raises the apparent temperature of R_r . This has nothing to do with the physical temperature of the antenna itself but is

related to the temperature of distant objects that the antenna is looking at. R_r may be thought of as virtual resistance that does not exist physically but is a quantity coupling the antenna to distant regions of space via a virtual transmission line.

Reciprocity-An antenna exhibits identical impedance during Transmission or Reception, same directional patterns during Transmission or Reception, same effective height while transmitting or receiving. Transmission and reception antennas can be used interchangeably. Medium must be linear, passive and isotropic (physical properties are the same in different directions.) Antennas are usually optimised for reception or transmission, not both.

Patterns

The radiation pattern or *antenna pattern* is the graphical representation of the radiation properties of the antenna as a function of space. That is, the antenna's pattern describes how the antenna radiates energy out into space (or how it receives energy). It is important to state that an antenna can radiate energy in all directions, so the antenna pattern is actually three-dimensional. It is common, however, to describe this 3D pattern with two planar patterns, called the *principal plane patterns*. These principal plane patterns can be obtained by making two slices through the 3D pattern, through the maximum value of the pattern. It is these principal plane patterns that are commonly referred to as the antenna patterns.



Radiation pattern or Antenna pattern is defined as the spatial distribution of a 'quantity' that characterizes the EM field generated by an antenna.

The 'quantity' may be Power, Radiation Intensity, Field amplitude, Relative Phase etc.

Normalized patterns

It is customary to divide the field or power component by its maximum value and plot the normalized function. Normalized quantities are dimensionless and are quantities with maximum value of unity

$$\text{Normalized Field Pattern } E_{\theta}(\theta, \phi)_n = \frac{E_{\theta}(\theta, \phi)}{E_{\theta}(\theta, \phi)_{\max}}$$

Half power level occurs at those angles (θ, Φ) for which $E_{\theta}(\theta, \Phi)_n = 0.707$
 At distance $d \gg \lambda$ and $d \gg$ size of the antenna, the shape of the field pattern is independent of the distance

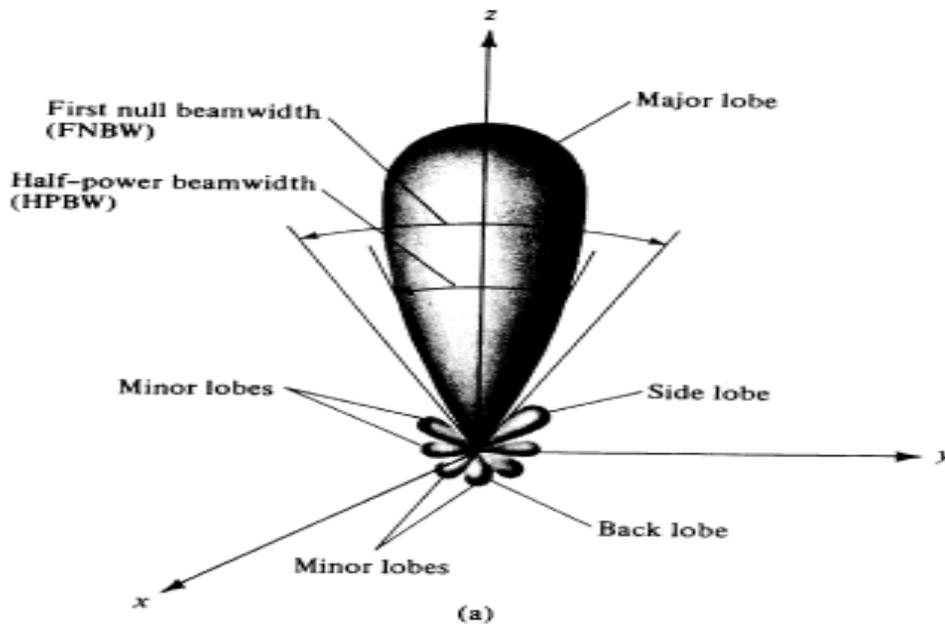
$$\text{Normalized Power Pattern } = P_n(\theta, \phi)_n = \frac{S(\theta, \phi)}{S(\theta, \phi)_{\max}}$$

where

$$S(\theta, \phi) = \frac{[E_{\theta}^2(\theta, \phi) + E_{\phi}^2(\theta, \phi)]}{Z_0} \text{ W/m}^2$$

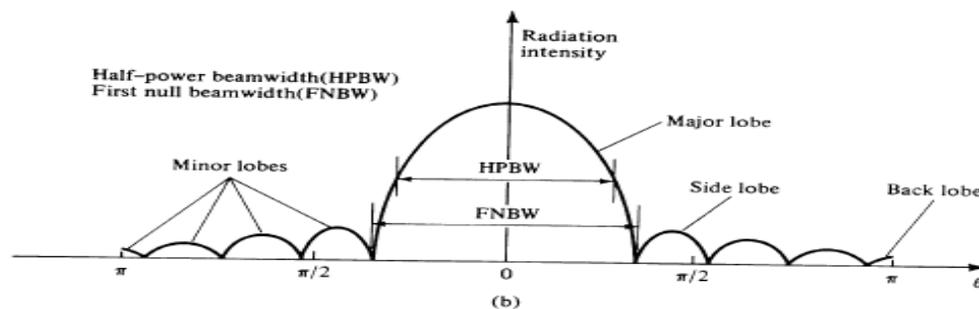
is the Poynting vector. Half power level occurs at those angles (θ, Φ) for which $P(\theta, \Phi)_n = 0.5$

Pattern lobes and beam widths



Pattern in spherical co-ordinate system

Beamwidth is associated with the lobes in the antenna pattern. It is defined as the angular separation between two identical points on the opposite sides of the main lobe. The most common type of beamwidth is the half-power (3 dB) beamwidth (HPBW). To find HPBW, in the equation, defining the radiation pattern, we set power equal to 0.5 and solve it for angles. Another frequently used measure of beamwidth is the first-null beamwidth (FNBW), which is the angular separation between the first nulls on either sides of the main lobe.



Pattern in Cartesian co-ordinate system

Beamwidth defines the resolution capability of the antenna: i.e., the ability of the system to separate two adjacent targets

Examples :

1. An antenna has a field pattern given by $E(\theta) = \cos^2 \theta$ for $0^\circ \leq \theta \leq 90^\circ$. Find the Half power beamwidth (HPBW)

$E(\theta)$ at half power = 0.707

Therefore, $\cos^2 \theta = 0.707$ at Halfpower point

i.e., $\theta = \cos^{-1}[(0.707)^{1/2}] = 33^\circ$

HPBW = $2\theta = 66^\circ$

2. Calculate the beamwidths in x-y and y-z planes of an antenna, the power pattern of which is given by

$$U(\theta, \phi) = \begin{cases} \sin^2 \theta \sin \phi; & 0 \leq \theta \leq \pi, 0 \leq \phi \leq \pi \\ 0; & \pi \leq \theta \leq 2\pi, \pi \leq \phi \leq 2\pi \end{cases}$$

soln: In the x-y plane, $\theta = \pi/2$ and power pattern is given by $U(\pi/2, \Phi) = \sin \Phi$

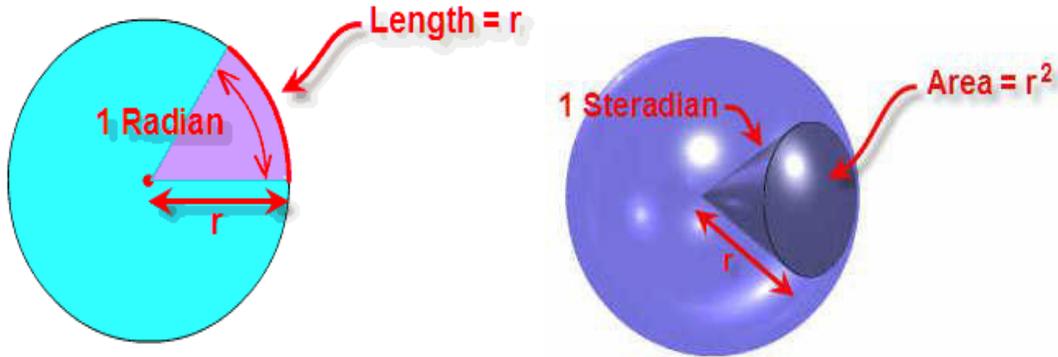
- Therefore half power points are at $\sin \Phi = 0.5$, i.e., at $\Phi = 30^\circ$ and $\Phi = 150^\circ$
- Hence 3dB beamwidth in x-y plane is $(150 - 30) = 120^\circ$
- In the y-z plane, $\Phi = \pi/2$ and power pattern is given by $U(\theta, \pi/2) = \sin^2 \theta$
- Therefore half power points are at $\sin^2 \theta = 0.5$, i.e., at $\theta = 45^\circ$ and $\theta = 135^\circ$
- Hence 3dB beamwidth in y-z plane is $(135 - 45) = 90^\circ$

Beam area or Beam solid angle Ω_A

Radian and Steradian: Radian is plane angle with its vertex at the centre of a circle of radius r and is subtended by an arc whose length is equal to r . Circumference of the circle is $2\pi r$. Therefore total angle of the circle is 2π radians.

Steradian is solid angle with its vertex at the centre of a sphere of radius r , which is subtended by a spherical surface area equal to the area of a square with side length r

Area of the sphere is $4\pi r^2$. Therefore the total solid angle of the sphere is 4π steradians



$$\begin{aligned}
 1 \text{ steradian} &= (1 \text{ radian})^2 \\
 &= (180 / \pi)^2 \\
 &= 3282.8 \text{ Square degrees}
 \end{aligned}$$

The infinitesimal area ds on a surface of a sphere of radius r in spherical coordinates (with θ as vertical angle and Φ as azimuth angle) is

$$ds = r^2 \sin \theta d\theta d\phi$$

By definition of solid angle $ds = r^2 d\Omega$

$$\therefore d\Omega = \sin \theta d\theta d\phi$$

Beam area is the solid angle Ω_A for an antenna, is given by the integral of the normalized power pattern over a sphere (4π steradians)

i.e.,

$$\Omega_A = \int_0^{2\pi} \int_0^\pi P_n(\theta, \phi) d\Omega$$

$$d\Omega = \sin \theta d\theta d\phi$$

Beam area is the solid angle through which all of the power radiated by the antenna would stream if $P(\theta, \Phi)$ maintained its maximum value over Ω_A and was zero elsewhere.

i.e., Power radiated = $P(\theta, \Phi) \Omega_A$ watts

Beam area is the solid angle Ω_A is often approximated in terms of the angles subtended by the Half Power points of the main lobe in the two principal planes (Minor lobes are neglected)

$$\Omega_A \approx \theta_{HP} \phi_{HP}$$

Example

An antenna has a field pattern given by $E(\theta) = \cos^2\theta$ for $0^\circ \leq \theta \leq 90^\circ$. Find the Beam area of the pattern. Also find Approximate beam area using Half Power Beamwidths

$$\Omega_A = \int_0^{2\pi} \int_0^\pi P_n(\theta, \phi) d\Omega$$

$$d\Omega = \sin\theta d\theta d\phi$$

$$\Omega_A = \int_0^{2\pi} \int_0^\pi \cos^4(\theta) \sin\theta d\theta d\phi$$

$$\Omega_A = -2\pi \left[\frac{1}{25} \cos^5(\theta) \right]_0^\pi = \frac{2\pi}{5} = 1.26 \text{ Sr}$$

Radiation Intensity

Definition: The power radiated from an Antenna per unit solid angle is called the Radiation Intensity. U Units: Watts/Steradians

Poyting vector or power density is dependant on distance from the antenna while Radiation intensity is independent of the distance

Beam efficiency

The total beam area Ω_A consists of Main beam area Ω_M and minor lobe area Ω_m

$$\therefore \Omega_A = \Omega_M + \Omega_m$$

'Beam efficiency' is defined by $\epsilon_M = \frac{\Omega_M}{\Omega_A}$

And 'stray factor' is $\epsilon_m = \frac{\Omega_m}{\Omega_A}$

$$\therefore \epsilon_M + \epsilon_m = 1$$

Directivity and Gain

From the field point of view, the most important quantitative information on the antenna is the directivity, which is a measure of the concentration of radiated power in a particular direction. It is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. The average radiation intensity is equal to the total radiated power divided by 4π . If the direction is not specified, the direction of maximum radiation is implied. Mathematically, the directivity (dimensionless) can be written as

$$D = \frac{U(\theta, \phi)_{\max}}{U(\theta, \phi)_{\text{average}}}$$

The directivity is a dimensionless quantity. The maximum directivity is always ≥ 1

Directivity and Beam area

$$P(\theta, \phi)_{\text{Av}} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} P(\theta, \phi) \sin\theta d\theta d\phi$$

$$= \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} P(\theta, \phi) d\Omega$$

$$\therefore D = \frac{P(\theta, \phi)_{\max}}{\frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} P(\theta, \phi) d\Omega}$$

$$D = \frac{1}{\frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} P_n(\theta, \phi) d\Omega}$$

$$\text{i.e., } D = \frac{4\pi}{\Omega_A}$$

Directivity is the ratio of total solid angle of the sphere to beam solid angle. For antennas with rotationally symmetric lobes, the directivity D can be approximated as

$$D \approx \frac{4\pi}{\theta_{HP} \phi_{HP}}$$

- Directivity of isotropic antenna is equal to unity, for an isotropic antenna Beam area $\Omega_A = 4\pi$
- Directivity indicates how well an antenna radiates in a particular direction in comparison with an isotropic antenna radiating same amount of power
- Smaller the beam area, larger is the directivity

Gain: Any physical Antenna has losses associated with it. Depending on structure both ohmic and dielectric losses can be present. Input power P_{in} is the sum of the Radiated power P_{rad} and losses P_{loss}

$$P_{in} = P_{rad} + P_{loss}$$

The Gain G of an Antenna is an actual or realized quantity which is less than Directivity D due to ohmic losses in the antenna. Mismatch in feeding the antenna also reduces gain

The ratio of Gain to Directivity is the Antenna efficiency factor k (dimensionless)

$$\therefore G = kD$$

$$0 \leq k \leq 1$$

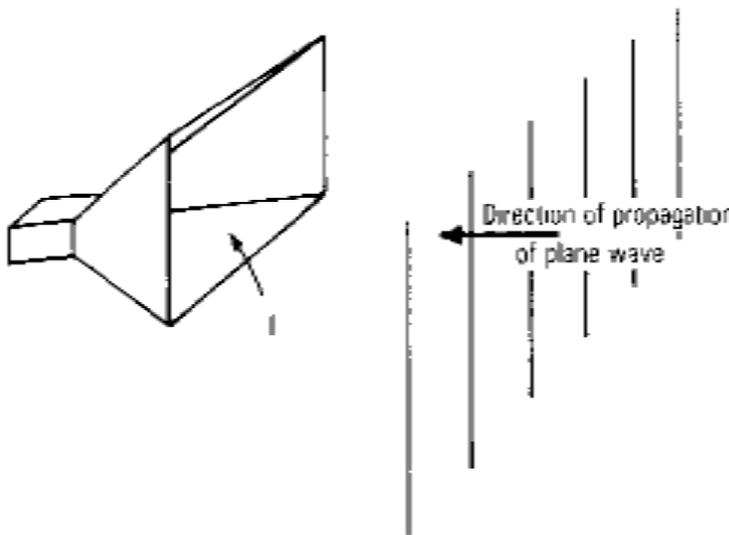
In practice, the total input power to an antenna can be obtained easily, but the total radiated power by an antenna is actually hard to get. The *gain of an antenna is introduced to solve* this problem. This is defined as the ratio of the radiation intensity in a given direction from the antenna to the total input power accepted by the antenna divided by 4π . *If the direction is not specified, the direction of maximum radiation is implied.* Mathematically, the gain (dimensionless) can be written as

$$G = \frac{4\pi U}{P_{in}}$$

Directivity and Gain: Directivity and Gain of an antenna represent the ability to focus its beam in a particular direction
 Directivity is a parameter dependant only on the shape of radiation pattern while gain takes ohmic and other losses into account

Effective Aperture

Aperture Concept: Aperture of an Antenna is the area through which the power is radiated or received. Concept of Apertures is most simply introduced by considering a Receiving Antenna. Let receiving antenna be a rectangular Horn immersed in the field of uniform plane wave as shown



Let the poynting vector or power density of the plane wave be S watts/sq –m and let the area or physical aperture be A_p sq-m.If the Horn extracts all the power from the Wave over its entire physical Aperture A_p , Power absorbed is given by

$$P=SA_p= (E^2/Z)A_p \text{ Watts,}$$

- S is poynting vector ,
- Z is intrinsic impedance of medium,
- E is rms value of electric field

But the Field response of Horn is not uniform across A_p because E at sidewalls must equal zero. Thus effective Aperture A_e of the Horn is less than A_p

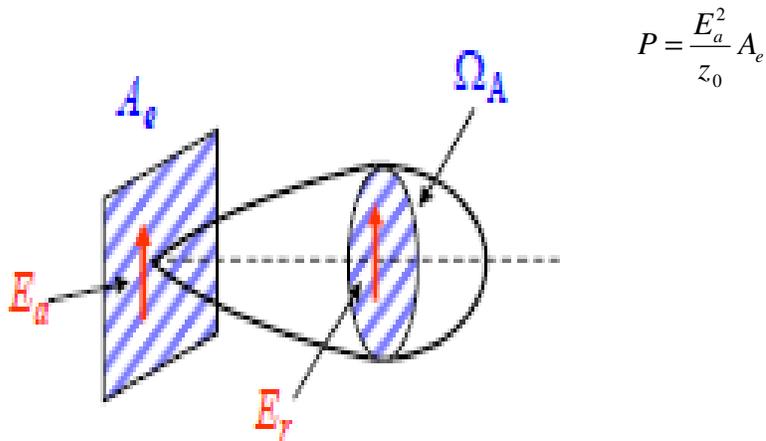
Aperture efficiency is defined as $\epsilon_{ap} = \frac{A_e}{A_p}$

The effective antenna aperture is the ratio of the available power at the terminals of the antenna to the power flux density of a plane wave incident upon the antenna, which is matched to the antenna in terms of polarization. If no direction is specified, the direction of maximum radiation is implied. Effective Aperture (A_e) describes the effectiveness of an Antenna in receiving mode, It is the ratio of power delivered to receiver to incident power density

It is the area that captures energy from a passing EM wave

An Antenna with large aperture (A_e) has more gain than one with smaller aperture (A_e) since it captures more energy from a passing radio wave and can radiate more in that direction while transmitting

Effective Aperture and Beam area: Consider an Antenna with an effective Aperture A_e which radiates all of it's power in a conical pattern of beam area Ω_A , assuming uniform field E_a over the aperture, power radiated is



$$P = \frac{E_a^2}{z_0} A_e$$

Assuming a uniform field E_r in far field at a distance r , Power Radiated is

also given by $P = \frac{E_r^2}{z_0} r^2 \Omega_A$

Equating the two and noting that $E_r = E_a A_e / r \lambda$ we get Aperture –Beam area relation

$$\lambda^2 = A_e \Omega_A$$

At a given wavelength if Effective Aperture is known, Beam area can be determined or vice-versa

Directivity in terms of beam area is given by $D = \frac{4\pi}{\Omega_A}$

Aperture and beam area are related by $\lambda^2 = A_e \Omega_A$

Directivity can be written as $D = \frac{4\pi}{\lambda^2} A_e$

Other antenna equivalent areas :

Scattering area : It is the area, which when multiplied with the incident wave power density, produces the re-radiated (scattered) power

Loss area : It is the area, which when multiplied by the incident wave power density, produces the dissipated (as heat) power of the antenna

Capture area: It is the area, which when multiplied with the incident wave power density, produces the total power intercepted by the antenna.

Effective height

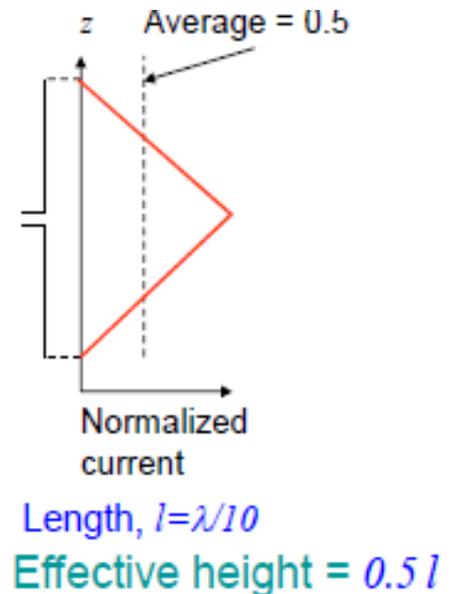
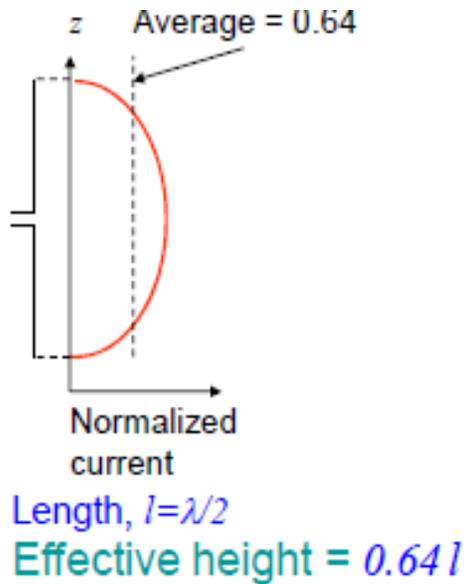
The effective height is another parameter related to the apertures.

Multiplying the effective height, h_e (meters), times the magnitude of the incident electric field E (V/m) yields the voltage V induced. Thus

$V = h_e E$ or $h_e = V / E$ (m). Effective height provides an indication as to how much of the antenna is involved in radiating (or receiving). To demonstrate this, consider the current distributions on a dipole antenna for two different lengths.

If the current distribution of the dipole were uniform, its effective height would be l . Here the current distribution is nearly sinusoidal with average value $2/\pi = 0.64$ (of the maximum) so that its effective height is $0.64l$. It is assumed that antenna is oriented for maximum response.

If the same dipole is used at longer wavelength so that it is only 0.1λ long, the current tapers almost linearly from the central feed point to zero at the ends in a triangular distribution. The average current is now 0.5 & effective height is $0.5l$



For an antenna of radiation resistance R_r matched to it's load , power delivered to load is $P = V^2 / (4R_r)$, voltage is given by $V = h_e E$.

Therefore $P = (h_e E)^2 / (4R_r)$

In terms of Effective aperture the same power is given by

$P = S A_e = (E^2 / Z_0) A_e$

Equating the two,

$$P = \frac{h_e^2 E^2}{4R_r} = \frac{E^2}{Z_0} A_e \Rightarrow h_e = \sqrt{\frac{4R_r A_e}{Z_0}} \text{ (m) and } A_e = \frac{h_e^2 Z_0}{4R_r} \text{ (m}^2\text{)}$$

Notes: the above calculations assume that the electric field is constant over the antenna Z_0 is the intrinsic impedance of free space = $120\pi \Omega$ or 377Ω

Bandwidth or frequency bandwidth

This is the range of frequencies, within which the antenna characteristics (input impedance, pattern) conform to certain specifications . Antenna characteristics, which should conform to certain requirements, might be: input impedance, radiation pattern, beamwidth, polarization, side-lobe level, gain, beam direction and width, radiation efficiency. Separate bandwidths may be introduced: impedance bandwidth, pattern bandwidth, etc.

The FBW of broadband antennas is expressed as the ratio of the upper to the lower frequencies, where the antenna performance is acceptable.

Based on Bandwidth antennas can be classified as

1. Broad band antennas-BW expressed as ratio of upper to lower frequencies of acceptable operation eg: 10:1 BW means f_H is 10 times greater than f_L
2. Narrow band antennas-BW is expressed as percentage of frequency difference over centre frequency eg:5% means $(f_H - f_L) / f_0$ is .05.
Bandwidth can be considered to be the range of frequencies on either sides of a centre frequency(usually resonant freq. for a dipole)

The FBW of broadband antennas is expressed as the ratio of the upper to the lower frequencies, where the antenna performance is acceptable

$$FBW = f_{\max} / f_{\min} .$$

Broadband antennas with FBW as large as 40:1 have been designed. Such antennas are referred to as *frequency independent antennas*.

For narrowband antennas, the FBW is expressed as a percentage of the frequency difference over the center frequency

$$FBW = \frac{f_{\max} - f_{\min}}{f_0} \cdot 100 \% .$$

Usually, $f_0 = (f_{\max} + f_{\min}) / 2$ or $f_0 = \sqrt{f_{\max} f_{\min}}$.

The characteristics such as Z_i , G , Polarization etc of antenna does not necessarily vary in the same manner. Some times they are critically affected by frequency Usually there is a distinction made between pattern and input impedance variations. Accordingly pattern bandwidth or impedance bandwidth are used .pattern bandwidth is associated with characteristics such as Gain, Side lobe level, Polarization, Beam area.

(large antennas)

Impedance bandwidth is associated with characteristics such as input impedance, radiation efficiency(Short dipole)

Intermediate length antennas BW may be limited either by pattern or impedance variations depending on application

If BW is Very large (like 40:1 or greater), Antenna can be considered frequency independent.

Radiation Efficiency

Total antenna resistance is the sum of 5 components

$$R_r + R_g + R_i + R_c + R_w$$

R_r is Radiation resistance

R_g is ground resistance

R_i is equivalent insulation loss

R_c is resistance of tuning inductance

R_w is resistance equivalent of conductor loss

Radiation efficiency = $R_r / (R_r + R_g + R_i + R_c + R_w)$. It is the ratio of power radiated from the antenna to the total power supplied to the antenna

Antenna temperature

The antenna noise can be divided into two types according to its physical source:

- noise due to the loss resistance of the antenna itself; and
- noise, which the antenna picks up from the surrounding environment

The noise power per unit bandwidth is proportional to the object's temperature and is given by Nyquist's relation

$$p_h = kT_p, \text{ W/Hz}$$

where

T_p is the physical temperature of the object in K (Kelvin degrees); and

k is Boltzmann's constant (1.38×10^{-23} J/K)

A resistor is a thermal noise source. The noise voltage (rms value) generated by a resistor R , kept at a temperature T , is given by

$$V_n = \sqrt{4kTBR}$$

Where

k is Boltzmann's constant (1.38×10^{-23} J/K). And

B is the bandwidth in Hz

Often, we assume that heat energy is evenly distributed in the frequency band Δf . Then, the associated heat power in Δf is

$$P_h = kT_P \Delta f, \text{ W.}$$

For a temperature distribution $T(\theta, \Phi)$ and radiation pattern $R(\theta, \Phi)$ of the antenna,

Then noise temperature T_A is given by

$$T_A = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi R(\theta, \phi) \cdot T(\theta, \phi) \sin \theta d\theta d\phi$$

The noise power P_{TA} received from an antenna at temperature T_A can be expressed in terms of Bandwidth B over which the antenna (and its Receiver) is operating as

$$P_{TA} = kT_A B$$

The receiver also has a temperature T_R associated with it and the total system noise temperature (i.e., Antenna + Receiver) has combined temperature given by $T_{\text{sys}} = T_A + T_R$

And total noise power in the system is $P_{\text{Total}} = kT_{\text{sys}} B$

Antenna Field Zones

The space surrounding the antenna is divided into three regions according to the predominant field behaviour. The boundaries between the regions are not distinct and the field behaviour changes gradually as these boundaries are crossed. In this course, we are mostly concerned with the far-field characteristics of the antennas.

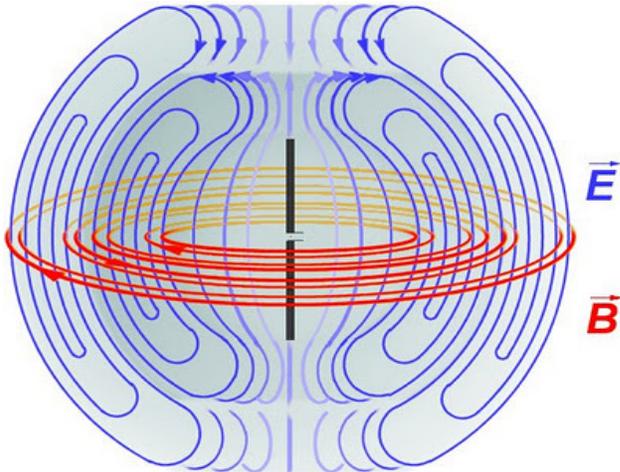


Fig: Radiation from a dipole

1. Reactive near-field region: This is the region immediately surrounding the antenna, where the reactive field dominates. For most antennas, it is assumed that this region is a sphere with the antenna at its centre

2. Radiating near-field (Fresnel) region : This is an intermediate region between the reactive near-field region and the far-field region, where the radiation field is more significant but the angular field distribution is still dependent on the distance from the antenna.

3. Far-field (Fraunhofer) region : Here $r \gg D$ and $r \gg \lambda$
The angular field distribution does not depend on the distance from the source any more, i.e., the far-field pattern is already well established.

-----X-----X-----X-----

The Electric Dipoles and Thin Linear Antennas

Short Electric dipole:

Any linear antenna may be considered as consisting of a large number of very short conductors connected in series. A short linear conductor is often called a 'short dipole'. A short dipole is always of finite length even though it may be very short. If the dipole is vanishingly short it is an infinite single dipole.

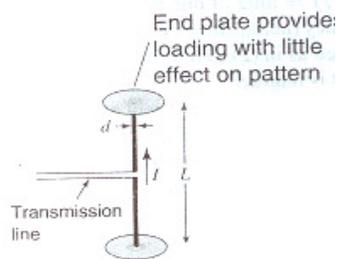


Fig 3.1: A short dipole antenna



Fig 3.2: Equivalent of short dipole antenna

Consider a short dipole as shown in figure 3.1, the length L is very short compared to the wavelength [$L \ll \lambda$]. The current I along the entire length is assumed to be uniform. The diameter d of the dipole is small compared to its length [$d \ll L$]. Thus the equivalent of short dipole is as shown in figure (b). It consists of a thin conductor of length L with uniform current I and point charges at the ends. The current and charge are related by

$$\frac{dq}{dt} = I \quad \text{-----(3.1)}$$

The fields of a short dipole:

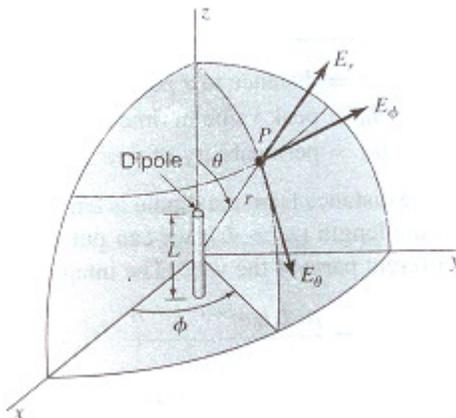


Fig3.3: Relation of dipole to co-ordinates

Consider a dipole of length 'L' placed coincident with the z-axis with its center at the origin. The electric and magnetic fields due to the dipoles can

be expressed in terms of vector and scalar potentials. The relation of electric field E_r , E_θ and E_ϕ is as shown in figure 3.3. It is assumed that the medium surrounding the dipole is air.

Retardation effect:

In dealing with antennas, the propagation time is a matter of great importance. Thus if a current is flowing in the short dipole. The effect of the current is not felt instantaneously at the point P, but only after an interval equal to the time required for the disturbance to propagate over the distance 'r'. This is known as retardation effect.

When retardation effect is considered instead of writing current I as $I = I_0 e^{j\omega t}$ which implies instantaneous propagation of the effect of the current, we introduce propagation time as

$$[I] = I_0 e^{j\omega(t - \frac{r}{c})} \quad \text{----- (3.2)}$$

Where [I] is called retarded current
 c - Velocity of propagation

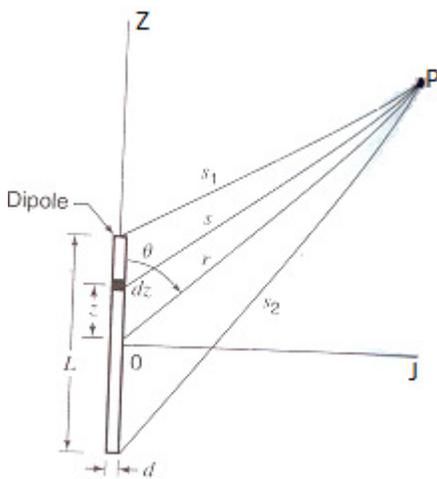


Fig 3.3: Geometry for short dipole

For a dipole as shown in the above figure the retarded vector potential of the electric current has only one component namely A_3 and it is given by

$$A_z = \frac{I_0}{4\pi} \int_{-L/2}^{L/2} \frac{[I]}{S} dz \quad \text{----- (3.3)}$$

[I] is the retarded current given by

$$[I] = I_0 e^{j\omega(t - \frac{S}{c})}$$

Z= distance to a point on the conductor

I_0 = peak value in the time of current

μ_0 = permeability of free space = $4\pi \times 10^{-7} \text{Hm}^{-1}$

If the distance from the dipole is large compare to its length ($r \gg L$) and wavelength is large compare to the length ($\lambda \gg L$), we can put $s=r$ and neglect the phase difference of the field contributions from different parts of the wire. The integrand in (2) can then be regarded as a constant. So that (2) becomes

$$A_z = \frac{\mu_0 L I_0 e^{j\omega(t-\frac{r}{c})}}{4\pi r} \quad \text{----- (3.4)}$$

The retarded scaled potential V of a charge distributed is

$$V = \frac{1}{4\pi\epsilon_0} \int_r \frac{[\rho]}{s} d\tau \quad \text{----- (3.5)}$$

Where $[\rho]$ is the retarded charge density given by

$$[\rho] = \rho_0 e^{j\omega(t-\frac{s}{c})}$$

$d\tau$ = infinitesimal volume element

μ_0 = permittivity of free space [= $8.854 \times 10^{-10} \text{ Fm}^{-1}$]

Since the region of charge in the case of the dipole being considered is confined to the points at the ends as in figure 3.2 equation 3.5 reduce to

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{[q]}{s_1} - \frac{[q]}{s_2} \right] \quad \text{----- (3.6)}$$

But,

$$[q] = \int [I] dt = I_0 \int e^{j\omega(t-\frac{s}{c})} dt = \frac{[I]}{j\omega} \quad \text{----- (3.7)}$$

Substituting equation (3.7) in (3.6)

$$V = \frac{I_0}{4\pi\epsilon_0 j\omega} \left[\frac{e^{j\omega(t-\frac{s_1}{c})}}{s_1} - \frac{e^{j\omega(t-\frac{s_2}{c})}}{s_2} \right] \quad \text{----- (3.8)}$$

Referring the figure

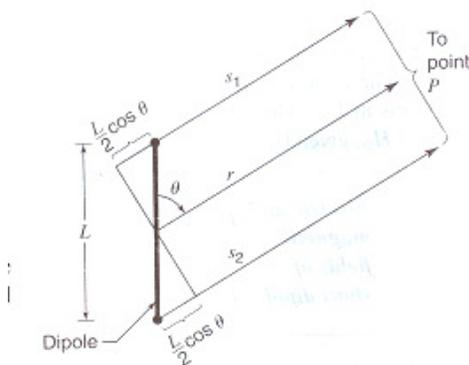


Fig 3.4: Relation for short dipole when $r \gg L$

When $r \gg L$, the lines connecting the ends of the dipole and the point 'p' may be consider as parallel so that

$$S_1 = r - \frac{L}{2} \cos \theta$$

$$S_2 = r + \frac{L}{2} \cos \theta$$

Sub S_1 and S_2 in the equation 3.8

$$V = \frac{I_0}{4\pi\epsilon_0 j\omega} \left[\frac{e^{j\omega(t - \frac{r}{c} + \frac{L}{2c} \cos \theta)}}{r - \frac{L}{2} \cos \theta} - \frac{e^{j\omega(t - \frac{r}{c} - \frac{L}{2c} \cos \theta)}}{r + \frac{L}{2} \cos \theta} \right]$$

$$V = \frac{I_0 e^{j\omega(t - \frac{r}{c})}}{4\pi\epsilon_0 j\omega} \left[\frac{e^{j\omega \frac{L}{2c} \cos \theta} \left[r + \frac{L}{2} \cos \theta \right] - e^{-j\omega \frac{L}{2c} \cos \theta} \left[r - \frac{L}{2} \cos \theta \right]}{r^2 - \frac{L^2}{4} \cos^2 \theta} \right]$$

The term $\frac{L^2}{4} \cos^2 \theta$ is negligible compare to r^2 assuming $r \gg L$

$$V = \frac{I_0 e^{j\omega(t - \frac{r}{c})}}{4\pi\epsilon_0 j\omega} \left[\left(\cos \frac{\omega L \cos \theta}{2c} + j \sin \frac{\omega L \cos \theta}{2c} \right) \left(r + \frac{L}{2} \cos \theta \right) - \left(\cos \frac{\omega L \cos \theta}{2c} - j \sin \frac{\omega L \cos \theta}{2c} \right) \left(r - \frac{L}{2} \cos \theta \right) \right]$$

If the wavelength is much greater than the length of dipole ($\lambda \gg L$) then,

$$\cos \left[\frac{\omega L \cos \theta}{2c} \right] = \cos \left[\frac{\pi L \cos \theta}{\lambda} \right] \approx 1$$

$$\sin \left[\frac{\omega L \cos \theta}{2c} \right] = \left[\frac{\omega L \cos \theta}{2c} \right]$$

Thus the above expression reduce to

$$V = \frac{I_0 e^{j\omega(t - \frac{r}{c})}}{4\pi\epsilon_0 (j\omega) r^2} \left[\left(1 + j \frac{\omega L \cos \theta}{2c} \right) \left(r + \frac{L}{2} \cos \theta \right) - \left(1 - j \frac{\omega L \cos \theta}{2c} \right) \left(r - \frac{L}{2} \cos \theta \right) \right]$$

$$V = \frac{I_0 e^{j\omega(t - \frac{r}{c})}}{4\pi\epsilon_0 (j\omega) r^2} \left[r + \frac{L}{2} \cos \theta + \frac{j\omega L r \cos \theta}{2c} + \frac{j\omega L^2 \cos^2 \theta}{4c} - r + \frac{L}{2} \cos \theta + \frac{j\omega L r \cos \theta}{2c} + \frac{j\omega L^2 \cos^2 \theta}{4c} \right]$$

$$V = \frac{I_0 e^{j\omega(t - \frac{r}{c})}}{4\pi\epsilon_0 (j\omega) r^2} \left[L \cos \theta + \frac{j\omega L r \cos \theta}{c} \right]$$

$$V = \frac{I_0 e^{j\omega(t - \frac{r}{c})} L \cos \theta}{4\pi\epsilon_0 (j\omega) r^2} \left[1 + \frac{j\omega r}{c} \right]$$

$$V = \frac{I_0 L \cos \theta e^{j\omega(t - \frac{r}{c})}}{4\pi\epsilon_0 c} \left[\frac{1}{r} + \frac{c}{j\omega r^2} \right] \quad \text{----- (3.9)}$$

Equation 3.4 and 3.9 express the vector and scalar potentials everywhere due to a short dipole. The only restrictions are $r \gg L$ and $\lambda \gg L$

These equations gives the vector and scalar potentials at a point P in terms of the distance 'r' to the point from the center of the dipole, the angle θ , the length of the dipole L the current on the dipole and some constants.

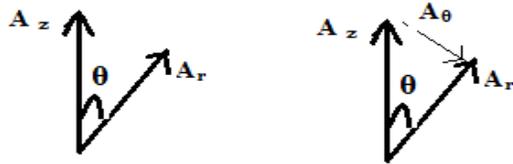


Fig 3.5: resolution of Vector potential into A_r and A_θ components

Knowing the vector potential A and the scalar potential V the electric and magnetic field may then be obtained from the relations

$$\left. \begin{aligned} E &= j\omega A - \nabla V \\ H &= \frac{1}{\mu} \nabla \times A \end{aligned} \right\} \text{----- (3.10)}$$

It will be desirable to obtain E x H components in polar coordinates. The polar coordinate components for the vector potential are

$$A = \hat{a}_r A_r + \hat{a}_\theta A_\theta + \hat{a}_\phi A_\phi \text{----- (3.11)}$$

Since the vector potential for the dipole has only 'z' components $A_\phi=0$ and A_r and A_θ are given by

$$\left. \begin{aligned} A_r &= A_z \cos\theta \\ A_\theta &= -A_z \sin\theta \end{aligned} \right\} \text{----- (3.12)}$$

In polar coordinates the gradient of V is

$$\nabla V = \hat{a}_r \frac{\partial V}{\partial r} + \hat{a}_\theta \frac{1}{r} \frac{\partial V}{\partial \theta} + \hat{a}_\phi \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} \text{----- (3.13)}$$

Expressing E in its polar coordinates components is

$$E = \hat{a}_r E_r + \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

Sub Eqn. (3.11) (3.12) and (3.13) in (3.10)

$$\hat{a}_r E_r + \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi = -j\omega[\hat{a}_r A_r + \hat{a}_\theta A_\theta + \hat{a}_\phi A_\phi] - \left[\hat{a}_r \frac{\partial V}{\partial r} + \hat{a}_\theta \frac{1}{r} \frac{\partial V}{\partial \theta} + \hat{a}_\phi \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} \right]$$

Equating the respective component on either side we get

$$\left. \begin{aligned} E_r &= -j\omega A_r - \frac{\partial V}{\partial r} \\ E_\theta &= -j\omega A_\theta - \frac{1}{r} \frac{\partial V}{\partial \theta} \\ E_\phi &= -j\omega A_\phi - \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} \end{aligned} \right\} \text{----- (3.14)}$$

Since $A_\phi=0$ and $\frac{\partial V}{\partial \phi} = 0$ $E_\phi = 0$ substituting A_r and A_θ from (3.12)

$$\left. \begin{aligned} E_r &= -j\omega A_z \cos\theta - \frac{\partial V}{\partial r} \\ E_\theta &= -j\omega A_z \sin\theta - \frac{1}{r} \frac{\partial V}{\partial \theta} \end{aligned} \right\} \text{----- (3.15)}$$

We have,

$$\begin{aligned} A_z &= \frac{\mu_0 L I_0 e^{j\omega(t-\frac{r}{c})}}{4\pi r} \\ V &= \frac{I_0 L \cos\theta e^{j\omega(t-\frac{r}{c})}}{4\pi \epsilon_0 c} \left[\frac{1}{r} + \frac{c}{j\omega r^2} \right] \\ \therefore \frac{\partial V}{\partial r} &= \frac{-I_0 L \cos\theta e^{j\omega(t-\frac{r}{c})}}{4\pi \epsilon_0 c} \left[\frac{2}{r^2} + \frac{j\omega}{cr} + \frac{2c}{j\omega r^3} \right] \\ \text{and } \frac{\partial V}{\partial \theta} &= \frac{-I_0 L \sin\theta e^{j\omega(t-\frac{r}{c})}}{4\pi \epsilon_0 c} \left[\frac{1}{r} + \frac{c}{j\omega r^2} \right] \end{aligned}$$

Substituting these values in equation (3.15)

$$\begin{aligned} E_r &= -j\omega A_z \cos\theta - \frac{\partial V}{\partial r} \\ &= -j\omega \left[\frac{\mu_0 L I_0 e^{j\omega(t-\frac{r}{c})}}{4\pi r} \cos\theta + \frac{I_0 L \cos\theta e^{j\omega(t-\frac{r}{c})}}{4\pi \epsilon_0 c} \left[\frac{2}{r^2} + \frac{j\omega}{cr} + \frac{2c}{j\omega r^3} \right] \right] \text{----- (3.16)} \end{aligned}$$

Expressing $\mu_0 = \frac{1}{\epsilon_0 c^2}$ the 1st and 3rd term are one and the same with opposite sign therefore

$$E_r = \frac{I_0 L \cos\theta e^{j\omega(t-\frac{r}{c})}}{2\pi \epsilon_0} \left[\frac{1}{cr^2} + \frac{1}{j\omega r^3} \right] \text{----- (3.17)}$$

And

$$E_\theta = \frac{j\omega \mu_0 L I_0 \sin\theta e^{j\omega(t-\frac{r}{c})}}{4\pi r} + \frac{I_0 L \sin\theta e^{j\omega(t-\frac{r}{c})}}{4\pi \epsilon_0 cr} \left[\frac{1}{r} + \frac{c}{j\omega r^2} \right]$$

Sub $\mu_0 = \frac{1}{\epsilon_0 c^2}$ in the first term we get

$$\begin{aligned} E_\theta &= \frac{j\omega L I_0 \sin\theta e^{j\omega(t-\frac{r}{c})}}{4\pi \epsilon_0 c^2 r} + \frac{I_0 L \sin\theta e^{j\omega(t-\frac{r}{c})}}{4\pi \epsilon_0 cr} \left[\frac{1}{r} + \frac{c}{j\omega r^2} \right] \\ E_\theta &= \frac{I_0 L \sin\theta e^{j\omega(t-\frac{r}{c})}}{4\pi \epsilon_0} \left[\frac{j\omega}{c^2 r} + \frac{j\omega}{cr^2} + \frac{1}{j\omega r^3} \right] \text{----- (3.18)} \end{aligned}$$

Magnetic Field Component: To find magnetic field component we use the relation

$$\vec{H} = \frac{1}{\mu} \nabla \times \vec{A} \text{----- (3.19)}$$

For Spherical Coordinate system $\nabla \times \vec{A}$ is given by

$$\nabla \times \vec{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix} \quad \text{--- (3.20)}$$

Since A_z has only two components i.e A_r and A_θ which are given by

$$A_r = A_z \cos\theta$$

$$A_\theta = -A_z \sin\theta$$

A_z has no components in A_ϕ direction therefore $\frac{\partial}{\partial \phi} = 0$ and substituting these

two values $r\sin\theta A_\phi = 0$

Substituting these two values

$$\begin{aligned} \nabla \times \vec{A} &= \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & 0 \\ A_r & rA_\theta & 0 \end{vmatrix} \\ \nabla \times \vec{A} &= \frac{1}{r^2 \sin\theta} r\sin\theta \hat{a}_\phi \left[\frac{\partial[rA_\theta]}{\partial r} - \frac{\partial[A_r]}{\partial \theta} \right] \\ &= \frac{\hat{a}_\phi}{r} \left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial(A_r)}{\partial \theta} \right] \quad \text{--- (3.21)} \end{aligned}$$

$$A_r = A_z \cos\theta$$

$$\therefore \frac{\partial A_r}{\partial \theta} = \frac{\partial(A_z \cos\theta)}{\partial \theta} = -A_z \sin\theta = -\frac{\mu_0 L I_0 e^{j\omega\left(\tau - \frac{r}{c}\right)}}{4\pi r} \sin\theta \quad \text{--- (3.22)}$$

Here A_z is independent of θ

$$rA_\theta = -rA_z \sin\theta$$

$$\frac{\partial A_\theta}{\partial r} = \frac{\partial \left[-\frac{\mu_0 L I_0 e^{j\omega\left(\tau - \frac{r}{c}\right)}}{4\pi r} \sin\theta \right]}{\partial r}$$

Substitute for A_z value

$$\begin{aligned} &= -\frac{\mu_0 L I_0 e^{j\omega\left(\tau - \frac{r}{c}\right)}}{4\pi} \sin\theta \left(-\frac{j\omega}{c} \right) \\ \frac{\partial[rA_\theta]}{\partial r} &= \frac{j\omega \mu_0 L I_0 e^{j\omega\left(\tau - \frac{r}{c}\right)}}{4\pi c} \sin\theta \quad \text{--- (3.23)} \end{aligned}$$

Substituting equations (3.22) and (3.23) in the equation (3.21)

$$\nabla \times \vec{A} = \frac{\hat{a}_\phi}{r} \left[\frac{j\omega \mu_0 L I_0 e^{j\omega\left(\tau - \frac{r}{c}\right)}}{4\pi c} + \frac{\mu_0 L I_0 e^{j\omega\left(\tau - \frac{r}{c}\right)}}{4\pi r} \sin\theta \right]$$

$$\vec{H}_\phi = \frac{1}{\mu_0} \nabla \times \vec{H} = \frac{1}{\mu_0} \frac{\mu_0 L I_0 e^{j\omega\left(\tau - \frac{r}{c}\right)}}{4\pi} \left[\frac{1}{r^2} + \frac{j\omega}{cr} \right]$$

$$\therefore \vec{H}_\phi = \frac{I_0 L \sin\theta e^{j\omega(t-\frac{r}{c})}}{4\pi} \left[\frac{1}{r^2} + \frac{j\omega}{cr} \right] \text{ --- (3.24)}$$

$$\vec{E}_\theta = \frac{I_0 L \sin\theta e^{j\omega(t-\frac{r}{c})}}{4\pi\epsilon_0} \left[\frac{j\omega}{c^2 r} + \frac{j\omega}{cr^2} + \frac{1}{j\omega r^3} \right] \text{ --- (3.25)}$$

The above two equation represents the total electric and magnetic fields due to short dipoles

When r is very large the terms $\frac{1}{r^2}$ and $\frac{1}{r^3}$ becomes negligible compare to $\frac{1}{r}$ [Er is also negligible]

$$\therefore E_\theta = \frac{j\omega I_0 L \sin\theta}{4\pi\epsilon_0 c^2 r} e^{j\omega(t-\frac{r}{c})} \text{ --- (3.26)}$$

$$|H| = H_\phi = \frac{j\omega I_0 L \sin\theta}{4\pi cr} e^{j\omega(t-\frac{r}{c})} \text{ --- (3.27)}$$

[As $H_r = H_\theta = 0$]

Taking the ratio of E and H as in the above equation

$$\frac{E_\theta}{H_\phi} = \frac{1}{\epsilon_0 c} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega \text{ --- (3.28)}$$

$$\therefore \eta = \frac{E}{H} = 377\Omega$$

Intrinsic impedance of free space [pure resistance]

Relation between E_r , E_θ and H_ϕ



Fig 3.6 (a): Near and far field pattern of E_θ and H_ϕ components for short dipole. (b): Near field component, E_r

From the equation (3.26) and (3.27), it is clear that E_θ and H_ϕ components are in phase in the field. The field pattern of both is proportional to $\sin\theta$. The pattern is independent of ϕ so that the space pattern is doughnut shaped.

When we consider near field ($\frac{1}{r^2}$ and $\frac{1}{r^3}$ is not neglected) for a small r, the electric field has two components E_θ and E_r , which are both in time phase quadrature with the magnetic field.

i.e.,

$$E_r = \frac{I_0 L \cos \theta}{2\pi \epsilon_0} e^{j\omega(t-\frac{r}{c})} \left[\frac{1}{cr^2} + \frac{1}{j\omega r^3} \right] \text{--- --- (3.28)}$$

$$E_\theta = \frac{I_0 L \sin \theta}{4\pi \epsilon_0} e^{j\omega(t-\frac{r}{c})} \left[\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3} \right] \text{--- --- (3.29)}$$

$$H_\phi = \frac{I_0 L \sin \theta}{4\pi} e^{j\omega(t-\frac{r}{c})} \left[\frac{1}{r^2} + \frac{j\omega}{cr} \right] \text{--- --- (3.30)}$$

At intermediate distance E_θ and E_r can approach time phase quadrature. So that total electric field vector rotates in a plane parallel to the direction of propagation thus referred as cross field.

For E_θ and H_ϕ components the near field patterns are same as the far field pattern [which is proportional to $\sin \theta$]

The near field pattern of E_r is proportional to $\cos \theta$ [far field $E_r=0$]

Quasi stationary or dc case:

It refers to low frequency of operation

The retarded current is given by

$$[I] = I_0 L \sin \theta e^{j\omega(t-\frac{r}{c})} = j\omega[q]$$

E_θ and E_r can be written as [equations (3.28) (3.29) and (3.30)]

$$E_r = \frac{[q] \cos \theta L}{2\pi \epsilon_0} \left[\frac{j\omega}{cr^2} + \frac{1}{r^3} \right]$$

$$E_\theta = \frac{[q] \sin \theta L}{4\pi \epsilon_0} \left[\frac{\omega^2}{c^2 r} + \frac{j\omega}{cr^2} + \frac{1}{r^3} \right]$$

And magnetic field

$$H_\phi = \frac{[I] L \sin \theta}{4\pi} \left[\frac{1}{r^2} + \frac{j\omega}{cr} \right]$$

At low frequency $f \rightarrow 0$ or $\omega \rightarrow 0$, so that

$$[q] = q_0 e^{j\omega(t-\frac{r}{c})} = q_0$$

$$[I] = I_0$$

$$E_r = \frac{q_0 L \cos \theta}{2\pi \epsilon_0 r^3}$$

$$E_\theta = \frac{q_0 L \sin \theta}{4\pi \epsilon_0 r^3}$$

$$H_\phi = \frac{I_0 L \sin \theta}{4\pi r^2} \text{--- --- (3.31)}$$

Table:
Fields of Short Dipole:-

Component	General Expression	Far Field	Quasi Stationary
E_r	$\frac{[I]L\cos\theta}{2\pi\epsilon_0} \left[\frac{1}{cr^2} + \frac{1}{j\omega r^3} \right]$	0	$\frac{q_0L\cos\theta}{2\pi\epsilon_0 r^2}$
E_θ	$\frac{[I]L\sin\theta}{4\pi\epsilon_0} \left[\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3} \right]$	$\frac{[I]Lj\omega\sin\theta}{4\pi\epsilon_0 c^2 r}$ or $\frac{j60\pi[I]\sin\theta L}{r\lambda}$	$\frac{q_0L\sin\theta}{4\pi\epsilon_0 r^2}$
H_ϕ	$\frac{[I]L\sin\theta}{4\pi} \left[\frac{j\omega}{cr} + \frac{1}{r^2} \right]$	$\frac{[I]j\omega L\sin\theta}{4\pi cr}$ Or $\frac{j[I]\sin\theta L}{2r \lambda}$	$\frac{I_0L\sin\theta}{4\pi r^2}$

Alternative Expression for field E:

In case of far field [E_r is negligible], the maximum value of Electric field is given by

$$E = \frac{\omega I_0 L \sin\theta}{4\pi\epsilon_0 c^2 r}$$

$$E = \frac{2\pi f I_0 L \sin\theta}{4\pi\epsilon_0 c \cdot r}$$

Substitute $c = f \lambda$

$$E = \frac{I_0 L \sin\theta}{2\pi\epsilon_0 \lambda cr}$$

Substitute $c = \frac{1}{\sqrt{\mu\epsilon_0}}$

$$E = \frac{I_0 L \sin\theta}{2\epsilon_0 \lambda \frac{1}{\sqrt{\mu_0 \epsilon_0}} r}$$

$$E = \frac{I_0 L \sin\theta}{2r\lambda \sqrt{\frac{\epsilon_0}{\mu_0}}}$$

But

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi = \eta$$

$$\therefore E = \left[\frac{60\pi I_0 L \sin\theta}{\lambda r} \right] \text{ --- (3.32)}$$

Expression for power: From pointing theorem

$$P = |\text{Poynting Vector}| = \frac{E^2}{\eta}$$

$$P = \frac{\left[\frac{60\pi I_0 L \sin\theta}{\lambda r}\right]^2}{120\pi}$$

$$P = \frac{30\pi I_0^2 L^2 \sin^2\theta}{\lambda^2 r^2} \text{ --- (3.33)}$$

Radiation resistance of short dipole:

Let, I = RMS value of current

P_T = Total power radiated.

R_r = Radiating resistance

The average pointing vector is given by

$$S_r = \vec{E} \times \vec{H} = \frac{E}{\eta} \quad \text{since } \eta = \frac{E}{H}$$

The total power radiated is

$$W = \oint S_r dA = \int_{\theta=0}^{2\pi} \int_{\phi=0}^{2\pi} \frac{E^2}{\eta} r^2 \sin\theta d\theta d\phi$$

Substitute E from equation (3.32)

i.e $E = \left[\frac{60\pi I_0 L \sin\theta}{\lambda r}\right]$ we get

$$W = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\left[\frac{60\pi I_0 L \sin\theta}{\lambda r}\right]^2}{120\pi} r^2 \sin\theta d\theta d\phi$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{30\pi I_0^2 L^2}{\lambda^2} \sin^3\theta d\theta d\phi$$

$$= \frac{30\pi I_0^2 L^2}{\lambda^2} \int_{\theta=0}^{\pi} \sin^3\theta d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \frac{80\pi^2 I_0^2 L^2}{\lambda^2} \text{ --- (3.34)}$$

$$\text{But total power radiated} = I_0^2 R_r \text{ --- (3.35)}$$

From Equations (3.34) and (3.35)

$$\frac{80\pi^2 I_0^2 L^2}{\lambda^2} = I_0^2 R_r$$

$$R_r = \left[\frac{80\pi^2 L^2}{\lambda^2}\right] \text{ --- (3.36)}$$

This gives the radiation resistance of short dipole

Field due to a thin linear antenna:

Thin antenna means its diameter is small compared to its wave length, i.e. $[d < \frac{\lambda}{100}]$ Where d = diameter of the antenna and λ = wavelength
 Antenna is fed at the center by a balanced two wire transmission line and assuming sinusoidal current distribution along various length of line as shown in figure 3.7

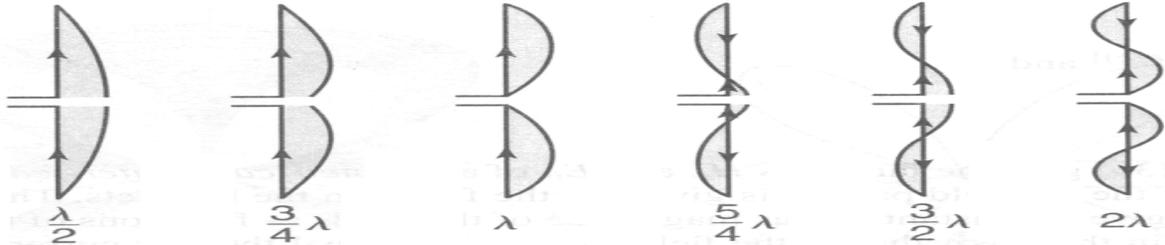


Figure 3.7: Approximate natural current distribution for thin linear center fed antenna of various length.

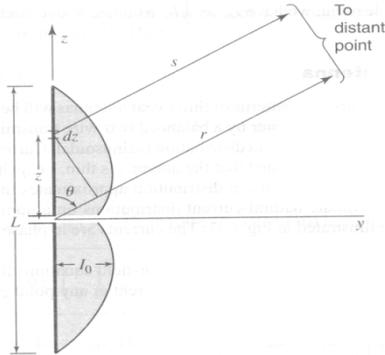


Fig 3.8: Relation for symmetrical thin linear center-fed antenna of length L. Relation for symmetrical thin linear center-fed antenna of length L is as shown in figure 3.8.

The magnitude of current at any point on the antenna is given by

$$I = I_0 \sin \left[\frac{2\pi}{\lambda} \left[\frac{L}{2} \pm z \right] \right] \text{ --- (3.37)}$$

The retarded current is given by

$$[I] = I_0 \sin \left[\frac{2\pi}{\lambda} \left[\frac{L}{2} \pm z \right] \right] e^{j\omega(t - r/c)} \text{ --- (3.38)}$$

The expression $\frac{L}{2} + z$ is used when $z < 0$ [-ve z]

$\frac{L}{2} - z$ is used when $z > 0$ [+ve z]

The total radiation field due to the antenna is obtained by considering the antenna as made up of a series of infinitesimal dipoles of length dz and integrating the field due to elementary dipole over the entire length. The far field $dE_\theta \times H_\theta$ at a distance ‘S’ from the infinitesimal dipole d_z are

$$dE_{\theta} = j60\pi \left[\frac{[I] \sin \theta dz}{s\lambda} \right] \text{----- (3.39)}$$

And

$$dH_{\phi} = \left[\frac{j[I] \sin \theta dz}{2s\lambda} \right] \text{----- (3.40)}$$

Since $E_{\theta} = \eta H_{\phi} = 120\pi H_{\phi}$. Thus either E_{θ} or H_{ϕ} calculate is enough

The value of the magnetic field H_{ϕ} for the entire antenna is the integral of dH_{ϕ} over the length of the antenna

$$\text{Thus } H_{\phi} = \int_{-L/2}^{L/2} dH_{\phi}$$

Substitute the value of dH_{ϕ} from equation (3.40) we get

$$H_{\phi} = \int_{-L/2}^{L/2} \frac{j[I] \sin \theta}{2s\lambda} dz \quad \text{substituting [I] from equation (3.38)}$$

$$H_{\phi} = \frac{jI_0 \sin \theta e^{j\omega t}}{2\lambda} \int_{-L/2}^{L/2} \frac{1}{s} \sin \left[\frac{2\pi}{\lambda} \left[\frac{L}{2} \pm z \right] \right] e^{-j\omega \frac{s}{c}} dz$$

$$H_{\phi} = \left[\frac{jI_0 \sin \theta e^{j\omega t}}{2\lambda} \right] \int_{-L/2}^{L/2} \frac{1}{s} \sin \left[\frac{2\pi}{\lambda} \left[\frac{L}{2} + z \right] \right] e^{-j\omega \frac{s}{c}} dz$$

$$+ \int_{-L/2}^{L/2} \frac{1}{s} \sin \left[\frac{2\pi}{\lambda} \left[\frac{L}{2} - z \right] \right] e^{-j\omega \frac{s}{c}} dz$$

At far distance the difference between s and r can be neglected for magnitude but can be considered for phase

[∴ s=r - zcosθ for phase s = r for magnitude]

$$H_{\phi} = \frac{jI_0 \sin \theta e^{j\omega(t - \frac{r}{c})}}{2\lambda r} \left[\int_{-L/2}^0 \sin \left[\frac{2\pi}{\lambda} \left[\frac{L}{2} + z \right] \right] e^{j\omega \frac{\cos \theta z}{c}} dz \right.$$

$$\left. + \int_0^{L/2} \sin \left[\frac{2\pi}{\lambda} \left[\frac{L}{2} - z \right] \right] e^{j\omega \frac{\cos \theta z}{c}} dz \right]$$

Since $\beta = \omega c = \frac{2\pi}{\lambda}$ the above equation can be written as

$$H_{\phi} = \frac{j\beta I_0 \sin \theta e^{j\omega(t - \frac{r}{c})}}{4\pi r} \left[\int_{-L/2}^0 e^{j\beta z \cos \theta} \sin \left[\beta \left[\frac{L}{2} + z \right] \right] dz \right.$$

$$\left. + \int_0^{L/2} e^{j\beta z \cos \theta} \sin \left[\beta \left[\frac{L}{2} - z \right] \right] dz \right] \text{--- (3.41)}$$

The integral is in the form

$$\int e^{ax} \sin[c + bx] dx = \frac{e^{ax}}{a^2 + b^2} [a \sin[c + bx] - b \cos[c + bx]]$$

For 1st integral $a = j\beta \cos\theta$, $b = \beta$ and $c = \frac{\beta L}{2}$

For 2nd integral $a = j\beta \cos\theta$, $b = \beta$, and $c = \frac{\beta L}{2}$

$$\therefore H_{\phi} = \frac{j[I_0]}{2\pi r} \left[\frac{\cos\left[\frac{\beta L \cos\theta}{2}\right] - \cos[\beta L/2]}{\sin\theta} \right] \quad (3.42)$$

Multiplying H_{ϕ} by $\eta = 120\pi$

$$E_{\theta} = \frac{j60[I_0]}{r} \left[\frac{\cos\left[\frac{\beta L \cos\theta}{2}\right] - \cos[\beta L/2]}{\sin\theta} \right] \quad (3.43)$$

Where $[I_0] = I e^{j\omega(t - r/c)}$ retarded current

Case 1: If the length of antenna is $\lambda/2$

$L = \lambda/2$, the magnitude of current distribution is given by

$$I_{mag} = I_0 \sin\left[\frac{2\pi}{\lambda} \left[\frac{\lambda}{4} \pm z\right]\right]$$

$$= I_0 \sin\left[\frac{\pi}{2} \pm \frac{2\pi z}{\lambda}\right] \quad (3.44)$$

When $z=0$ $I_{mag} = I_0$ [i.e., The maximum value of current at the center]

When $z = \pm \frac{\lambda}{4}$, $I_{mag} = 0$ [i.e., The minimum current at the end of dipole]

When $L = \lambda/2$, the pattern factor becomes

$$E = \left[\frac{\cos\left[\frac{\pi}{2} \cos\theta\right]}{\sin\theta} \right] \quad (3.45)$$

The pattern is as shown in figure 3.9(a), it is slightly more directional than the pattern of infinitesimal or short dipole [which is given by $\sin\theta$]. The beam width between half power points of $\lambda/2$ antenna is 78° as compared to 90° for a short dipole

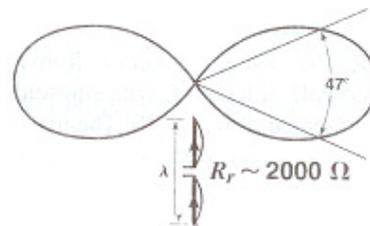
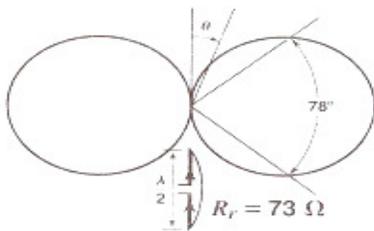


Fig 3.9: (a) Far field pattern of $\lambda/2$ antenna b) Far field pattern of full wave antenna

Case 2: If the length of antenna is λ

For full wave antenna [$L = \lambda$], the magnitude of current is given by

$$I_{mag} = I_0 \sin \left[\frac{2\pi}{\lambda} \left[\frac{\lambda}{2} \pm z \right] \right]$$

$$I_{mag} = I_0 \sin \left[\pi \mp \frac{2\pi}{\lambda} z \right] \text{ --- (3.46)}$$

When $z=0$, $I_{mag}=0$ current minimum at the center.

$$z = \pm \frac{\lambda}{2} \quad I_{mag} = 0 \text{ (again zero)}$$

Maximum current at $z = \pm \frac{\lambda}{4}$

The pattern factor is given by

$$E = \left[\frac{\cos[\pi \cos \theta] + 1}{\sin \theta} \right] \text{ --- (3.47)}$$

The pattern is as shown in figure 3.8(b) The half power beam width is 47°

Case 3: If the length of antenna is $3\lambda/2$

The pattern factor is

$$E = \left[\frac{\cos\left[\frac{3}{2} \pi \cos \theta\right]}{\sin \theta} \right] \text{ --- (3.48)}$$

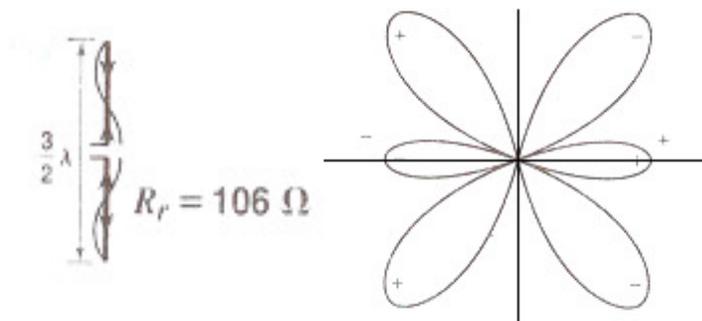


Fig 3.9: Far field pattern of $L = \frac{3}{2}\lambda$ antenna

The pattern is as shown in figure 3.9 and 3.10 It has been observed that increasing the length up to $L=\lambda$ increases the directivity in the H plane. For length $L>\lambda$ the H field strength decreases and the major part of the radiated energy is directed at some angle to the horizontal. For $L<\lambda$ the radiation pattern has no side lobes. This is key point in the design of directional array.

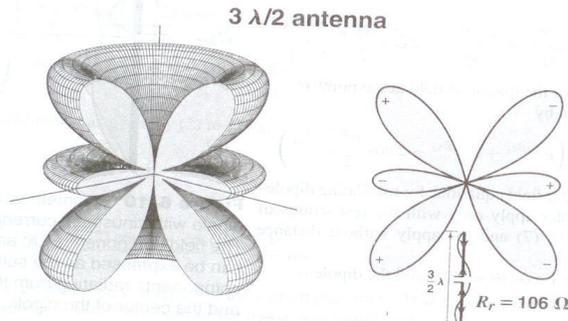


Fig 3.10: Far field pattern of $L = \frac{3}{2}\lambda$ antenna

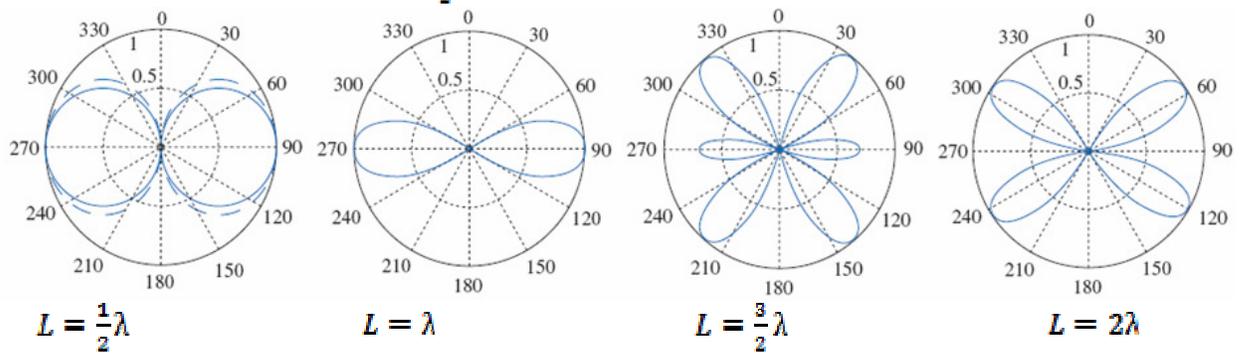


Fig 3.11: The E -plane radiation patterns for dipoles of different lengths

The E -plane radiation patterns for dipoles of different lengths, [length varies from $\lambda/2$ to 2λ] is as shown in Fig 3.11

Microstrip antenna:



Fig 3.12: Single Microstrip antenna

It is also called “patch antennas” as shown in figure3.12

- One of the most useful antennas at microwave frequencies ($f > 1 \text{ GHz}$).
- It consists of a metal “patch” on top of a grounded dielectric substrate.

The patch may be in a variety of shapes, but rectangular and circular are the most common

Advantages of Microstrip Antennas:

The advantages of microstrip antenna are

- Easy to fabricate (use etching and photolithography).
- Easy to feed (coaxial cable, Microstrip line, etc.) .
- Easy to use in an array or incorporate with other Microstrip circuit elements.
- Patterns are somewhat hemispherical, with a moderate directivity (about 6-8 dB is typical).
- Light weight, smaller size and lesser volume

Disadvantages of Microstrip Antennas:

- Low bandwidth
- Low efficiency
- Low gain

Long wire antenna

A linear wire antenna, many wavelengths long may be regarded as an array of $\lambda/2$ length but connected in a continuous linear fashion.

The long antennas are

- 1 V antenna
- 2 Rhombic antenna
- 3 Beverage antenna

V antenna:

- It is a type of transmitting or receiving antenna for providing a low angle beam for fixed frequency working in the HF band.
- It consists of two wires in the form of V fed at the apex.
- Length of the extending from 2λ to 8λ . The main lobe of radiation lies along the axis of the wire.
- The beam width of the main lobe is approximately equal to angle between wires.
- Due to reflection at the far end [as there is no load] a similar backward beam is also produced. This is minimized and the forward gain is increased by placing a similar V antenna in the plane of first V at an odd number of quarter wavelength behind the first V antenna.

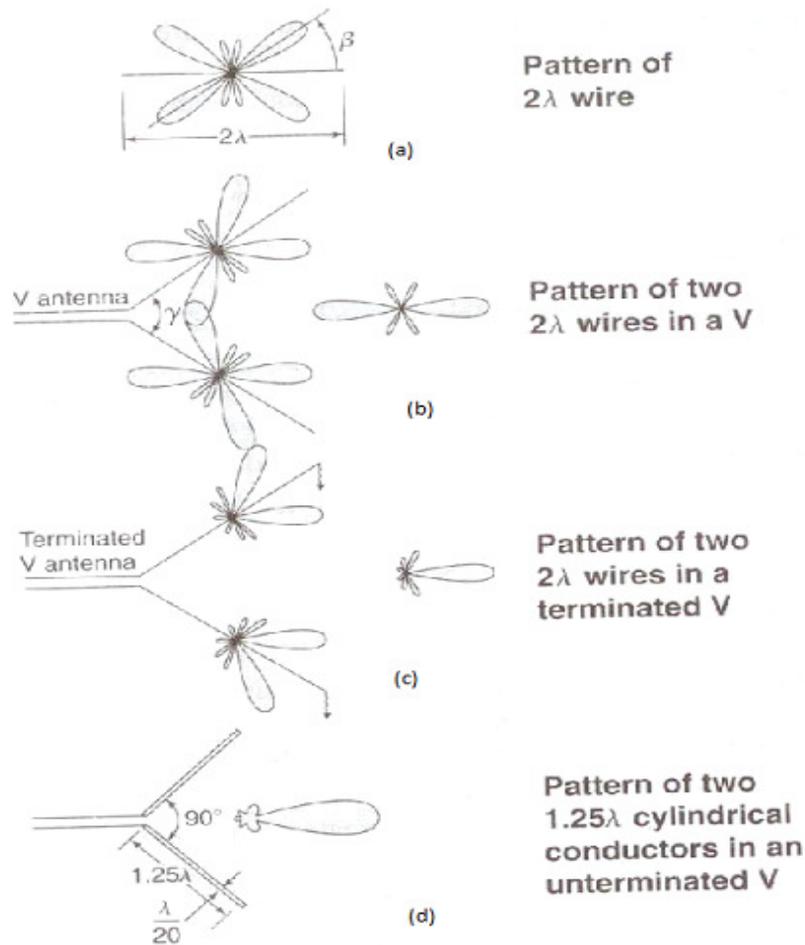


Fig 3.13: a) Calculated pattern of 2λ wire with standing wave, (b) v antenna of two such wires, (c) terminated V antenna with legs 2λ long and (d) V antenna of cylindrical conductors 1.25λ long with measured pattern.

Rhombic antenna

The rhombic antenna is as shown in fig 3.14. It consists of four wires in the form of a parallelogram in the horizontal plane above the earth the length of the wire is 2λ to 10λ .

For transmission purpose the radio frequency energy is fed through a balanced line at one end and the resistor at the other end. In free space the maximum gain is along the main axis and the polarization horizontal.

The antenna could be used for both transmission and reception. Because of its simplicity it is a very popular antenna for HF transmission.

Each wire produces a main beam of radiation and a number of side lobes. The design of rhombic antenna consists of the determination of the three factors, the length L , the tilt angle ϕ and the height h .

The earth serves to deflect the main beam upward at some angle of elevation. Combining with the earth plane the antenna produces a vertical pattern with an angle of elevation α .

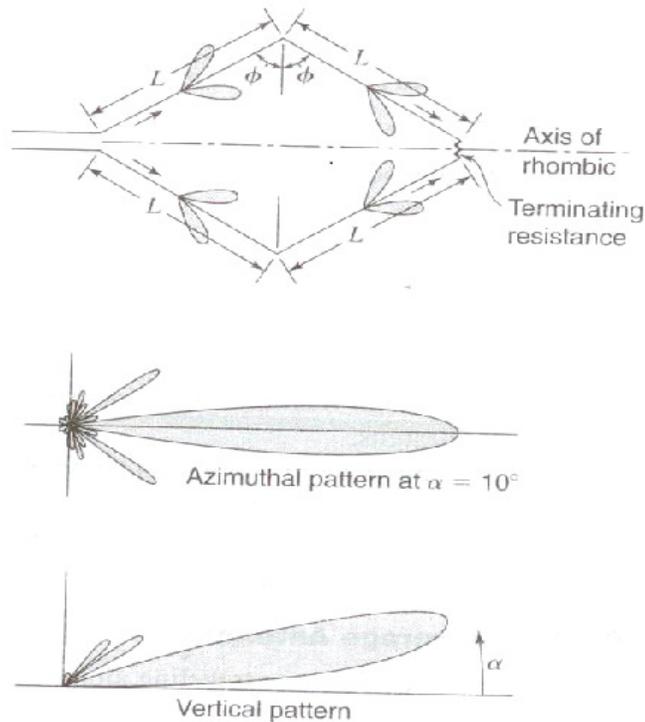


Figure 3.14: Rhombic antenna

Advantages

- It is a wideband antenna.
- It is a high gain antenna and the required angle of equation of main lobe be obtained.
- Design and structure are simple.
- Input impedance is constant for a range of frequency.

Disadvantages

- As terminating resistance is used efficiency is less.
- The width of the main lobe changes with frequency.

Folded dipole

A folded dipole antenna is formed by joining two half wave dipole at both ends and splitting one of them in the middle. The split dipole is fed at the center by a balanced transmission line as shown in figure.15.

This configuration is essentially two dipoles in parallel and therefore has the same voltages at the ends. The radiation pattern is same as that of a single dipole, but the input impedance is higher. Since the circuit in the two dipoles is half of that in a single dipole antenna and the radiated power is the same, it follows that the input impedance is four times that of a single dipole.

The input impedance [radiation resistance] for a dipole is around 73Ω , hence for the folded dipole with 2 arms the radiation resistance will be $4 \times 73 \Omega = 292 \Omega$.

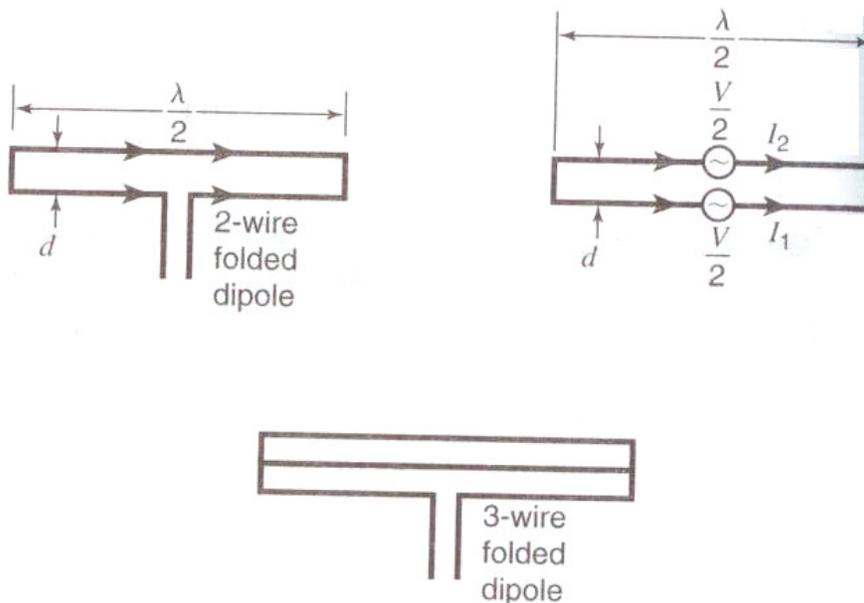


Fig 3.15: Folded dipole

If 3 arms are used the resistance will be $3^2 \times 73 = 657 \Omega$.

For impedance which is not a multiple of 73Ω (of a single dipole) the radii of the arms is made unequal. By doing so, the thicker dipole carries more current compared to the other one. A transformation ratio of 1.5 to 25 can be achieved with only two dipoles.

The folded dipole may be considered as two short circuited quarter wave transmission lines connected together and fed in series.

At the resonant frequency, the impedance presented at the fed point is very high (ideally ∞) and hence does not affect the total impedance seen by the

feeder on either side of their frequency, the impedance falls and is reactive, but since the Q is low. The fall is not steep and the antenna works satisfactorily over a broad-band of frequencies.

It is used for television reception along with same parasitic elements. The antenna than is referred as Yagi-Uda antenna



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Department of ECE**

**Course: Antennas & Wave Propagation
Unit-2**

1. Introduction

Loop antennas feature simplicity, low cost and versatility. They may have various shapes: circular, triangular, square, elliptical, etc. They are widely used in communication links up to the microwave bands (up to ≈ 3 GHz). They are also used as electromagnetic (EM) field probes in the microwave bands.

Loop antennas are usually classified as electrically small ($C < 0.1\lambda$) and electrically large ($C \sim \lambda$). Here, C denotes the loop's circumference. Electrically small loops of a single turn have very small radiation resistance (comparable to their loss resistance). Their radiation resistance can be substantially improved by adding more turns. Multi-turn loops have better radiation resistance although their efficiency is still poor. That is why they are used mostly as receiving antennas provided losses are not important. The radiation characteristics of a small loop antenna can be additionally improved by inserting a ferromagnetic core. Radio receivers of AM broadcast are usually equipped with ferrite-loop antennas. Such antennas are used in pagers, too.

The small loops, regardless of their shape, have a far-field pattern very similar to that of a small electric dipole normal to the plane of the loop. This is expected because they are equivalent to a magnetic dipole. Note, however, that the field polarization is orthogonal to that of the electric dipole.

As the circumference of the loop increases, the pattern maximum shifts towards the loop's normal, and when $C \approx \lambda$, the maximum of the pattern is along the loop's normal.

2. Radiation Characteristics of a Small Loop

A small loop is by definition a loop of constant current. Its radius satisfies

$$a < \frac{\lambda}{6\pi}, \quad (12.1)$$

or, equivalently, $C < \lambda/3$. The limit (12.1) is mathematically derived later in this Lecture from the first-order approximation of the Bessel function of the

first order $J_1(x)$ in the general solution for a loop of constant current. Actually, to make sure that the current has near-constant distribution along the loop, a tighter limit must be imposed:

$$a < 0.03\lambda, \quad (12.2)$$

or, $C < \lambda/5$. A good approximate model of a small loop is provided by the infinitesimal loop (or the infinitesimal magnetic dipole).

The expressions for the field components of an infinitesimal loop of electric current of area A were already derived in Lecture 3. Here, we give only the far-field components of the loop the axis of which is along the z :

$$E_\phi = \eta\beta^2 \cdot (IA) \cdot \frac{e^{-j\beta r}}{4\pi r} \cdot \sin\theta, \quad (12.3)$$

$$H_\theta = -\beta^2 \cdot (IA) \cdot \frac{e^{-j\beta r}}{4\pi r} \cdot \sin\theta. \quad (12.4)$$

It is obvious that the far-field pattern,

$$\bar{E}_\phi(\theta) = \sin\theta, \quad (12.5)$$

is identical to that of a z -directed infinitesimal electric dipole although the polarization is orthogonal. The power pattern is identical to that of the infinitesimal electric dipole:

$$F(\theta) = \sin^2\theta. \quad (12.6)$$

Radiated power:

$$\begin{aligned} \Pi &= \oint\oint \frac{1}{2\eta} |E_\phi|^2 \cdot \underbrace{r^2 \sin\theta d\theta d\phi}_{ds}, \\ \Pi &= \frac{1}{12\pi} \eta\beta^4 (IA)^2. \end{aligned} \quad (12.7)$$

Radiation resistance:

$$R_r = \eta \frac{8}{3} \pi^3 \left(\frac{A}{\lambda^2} \right)^2. \quad (12.8)$$

In free space, $\eta = 120\pi \Omega$, and

$$R_r \approx 31171 (A/\lambda^2)^2. \quad (12.9)$$

Equation (12.9) gives the radiation resistance of a single loop. If the loop antenna has N turns, then the radiation resistance increases with a factor of N^2 (because the radiated power increases as I^2):

$$R_r = \eta \frac{8}{3} \pi^3 \left(N \frac{A}{\lambda^2} \right)^2. \quad (12.10)$$

The relation in (12.10) provides a handy mechanism to increase R_r and the radiated power Π . Unfortunately, the losses of the loop antenna also increase (although only as $\sim N$) and this results in low efficiency.

The directivity is the same as that of an infinitesimal dipole:

$$D_0 = 4\pi \frac{U_{\max}}{\Pi_{rad}} = 1.5. \quad (12.11)$$

3. Circular Loop of Constant Current – General Solution

So far, we have assumed that the loop is of infinitesimal radius a , which allows the use of the expressions for the infinitesimal magnetic dipole. Now, we derive the far field of a circular loop, which might not be necessarily very small, but still has constant current distribution. This derivation illustrates the general loop-antenna analysis as the approach is used in the solutions to circular loop problems of nonuniform distributions, too.

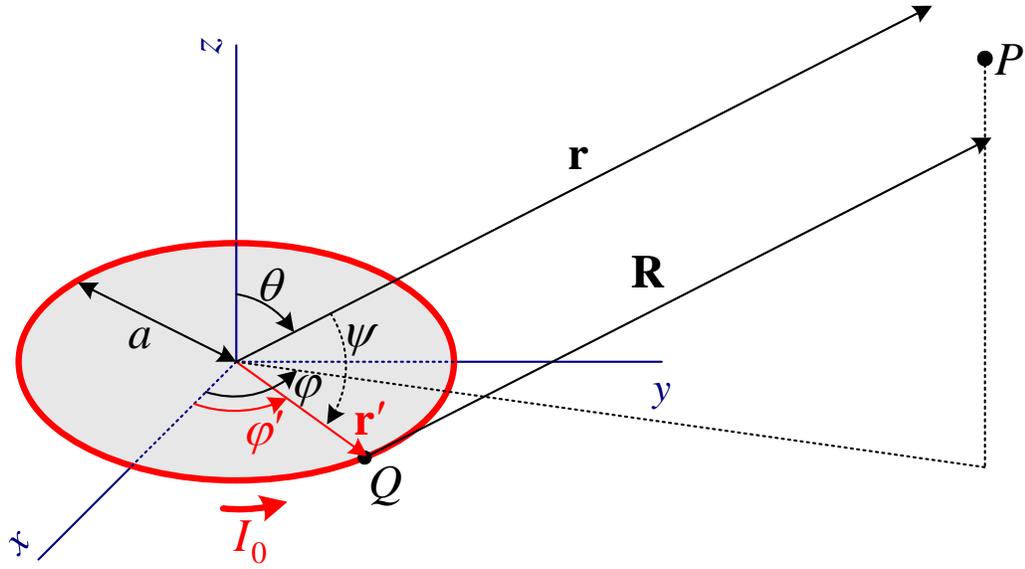
The circular loop can be divided into an infinite number of infinitesimal current elements. With reference to the figure below, the position of a current element in the xy plane is characterized by $0^\circ \leq \varphi' < 360^\circ$ and $\theta' = 90^\circ$. The position of the observation point P is defined by (θ, φ) .

The far-field approximations are

$$\begin{cases} R \approx r - a \cos \psi, & \text{for the phase term,} \\ \frac{1}{R} \approx \frac{1}{r}, & \text{for the amplitude term.} \end{cases} \quad (12.12)$$

In general, the solution for \mathbf{A} does not depend on φ because of the cylindrical symmetry of the problem. Here, we set $\varphi = 0$. The angle between the position vector of the source point Q and that of the observation point P is determined as

$$\cos \psi = \hat{\mathbf{r}} \cdot \hat{\mathbf{r}}' = (\hat{\mathbf{x}} \sin \theta \cos \varphi + \hat{\mathbf{y}} \sin \theta \sin \varphi + \hat{\mathbf{z}} \cos \theta) \cdot (\hat{\mathbf{x}} \cos \varphi' + \hat{\mathbf{y}} \sin \varphi'),$$



$$\Rightarrow \cos \psi = \sin \theta \cos \varphi'. \quad (12.13)$$

Now the vector potential integral can be solved for the far zone:

$$\mathbf{A}(r, \theta, \varphi) = \frac{\mu}{4\pi} \oint_C I_0 \frac{e^{-j\beta(r-a\sin\theta\cos\varphi')}}{r} d\mathbf{l} \quad (12.14)$$

where $d\mathbf{l} = \hat{\boldsymbol{\phi}}' a d\varphi'$ is the linear element of the loop contour. The current element changes its direction along the loop and its contribution depends on the angle between its direction and the respective \mathbf{A} component. Since all current elements are directed along $\hat{\boldsymbol{\phi}}$, we conclude that the vector potential has only A_φ component, i.e., $\mathbf{A} = A_\varphi \hat{\boldsymbol{\phi}}$, where

$$A_\varphi(r, \theta, \varphi) = \hat{\boldsymbol{\phi}} \cdot \mathbf{A}(r, \theta, \varphi) = \frac{\mu}{4\pi} (I_0 a) \frac{e^{-j\beta r}}{r} \int_0^{2\pi} (\hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}}') e^{j\beta a \sin\theta \cos\varphi'} d\varphi'. \quad (12.15)$$

Since

$$\begin{aligned} \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}}' &= (\hat{\mathbf{x}} \cos \varphi + \hat{\mathbf{y}} \sin \varphi) \cdot (\hat{\mathbf{x}} \cos \varphi' + \hat{\mathbf{y}} \sin \varphi') = \\ &= \cos \varphi \cos \varphi' + \sin \varphi \sin \varphi' = \\ &= \cos(\varphi - \varphi') \Big|_{\varphi=0} = \cos \varphi', \end{aligned} \quad (12.16)$$

the vector potential is

$$A_\varphi(\theta, 0) = \frac{\mu}{4\pi} (I_0 a) \frac{e^{-j\beta r}}{r} \int_0^{2\pi} \cos \varphi' \cdot e^{j\beta a \sin \theta \cos \varphi'} d\varphi', \quad (12.17)$$

$$A_\varphi(\theta) = \frac{\mu}{4\pi} (I_0 a) \frac{e^{-j\beta r}}{r} \left[\int_0^\pi \cos \varphi' \cdot e^{j\beta a \sin \theta \cos \varphi'} d\varphi' + \int_\pi^{2\pi} \cos \varphi' \cdot e^{j\beta a \sin \theta \cos \varphi'} d\varphi' \right].$$

We apply the following substitution in the second integral: $\varphi' = \varphi'' + \pi$. Then,

$$A_\varphi(\theta) = \frac{\mu I_0 a}{4\pi} \frac{e^{-j\beta r}}{r} \left[\int_0^\pi \cos \varphi' \cdot e^{j\beta a \sin \theta \cos \varphi'} d\varphi' - \int_0^\pi \cos \varphi'' \cdot e^{-j\beta a \sin \theta \cos \varphi''} d\varphi'' \right]. \quad (12.18)$$

The integrals in (12.18) can be expressed in terms of Bessel functions, which are defined as

$$\int_0^\pi \cos(n\varphi) e^{jz \cos \varphi} d\varphi = \pi j^n J_n(z). \quad (12.19)$$

Here, $J_n(z)$ is the Bessel function of the first kind of order n . From (12.18) and (12.19), it follows that

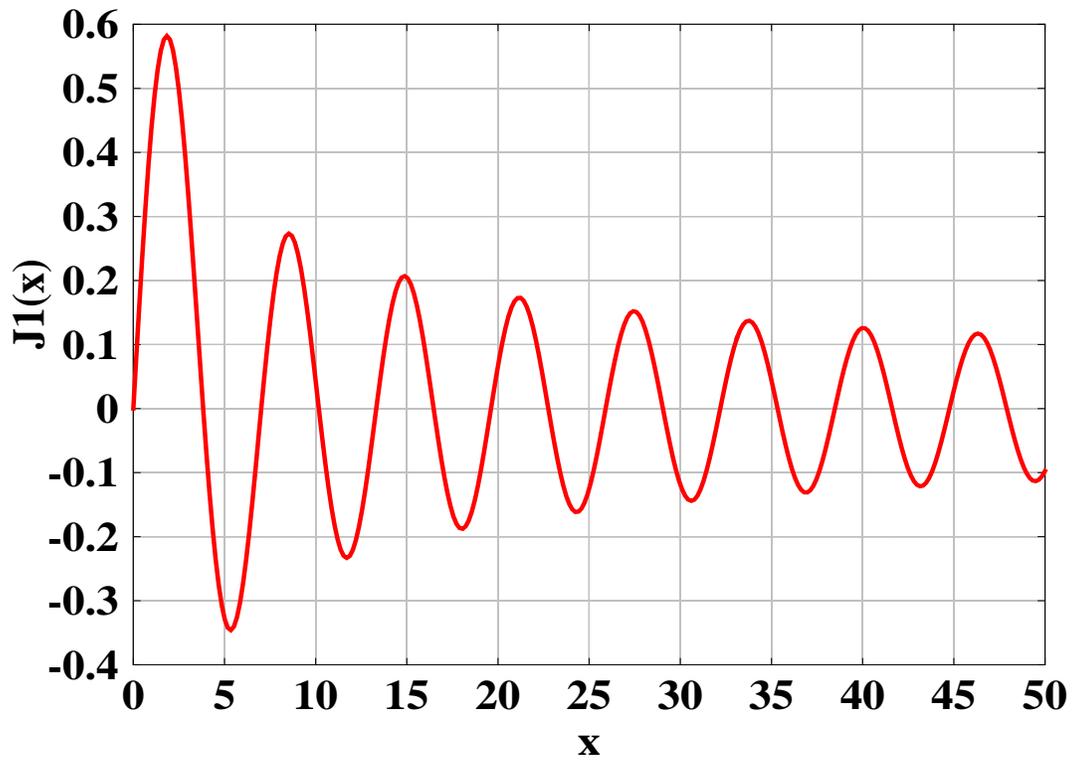
$$A_\varphi(\theta) = \frac{\mu}{4\pi} (I_0 a) \frac{e^{-j\beta r}}{r} \pi j \left[J_1(\beta a \sin \theta) - J_1(-\beta a \sin \theta) \right]. \quad (12.20)$$

Since

$$J_n(-z) = (-1)^n J_n(z), \quad (12.21)$$

equation (12.20) reduces to

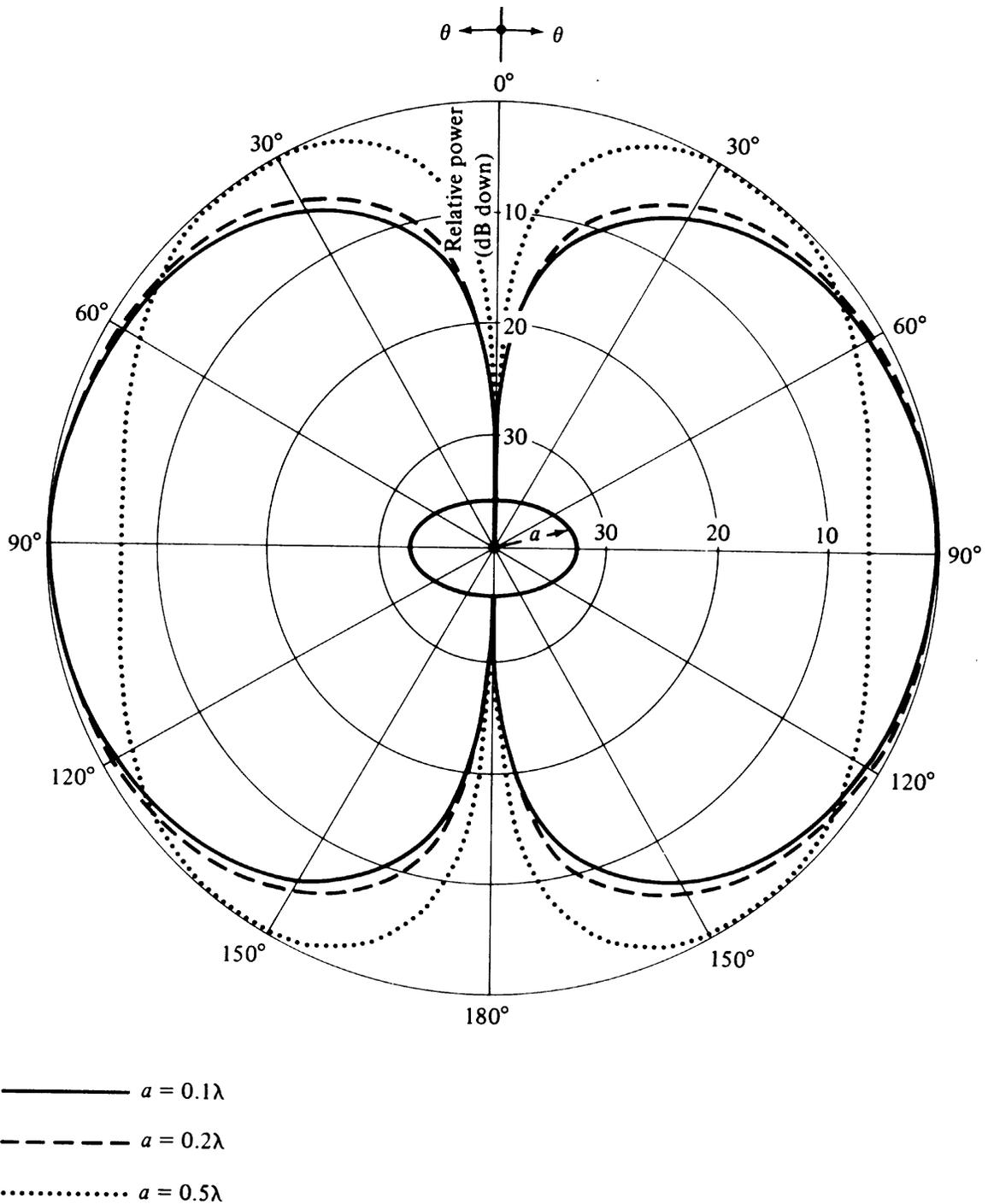
$$A_\varphi(\theta) = j \frac{\mu}{2} (I_0 a) \frac{e^{-j\beta r}}{r} J_1(\beta a \sin \theta). \quad (12.22)$$



The far-zone fields are derived as

$$\begin{cases} E_{\varphi}(\theta) = \beta\eta(I_0a)\frac{e^{-j\beta r}}{2r}J_1(\beta a \sin\theta), \\ H_{\theta}(\theta) = -\frac{E_{\varphi}}{\eta} = -\beta(I_0a)\frac{e^{-j\beta r}}{2r}J_1(\beta a \sin\theta). \end{cases} \quad (12.23)$$

The patterns of constant-current loops obtained from (12.23) are shown below:



[Balanis]

The small-loop field solution in (12.3)-(12.4) is actually a first-order approximation of the solution in (12.23). This becomes obvious when the Bessel function is expanded in series as

$$J_1(\beta a \sin \theta) = \frac{1}{2}(\beta a \sin \theta) - \frac{1}{16}(\beta a \sin \theta)^3 + \dots \quad (12.24)$$

For small values of the argument ($\beta a < 1/3$), the first-order approximation is acceptable, i.e.,

$$J_1(\beta a \sin \theta) \approx \frac{1}{2}(\beta a \sin \theta). \quad (12.25)$$

The substitution of (12.25) in (12.23) yields (12.3)-(12.4).

It can be shown that the maximum of the pattern given by (12.23) is in the direction $\theta = 90^\circ$ for all loops, which have circumference $C < 1.84\lambda$.

Radiated power and radiation resistance

We substitute the E_φ expression (12.23) in

$$\Pi = \oint\oint \frac{1}{2\eta} |E_\varphi|^2 \cdot \underbrace{r^2 \sin \theta d\theta d\varphi}_{ds},$$

which yields

$$\Pi = \frac{(\omega\mu)^2}{4\eta} (I_0^2 A) \cdot \int_0^\pi J_1^2(\beta a \sin \theta) \sin \theta d\theta. \quad (12.26)$$

Here, $A = \pi a^2$ is the loop's area. The integral in (12.26) does not have a closed form solution. Often, the following transformation is applied:

$$\int_0^\pi J_1^2(\beta a \sin \theta) \sin \theta d\theta = \frac{1}{\beta a} \int_0^{2\beta a} J_2(x) dx. \quad (12.27)$$

The second integral in (12.27) does not have a closed form solution either but it can be approximated with a highly convergent series:

$$\int_0^{2\beta a} J_2(x) dx = 2 \sum_{m=0}^{\infty} J_{2m+3}(2\beta a). \quad (12.28)$$

The radiation resistance is obtained as

$$R_r = \frac{2\Pi}{I_0^2} = \frac{(\omega\mu)^2}{2\eta} A \cdot \int_0^\pi J_1^2(\beta a \sin \theta) \sin \theta d\theta. \quad (12.29)$$

The radiation resistance of small loops is very small. For example, for $\lambda/100 < a < \lambda/30$ the radiation resistance increases from $\approx 3 \times 10^{-3} \Omega$ up to

$\approx 0.5 \Omega$. This is often less than the loss resistance of the loop. That is why small loop antennas are constructed with multiple turns and on ferromagnetic cores. Such loop antennas have large inductive reactance, which is compensated by a capacitor. This is convenient in narrowband receivers, where the antenna itself is a very efficient filter (together with the tuning capacitor), which can be tuned for different frequency bands.

4. Circular Loop of Nonuniform Current

When the loop radius becomes larger than 0.2λ , the constant-current assumption does not hold. A common assumption is the cosine distribution.^{1,2} Lindsay, Jr.,³ considers the circular loop to be a deformation of a shorted parallel-wire line. If I_s is the current magnitude at the “shorted” end, i.e., the point opposite to the feed point where $\varphi' = \pi$, then

$$I(\alpha) = I_s \cosh(\gamma a \alpha) \quad (12.30)$$

where $\alpha = \pi - \varphi'$ is the angle with respect to the shorted end, γ is the line propagation constant and a is the loop radius. If we assume loss-free transmission-line model, then $\gamma = j\beta$ and $\cosh(\gamma a \alpha) = \cos(\beta a \alpha)$. For a loop in open space, β is assumed to be the free-space wave number ($\beta = \omega\sqrt{\mu_0\epsilon_0}$).

The cosine distribution is not very accurate, especially close to the terminals, and this has a negative impact on the accuracy of the computed input impedance. That is why the current is often represented by a Fourier series:^{4,5}

$$I(\varphi') = I_0 + 2 \sum_{n=1}^N I_n \cos(n\varphi'). \quad (12.31)$$

Here, φ' is measured from the feed point. This way, the derivative of the current distribution with respect to φ' at $\varphi' = \pi$ (the point diametrically opposite to the feed point) is always zero. This imposes the requirement for a symmetrical current distribution on both sides of the diameter from $\varphi' = 0$ to $\varphi' = \pi$. The complete analysis of this general case will be left out, and only

¹ E.A. Wolff, *Antenna Analysis*, Wiley, New York, 1966.

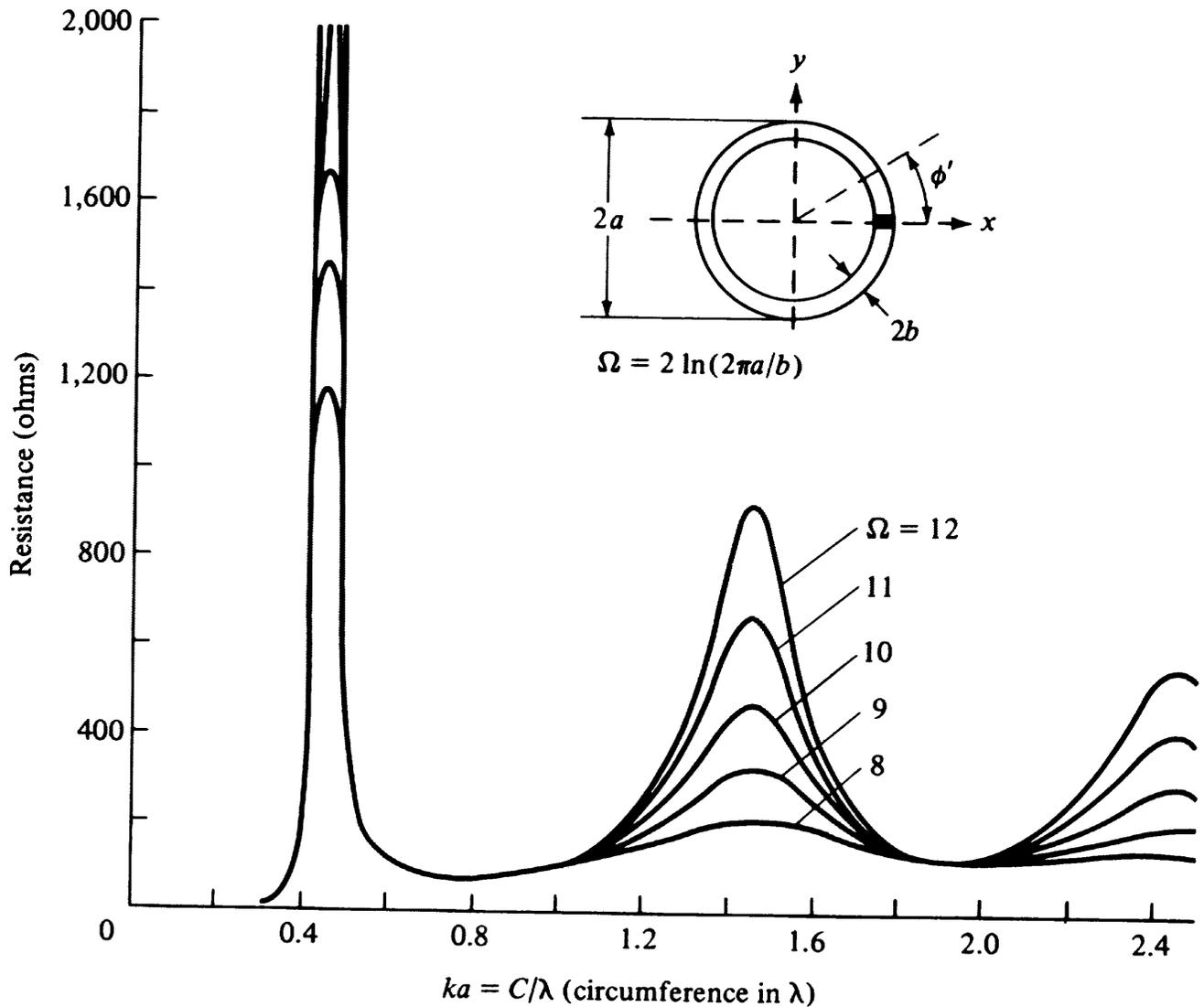
² A. Richtscheid, “Calculation of the radiation resistance of loop antennas with sinusoidal current distribution,” *IEEE Trans. Antennas Propagat.*, Nov. 1976, pp. 889-891.

³ J. E. Lindsay, Jr., “A circular loop antenna with non-uniform current distribution,” *IRE Trans. Antennas Propagat.*, vol. AP-8, No. 4, July 1960, pp. 439-441.

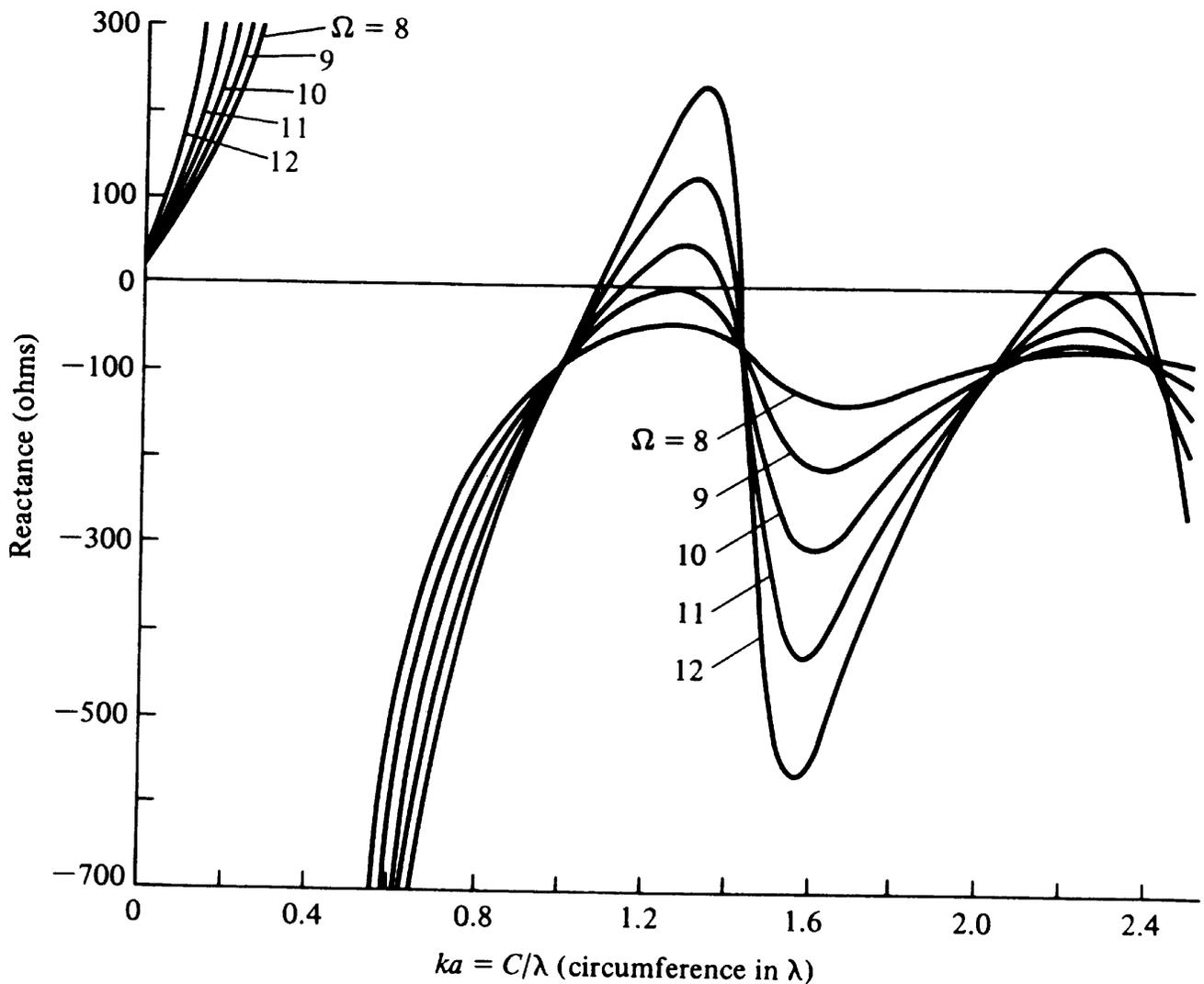
⁴ H. C. Pocklington, “Electrical oscillations in wire,” in *Cambridge Phil. Soc. Proc.*, vol. 9, 1897, pp. 324–332.

⁵ J. E. Storer, “Input impedance of circular loop antennas,” *Am. Inst. Electr. Eng. Trans.*, vol. 75, Nov. 1956.

some important results will be given. When the circumference of the loop approaches λ , the maximum of the radiation pattern shifts exactly along the loop's normal. Then, the input resistance of the antenna is also good (about 50 to 70 Ω). The maximum directivity occurs when $C \approx 1.4\lambda$ but then the input impedance is too large. The input resistance and reactance of the large circular loop are given below.



(a) Resistance



(b) Reactance

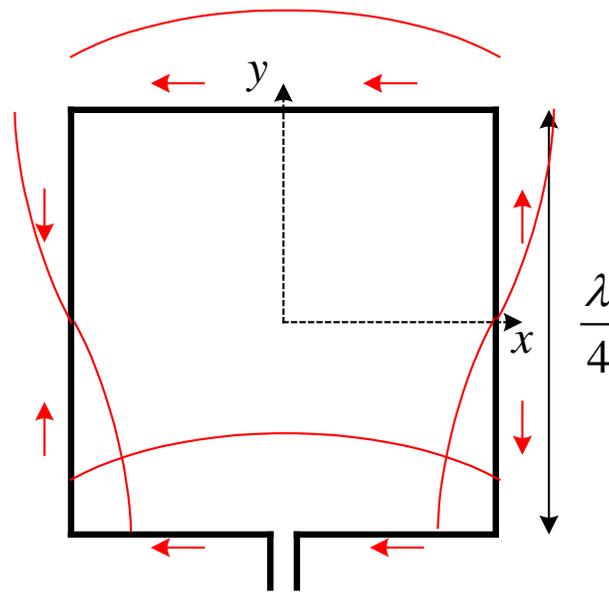
Figure 5.11 Input impedance of circular loop antennas. (SOURCE: J. E. Storer, "Impedance of Thin-Wire Loop Antennas," *AIEE Trans.*, Vol. 75, November 1956. © 1956 IEEE).

(Note: typo in author's name, read as J. E. Storer)

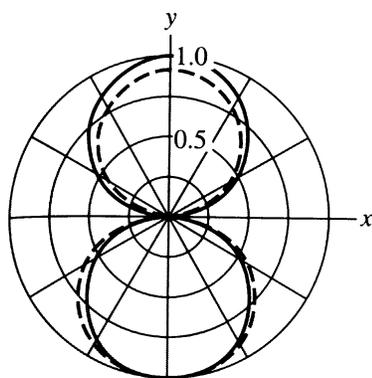
The large circular loop is very similar in its performance to the large square loop. An approximate solution of very good accuracy for the square-loop antenna can be found in

W.L. Stutzman and G.A. Thiele, *Antenna Theory and Design*, 2nd Ed., John Wiley & Sons, New York, 1998.

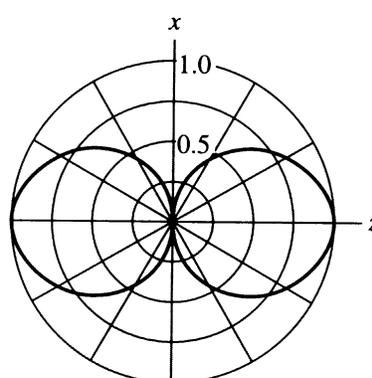
There, it is assumed that the total antenna loop is exactly one wavelength and has a cosine current distribution along the loop's wire.



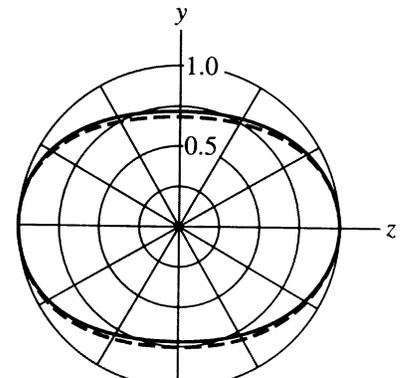
The principal plane patterns obtained through the cosine-current assumption (solid line) and using numerical methods (dash line) are shown below:



(a) The xy -plane (the plane of the loop and an E -plane) normalized pattern plot of E_ϕ . In this plane, $HP = 94^\circ$.

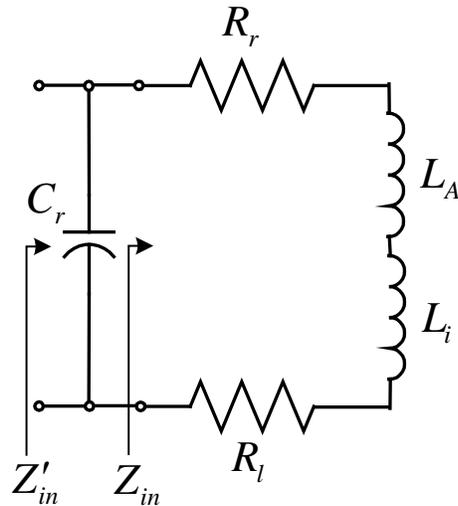


(b) The xz -plane (an E -plane) normalized pattern plot of E_θ . In this plane, $HP = 85^\circ$. The patterns from the two methods coincide in this plane.



(c) The yz -plane (the H -plane) pattern plot of E_ϕ .

5. Equivalent Circuit of a Loop Antenna



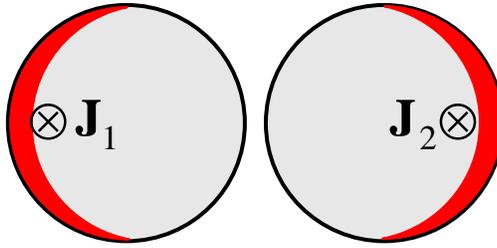
- C_r - resonance capacitor
- R_l - loss resistance of the loop antenna
- R_r - radiation resistance
- L_A - inductance of the loop
- L_i - inductance of the loop conductor (wire)

(a) Loss resistance

Usually, it is assumed that the loss resistance of loosely wound loop equals the high-frequency loss resistance of a straight wire of the same length as the loop and of the same current distribution. In the case of a uniform current distribution, the high-frequency resistance is calculated as

$$R_{hf} = \frac{l}{p} R_s, \quad R_s = \sqrt{\frac{\pi f \mu}{\sigma}}, \quad \Omega \quad (12.32)$$

where l is the length of the wire, and p is the perimeter of the wire's cross-section. We are not concerned with the current distribution now because it can be always taken into account in the same way as it is done for the dipole/monopole antennas. However, another important phenomenon has to be taken into account, namely the *proximity effect*.



When the spacing between the wound wires is very small, the loss resistance due to the proximity effect might be larger than that due to the skin effect. The following formula is used to calculate exactly the loss resistance of a loop with N turns, wire radius b , and loop separation $2c$:

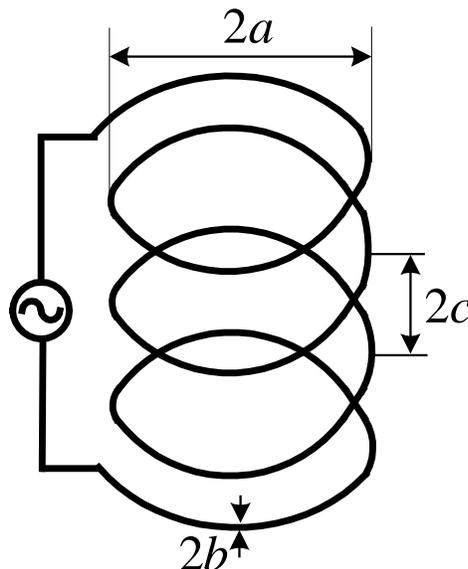
$$R_l = \frac{Na}{b} R_s \left(\frac{R_p}{R_0} + 1 \right) \quad (12.33)$$

where

R_s , Ω , is the surface resistance (see (12.32)),

R_p , Ω / m , is the ohmic resistance per unit length due to the proximity effect,

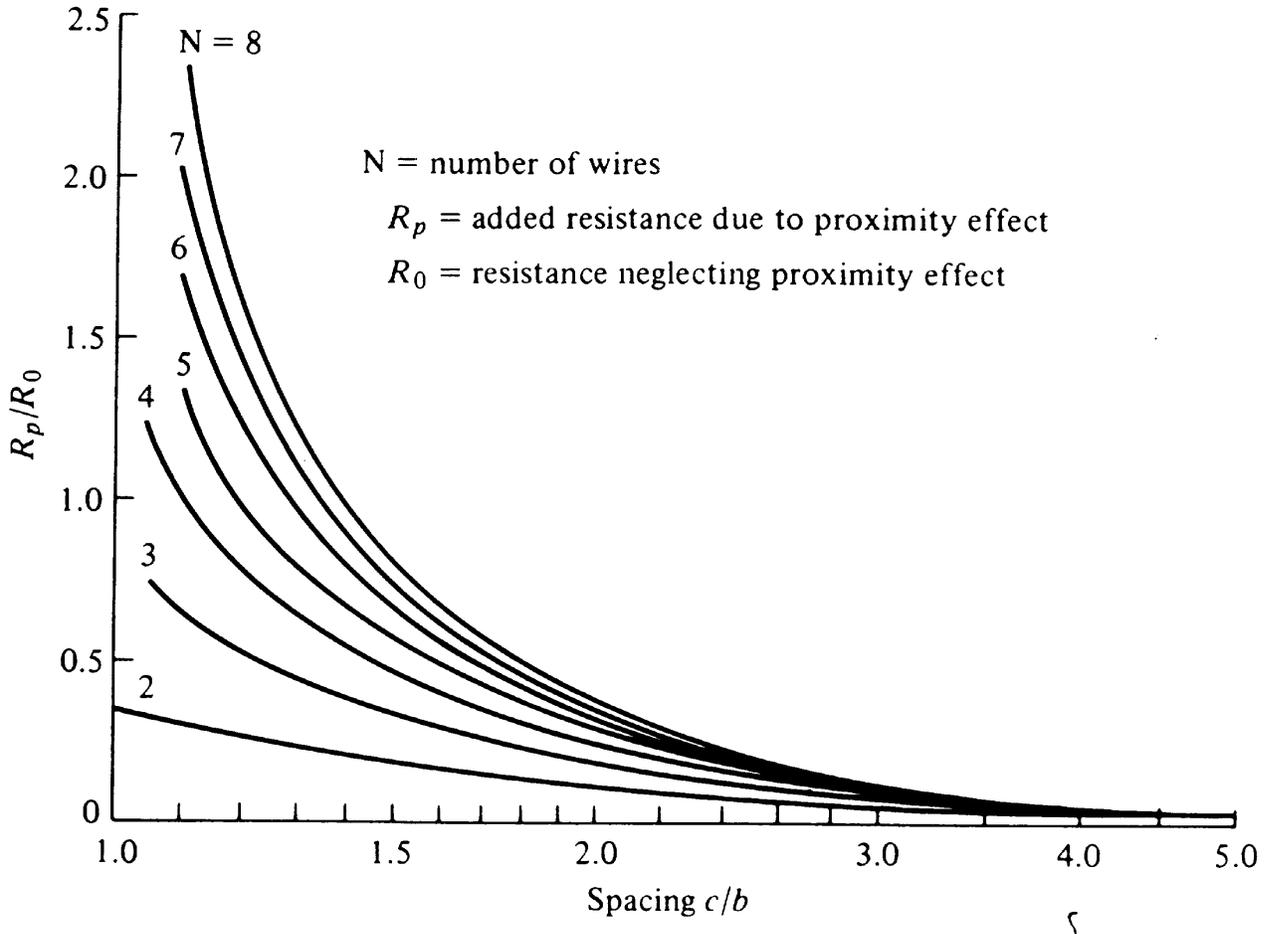
$R_0 = \frac{NR_s}{2\pi b}$, Ω / m , is the ohmic resistance per unit length due to the skin effect.



The ratio R_p / R_0 has been calculated for different relative spacings c / b , for loops with $2 \leq N \leq 8$ in:

G.N. Smith, "The proximity effect in systems of parallel conductors," *J. Appl. Phys.*, vol. 43, No. 5, May 1972, pp. 2196-2203.

The results are shown below:



(b) Ohmic resistance due to proximity (after G. N. Smith)

(b) Loop inductance

The inductance of a single circular loop of radius a made of wire of radius b is

$$L_{A_1} = \mu a \left[\ln \left(\frac{8a}{b} \right) - 2 \right]. \quad (12.34)$$

The inductance of a square loop with sides a and wire radius b is calculated as

$$L_{A_1}^{sq} = 2\mu \frac{a}{\pi} \left[\ln\left(\frac{a}{b}\right) - 0.774 \right]. \quad (12.35)$$

The inductance of a multi-turn coil is obtained from the inductance of a single-turn loop multiplied by N^2 , where N is the number of turns.

The inductance of the wire itself is very small and is often neglected. It can be shown that the DC self-inductance of a straight wire of length l is

$$L_i = \frac{\mu_0}{8\pi} \cdot l. \quad (12.36)$$

For a single loop, $l = 2\pi a$.

(c) Tuning capacitor

The susceptance of the capacitor B_r must be chosen to eliminate the susceptance of the loop. Assume that the equivalent admittance of the loop is

$$Y_{in} = \frac{1}{Z_{in}} = \frac{1}{R_{in} + jX_{in}} \quad (12.37)$$

where

$$R_{in} = R_r + R_l,$$

$$X_{in} = j\omega(L_A + L_i).$$

The following transformation holds:

$$Y_{in} = G_{in} + jB_{in} \quad (12.38)$$

where

$$\begin{cases} G_{in} = \frac{R_{in}}{R_{in}^2 + X_{in}^2}, \\ B_{in} = \frac{-X_{in}}{R_{in}^2 + X_{in}^2}. \end{cases} \quad (12.39)$$

The susceptance of the capacitor is

$$B_r = \omega C_r. \quad (12.40)$$

For resonance to occur at $f_0 = \omega_0 / (2\pi)$ when the capacitor is in parallel with the loop, the condition

$$B_r = -B_{in} \quad (12.41)$$

must be fulfilled. Therefore,

$$2\pi f_0 C_r = \frac{X_{in}}{R_{in}^2 + X_{in}^2}, \quad (12.42)$$

$$\Rightarrow C_r = \frac{1}{2\pi f} \frac{X_{in}}{(R_{in}^2 + X_{in}^2)}. \quad (12.43)$$

Under resonance, the input impedance Z'_{in} becomes

$$Z'_{in} = R'_{in} = \frac{1}{G'_{in}} = \frac{1}{G_{in}} = \frac{R_{in}^2 + X_{in}^2}{R_{in}}, \quad (12.44)$$

$$\Rightarrow Z'_{in} = R_{in} + \frac{X_{in}^2}{R_{in}}, \quad \Omega. \quad (12.45)$$

5. The Small Loop as a Receiving Antenna

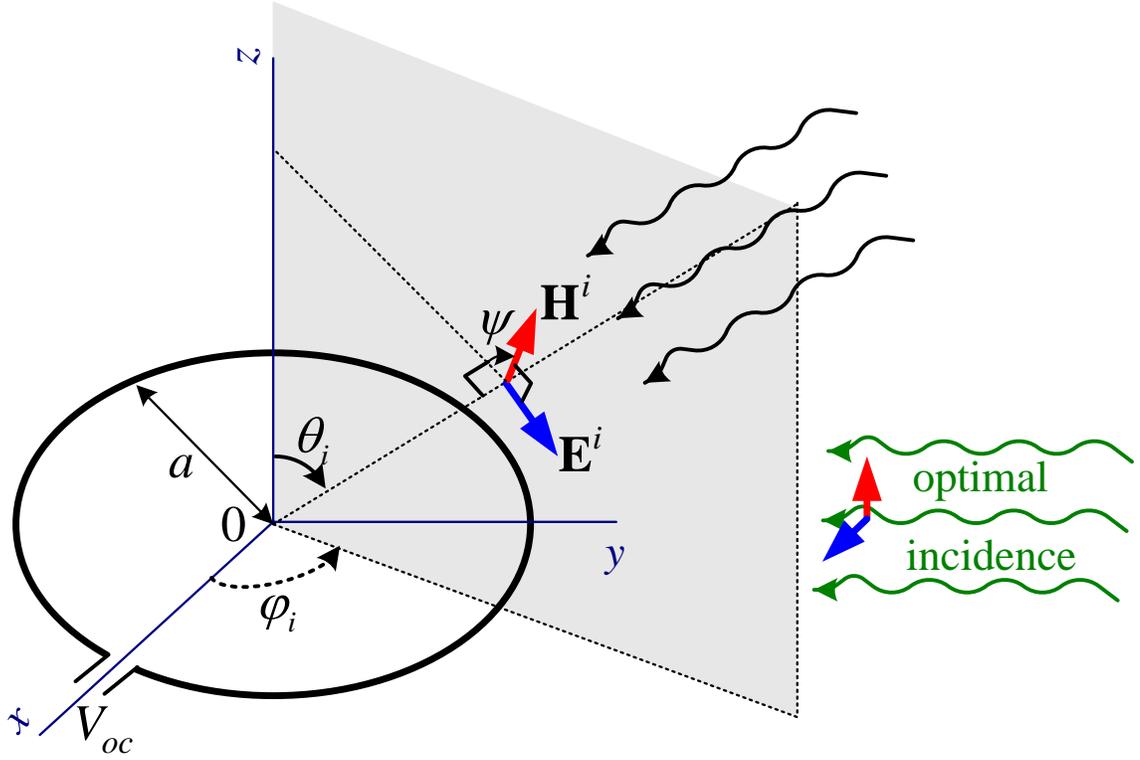
The small loop antennas have the following features:

- 1) high radiation resistance provided multi-turn ferrite-core constructions are used;
- 2) high losses, therefore, low radiation efficiency;
- 3) simple construction, small size and weight.

Small loops are usually not used as transmitting antennas due to their low efficiency e_{cd} . However, they are much preferred as receiving antennas in AM radio-receivers because of their high signal-to-noise ratio (they can be easily tuned to form a very high- Q resonant circuit), their small size and low cost.

Loops are constructed as magnetic field probes to measure magnetic flux densities. At higher frequencies (UHF and microwave), loops are used to measure the EM field intensity. In this case, ferrite rods are not used.

Since the loop is a typical linearly polarized antenna, it has to be oriented properly to optimize reception. The optimal case is a linearly polarized wave with the **H**-field aligned with the loop's axis.



The open-circuit voltage at the loop terminals is induced by the time-varying magnetic flux through the loop:

$$V_{oc} = j\omega\Psi_m = j\omega\mathbf{B} \cdot \mathbf{s} = j\omega\mu H_z \cdot \pi a^2, \quad (12.46)$$

$$H_z = H^i \cos\psi \sin\theta_i. \quad (12.47)$$

Here,

Ψ_m is the magnetic flux, Wb;

(θ_i, φ_i) are the angles specifying the direction of incidence;

ψ is the angle between the \mathbf{H}^i vector and the plane of incidence.

Finally, the open-circuit voltage can be expressed as

$$V_{oc} = j\omega\mu S H^i \cos\psi \sin\theta_i = j\beta S E^i \cos\psi \sin\theta_i. \quad (12.48)$$

Here, $S = \pi a^2$ denotes the area of the loop, and $\beta = \omega\sqrt{\mu\epsilon}$ is the phase constant. V_{oc} is maximum for $\theta_i = 90^\circ$ and $\psi = 0^\circ$.

6. Ferrite Loops

The radiation resistance and radiation efficiency can be raised by inserting a ferrite core, which has high magnetic permeability in the operating frequency band. Large magnetic permeability $\mu = \mu_0 \mu_r$ means large magnetic flux Ψ_m , and therefore large induced voltage V_{oc} . The radiation resistance of a small loop was already derived in (12.10) to include the number of turns, and it was shown that it increases as $\sim N^2$. Now the magnetic properties of the loop will be included in the expression for R_r .

The magnetic properties of a ferrite core depend not only on the relative magnetic permeability μ_r of the material it is made of but also on its geometry. The increase in the magnetic flux is then more realistically represented by the *effective relative permeability (effective magnetic constant)* $\mu_{r_{eff}}$. We show next that the radiation resistance of a ferrite-core loop is $(\mu_{r_{eff}})^2$ times larger than the radiation resistance of the air-core loop of the same geometry. When we calculated the far fields of a small loop, we used the equivalence between an electric current loop and a magnetic current element:

$$j\omega\mu(IA) = I_m l. \quad (12.49)$$

From (12.49) it is obvious that the equivalent magnetic current is proportional to μ . The field magnitudes are proportional to I_m , and therefore they are proportional to μ as well. This means that the radiated power Π_{rad} is proportional to μ^2 , and therefore the radiation resistance increases as $\sim (\mu_{r_{eff}})^2$.

Finally, we can express the radiation resistance as

$$R_r = \eta_0 \frac{8}{3} \pi^3 \left(N \mu_{r_{eff}} \frac{A}{\lambda^2} \right)^2. \quad (12.50)$$

Here, $A = \pi a^2$ is the loop area, and $\eta_0 = \sqrt{\mu_0 / \epsilon_0}$ is the intrinsic impedance of vacuum.

Some notes are made below with regard to the properties of ferrite cores:

- The effective magnetic constant of a ferrite core is always less than the magnetic constant of the ferromagnetic material it is made of, i.e., $\mu_{r_{eff}} < \mu_r$. Toroidal cores have the highest $\mu_{r_{eff}}$, and ferrite-stick cores have the lowest $\mu_{r_{eff}}$.

- The effective magnetic constant is frequency dependent. One has to be careful when picking the right core for the application at hand.
- The magnetic losses of ferromagnetic materials increase with frequency. At very high (microwave) frequencies, the magnetic losses are very high. They have to be calculated and represented in the equivalent circuit of the antenna as a shunt conductance G_m .

Helical Antenna

Helical Antenna consists of a conducting wire wound in the form of a screw thread forming a helix as shown in figure 6.1. In the most cases the helix is used with a ground plane. The helix is usually connected to the center conductor of a co-axial transmission line and the outer conductor of the line is attached to the ground plane

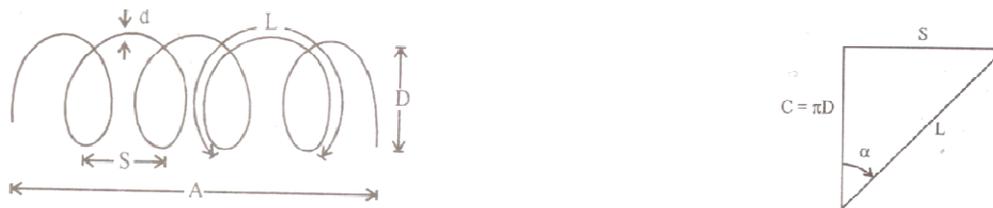


Fig 6.1: Helical Antenna

The helix parameters are related by

$$(\pi D)^2 = L^2 - S^2$$

Let S = Spacing between each turns

N = No. of Turns

D = Diameter of the helix

$L' = A = Ns$ = Total length of the antenna

L = Length of the wire between each turn = $\sqrt{(\pi D)^2 + s^2}$

$L_n = LN$ = Total length of the wire

$C = \pi D$ = Circumference of the helix

α = Pitch angle formed by a line tangent to the helix wire and a plane perpendicular to the helix axis.

$$\alpha = \tan^{-1} \frac{s}{c} = \tan^{-1} \frac{s}{\pi D} \quad - (6.1)$$

The radiation characteristics of the antenna can be varied by controlling the size of its geometrical properties compared to the wavelength.

Mode of Operation

- Normal Mode
- Axial Mode

Normal Mode:-

If the circumference, pitch and length of the helix are small compared to the wavelength, so that the current is approximately uniform in magnitude and phase in all parts of the helix, the normal mode of radiation is excited.

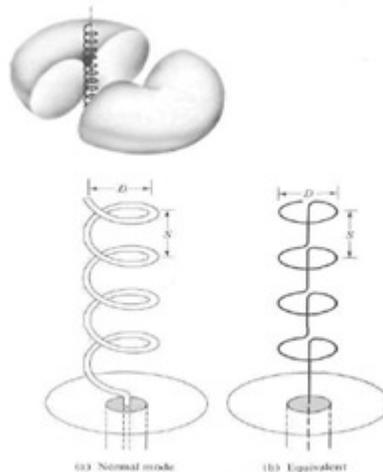


Fig 6.2: Normal mode Helical antenna and its equivalent

In normal mode as shown in fig 6.2 the radiation is maximum in the plane normal to the helix axis. The radiation may be elliptically or circularly polarized depending upon helix dimensions.

Disadvantages:

- Narrow Bandwidth
- Poor Efficiency

The radiation pattern in this mode is a combination of the equivalent radiation form a short dipole positioned along the axis of the helix and a small co-axial loop.

The radiation pattern of these two equivalent radiators is the same with the polarization at right angles and the phase angle at a given point in space is at 90° apart. Therefore the radiation is either elliptically polarized or circularly polarized depending upon the field strength ratio of the two components. This depends on the pitch angle α

When ' α ' is very small, the loop type of radiation predominates, when it becomes very large, the helix becomes essentially a short dipole. In these two limiting cases the polarization is linear. For intermediate value of the polarization is elliptical and at a particular value of ' α ' the polarization is circular

Analysis of normal mode

Field due to short dipole is given by

$$E_\theta(\theta) = \frac{j60\pi I s \sin \theta}{\lambda r} \text{----- (6.2)}$$

Field of a small loop

$$E_\phi(\theta) = \frac{j60\pi^2 I A \sin \theta}{\lambda^2 r} \text{----- (6.3)}$$

Magnitude of $E_\theta(\theta)$ and $E_\phi(\theta)$ ratio defines axial ratio

$$\text{Axial ratio} = \frac{|E_\theta|}{|E_\phi|} = \frac{s\lambda}{2\pi A} = \frac{s}{\beta A} \text{----- (6.4)}$$

The field is circularly polarized if $S = \beta A$

$$\therefore s = \frac{2\pi}{\lambda} \frac{\pi D^2}{4} = \frac{(\pi D)^2}{2\lambda}$$

$$\frac{2s}{\lambda} = \left(\frac{\pi D}{\lambda}\right)^2 \text{ From figure 6.1 } L^2 - s^2 = (\pi D)^2$$

$$\therefore \left(\frac{L}{\lambda}\right)^2 - \left(\frac{s}{\lambda}\right)^2 = \left(\frac{\pi D}{\lambda}\right)^2 = \frac{2s}{\lambda}$$

$$1 + \left(\frac{L}{\lambda}\right)^2 = 1 + \frac{2s}{\lambda} + \left(\frac{s}{\lambda}\right)^2 = \left(1 + \frac{s}{\lambda}\right)^2$$

$$1 + \frac{s}{\lambda} = \sqrt{1 + \left(\frac{L}{\lambda}\right)^2}$$

$$\left(\frac{s}{\lambda}\right) = -1 + \sqrt{1 + \left(\frac{L}{\lambda}\right)^2} \text{ ----- (6.5)}$$

This is the condition for circular polarization

The pitch angle is given by

$$\tan \alpha = \frac{s}{\pi D} \text{ but } s = \frac{(\pi D)^2}{2\lambda}$$

$$\tan \alpha = \frac{(\pi D)^2}{2\lambda \pi D} = \frac{\pi D}{2\lambda} \text{ ----- (6.6)}$$

Axial Mode:-

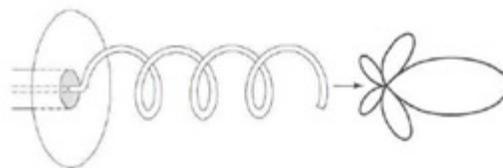


Fig 6.3: Axial mode of helix

If the dimensions of the helix are such that the circumference of one turn is approximately λ , the antenna radiates in the axial mode, which is as shown in fig 6.3.

Advantages:

Large Bandwidth and Good Efficiency

The Radiation is circularly polarized and has a max value in the direction of helix axis. The directivity increase linearly with the length of the helix. It also referred as “helix beam antenna”.

It acts like end fire array. The far field pattern of the helix can be developed by assuming that the helix consists of an array of N identical turns with an uniform spacing ‘s’ between them.

The 3db bandwidth is given by $f_{3db} = \frac{52}{C} \sqrt{\frac{\lambda^3}{NS}} \text{ deg}$ ----- (6.7)

Directivity is given by $D_{\max} = \frac{15N S C^2}{\lambda^3}$ ----- (6.8)

N= Number of turns

C= Circumference

S=Spacing between turns

λ =Wavelength

Applications:-

Used in space telemetry application at the ground end of the telemetry link for satellite and space probes at HF and VHF.

Low Frequency, Medium Frequency and High Frequency Antennas:

The choice of an antenna for a particular frequency depends on following factors.

- Radiation Efficiency to ensure proper utilization of power.
- Antenna gain and Radiation Pattern
- Knowledge of antenna impedance for efficient matching of the feeder.
- Frequency characteristics and Bandwidth
- Structural consideration

Yagi uda array:

Yagi-Uda or Yagi is named after the inventors Prof. S.Uda and Prof. H.Yagi around 1928.

The basic element used in a Yagi is $\lambda/2$ dipole placed horizontally known as driven element or active element. In order to convert bidirectional dipole into unidirectional system, the passive elements are used which include reflector and director. The passive or parasitic elements are placed parallel to driven element, collinearly placed close together as shown in fig 6.4.

The Parasitic element placed in front of driven element is called director whose length is 5% less than the drive element. The element placed at the back of driven element is called reflector whose length is 5% more than that of driver element. The space between the element ranges between 0.1λ to 0.3λ .

Seven element Yagi-Uda

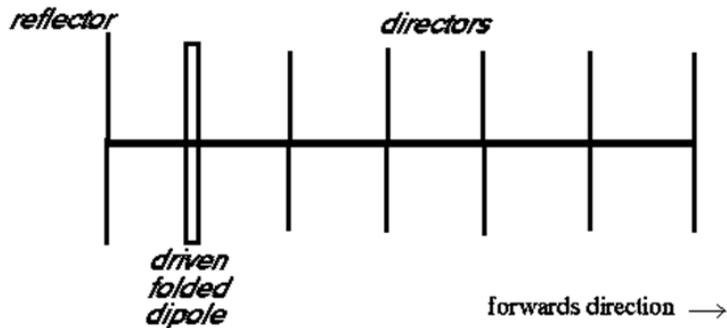


Fig 6.4: Seven segment yagi-uda antenna

For a three element system,

Reflector length = $500/f$ (MHz) feet

Driven element length = $475/f$ (MHz) feet

Director length = $455/f$ (MHz) feet.

The above relations are given for elements with length to diameter ratio between 200 to 400 and spacing between 0.1λ to 0.2λ .

With parasitic elements the impedance reduces less than 73Ω and may be even less than 25Ω . A folded $\lambda/2$ dipole is used to increase the impedance.

System may be constructed with more than one director. Addition of each director increases the gain by nearly 3 dB. Number of elements in a yagi is limited to 11.

Basic Operation:

The phases of the current in the parasitic element depends upon the length and the distance between the elements. Parasitic antenna in the vicinity of radiating antenna is used either to reflect or to direct the radiated energy so that a compact directional system is obtained.

A parasitic element of length greater than $\lambda/2$ is inductive which lags and of length less than $\lambda/2$ is capacitive which leads the current due to induced voltage. Properly spaced elements of length less than $\lambda/2$ act as director and add the fields of driven element. Each director will excite the next. The reflector adds the fields of driven element in the direction from reflector towards the driven element.

The greater the distance between driven and director elements, the greater the capacitive reactance needed to provide correct phasing of parasitic elements. Hence the length of element is tapered-off to achieve reactance.

A Yagi system has the following characteristics.

1. The three element array (reflector, active and director) is generally referred as “beam antenna”
 2. It has unidirectional beam of moderate directivity with light weight, low cost and simplicity in design.
 3. The band width increases between 2% when the space between elements ranges between 0.1λ to 0.15λ .
 4. It provides a gain of 8 dB and a front-to-back ratio of 20dB.
 5. Yagi is also known as super-directive or super gain antenna since the system results a high gain.
 6. If greater directivity is to be obtained, more directors are used. Array up to 40 elements can be used.
 7. Arrays can be stacked to increase the directivity.
 8. Yagi is essentially a fixed frequency device. Frequency sensitivity and bandwidth of about 3% is achievable.
 9. To increase the directivity Yagi's can be staked one above the other or one by side of the other.
-

HORN ANTENNAS

Flared waveguides that produce a nearly uniform phase front larger than the waveguide itself. Constructed in a variety of shapes such as sectoral E-plane, sectoral H-plane, pyramidal, conical, etc. as shown in figure 4.4.

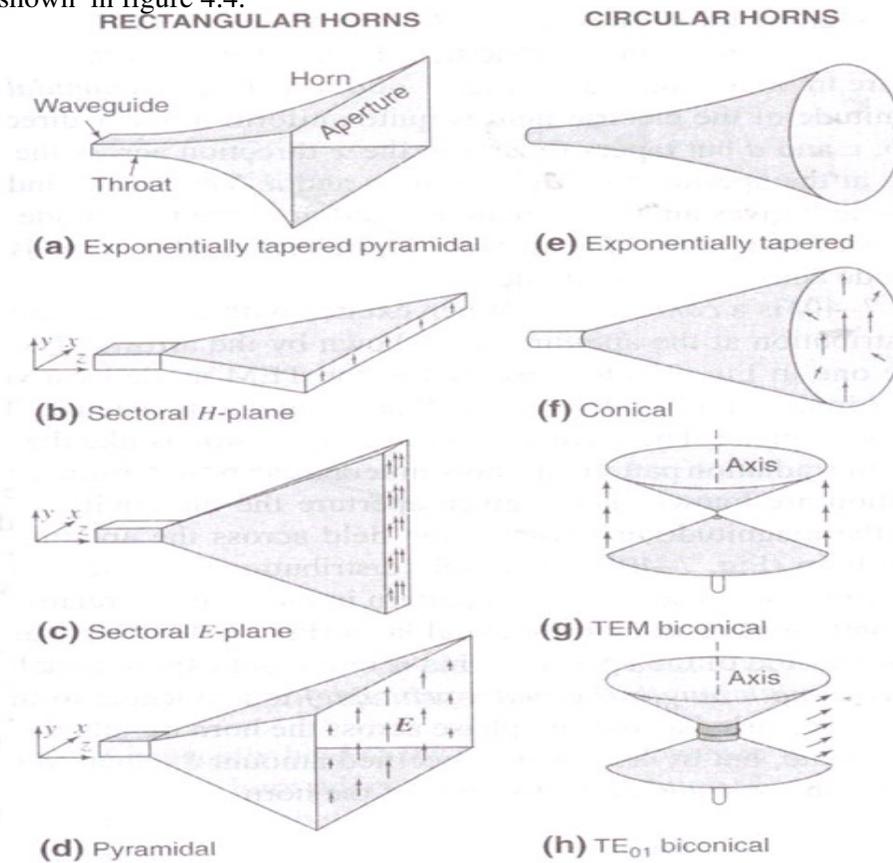


Figure:4.4: Different types of Horn antenna

Horn Antennas -Application Areas

- Used as a feed element for large radio astronomy, satellite tracking and communication dishes
- A common element of phased arrays
- Used in the calibration, other high-gain antennas
- Used for making electromagnetic interference measurements

Rectangular Horn antenna:

A rectangular horn antenna is as shown in figure 4.6. This is an extension of rectangular wave guide. TE_{10} mode is preferred for rectangular horns.

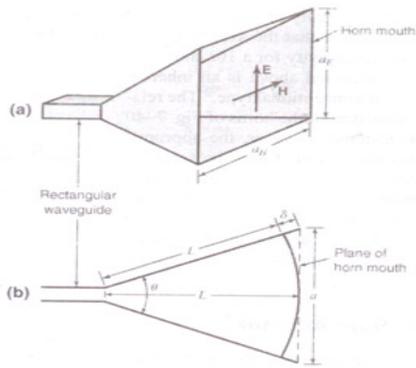


Fig 4.6: Rectangular Horn antenna

Consider a rectangular horn as in figure 4.6(b). from figure

$$\cos \frac{\theta}{2} = \frac{L}{L + \delta}$$

$$\sin \frac{\theta}{2} = \frac{a}{2(L + \delta)}$$

$$\tan \frac{\theta}{2} = \frac{a}{2L}$$

Where

L= length of the horn

side length=L+δ

A=Aperture

θ=Flare angle

From the geometry, we have also that

$$L = \frac{a^2}{8\delta} \quad (\delta \ll L)$$

and

$$\theta = 2 \tan^{-1} \frac{a}{2L} = 2 \cos^{-1} \frac{L}{L + \delta}$$

Optimum horn dimensions

$$\delta_0 = \frac{L}{\cos(\theta/2)} - L = \text{optimum } \delta$$

$$L = \frac{\delta_0 \cos(\theta/2)}{1 - \cos(\theta/2)} = \text{optimum length}$$

Table:

Type of Aperture	Beam width, deg	
	Between First nulls	Between Half power points
Uniformly illuminated rectangular aperture or linear array	$\frac{115}{L_\lambda}$	$\frac{51}{L_\lambda}$
Uniformly illuminated circular aperture	$\frac{140}{D_\lambda}$	$\frac{58}{D_\lambda}$
Optimum E-plane rectangular horn	$\frac{115}{a_{E\lambda}}$	$\frac{56}{a_{E\lambda}}$
Optimum H-plane rectangular horn	$\frac{172}{a_{H\lambda}}$	$\frac{67}{a_{H\lambda}}$

Problem:

- Determine the length L, H-Plane aperture and flare angles θ_E and θ_H (in the E and H planes, respectively) of a pyramidal horn for which the E-plane aperture $a_E = 10\lambda$. The horn is fed by a rectangular waveguide with TE₁₀ mode. Let $\delta = 0.2\lambda$ in the E plane and 0.375λ in the H plane.
- What are the beam widths?
- What is the directivity?

Solution:

Taking $\delta = \lambda/5$ in the E plane, the required horn length

$$L = \frac{a^2}{8\delta} = \frac{100\lambda}{8/5} = 62.5\lambda$$

and the flare angle in the E plane

$$\theta_E = 2 \tan^{-1} \frac{a}{2L} = 2 \tan^{-1} \frac{10}{125} = 9.1^\circ$$

Taking $\delta = 3\lambda/8$ in the H plane, the flare angle in the H plane

$$\theta_H = 2 \cos^{-1} \frac{L}{L + \delta} = 2 \cos^{-1} \frac{62.5}{62.5 + 0.375} = 12.52^\circ$$

and the H-plane aperture

$$a_H = 2L \tan^{-1} \frac{\theta_H}{2} = 2 \times 62.5 \lambda \tan 6.26^\circ = 13.7 \lambda$$

$$HPBW (EPlane) = \frac{56^\circ}{a_{E\lambda}} = \frac{56^\circ}{10} = 5.6^\circ$$

$$HPBW (HPlane) = \frac{67^\circ}{a_{H\lambda}} = \frac{67^\circ}{13.7} = 4.9^\circ$$

$$D \cong 10 \log \left(\frac{7.5 A_p}{\lambda^2} \right) = 10 \log (7.5 \times 10 \times 13.5) = 30.1 \text{ dBi}$$

Problems:

The radius of a circular loop Antenna is 0.02λ . How many turns of the antenna will give a radiation resistance of 35Ω .

$$A = \pi (0.02\lambda)^2$$

$$A < \frac{\lambda^2}{100} \text{ or } \frac{A}{\lambda^2} < 0.01$$

$$\frac{A}{\lambda^2} = \pi (0.02)^2 = 0.001256 < 0.01$$

$$R_r = 31200 \left(\frac{An}{\lambda^2} \right)^2 = 35$$

$$\text{hence } n = 27$$

Problem:

The impedance of an infinitesimally thin $\lambda/2$ antenna is $73 + j42.5 \Omega$ Find the terminal impedance of

infinitesimally thin $\lambda/2$ slot antenna.

$$(L = 0.5\lambda \text{ and } L/D = \infty)$$

$$(L = 0.5\lambda \text{ and } L/w = \infty)$$

$$Z_1 = \frac{35,476}{73 + j42.5} = 363 - j211 \Omega$$

Problem:

The impedance of a thin cylindrical antenna is resistive and equal to 67Ω . Find the terminal impedance of complementary slot antenna

$$(L = 0.475\lambda \text{ and } L/D = 100)$$

$$(w = 2D = 2L/100 = 0.01\lambda)$$

$$Z_1 = \frac{35.476}{67} = 530 + j0\Omega$$



**G.PULLAIAH COLLEGE OF ENGINEERING AND
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DEPARTMENT OF ECE

COURSE: ANTENNAS & WAVE PROPAGATION

UNIT-III

UNIT-3

MICROSTRIP ANTENNAS

REFLECTOR ANTENNAS

LENS ANTENNAS

In telecommunication, a microstrip antenna (also known as a printed antenna) usually means an antenna fabricated using microstrip techniques on a printed circuit board (PCB). They are mostly used at microwave frequencies. An individual microstrip antenna consists of a patch of metal foil of various shapes (a patch antenna) on the surface of a PCB, with a metal foil ground plane on the other side of the board. Most microstrip antennas consist of multiple patches in a two-dimensional array. The antenna is usually connected to the transmitter or receiver through foil microstrip transmission lines. The radio frequency current is applied (or in receiving antennas the received signal is produced) between the antenna and ground plane. Microstrip antennas have become very popular in recent decades due to their thin planar profile which can be incorporated into the surfaces of consumer products, aircraft and missiles; their ease of fabrication using printed circuit techniques; the ease of integrating the antenna on the same board with the rest of the circuit, and the possibility of adding active devices such as microwave integrated circuits to the antenna itself to make active antennas.

Microstrip antenna:



Fig 3.12: Single Microstrip antenna

It is also called “patch antennas” as shown in figure3.12

- One of the most useful antennas at microwave frequencies ($f > 1$ GHz).
- It consists of a metal “patch” on top of a grounded dielectric substrate.

The patch may be in a variety of shapes, but rectangular and circular are the most common

Advantages of Microstrip Antennas:

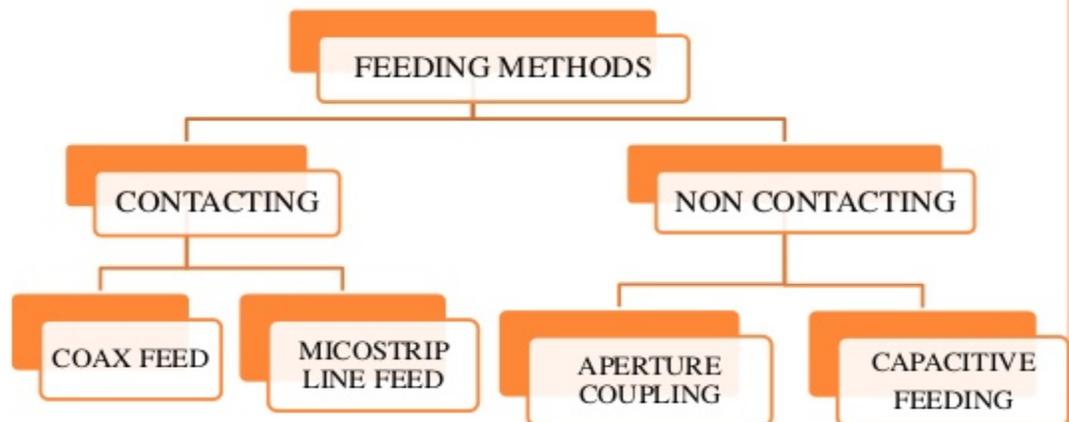
The advantages of microstrip antenna are

- Easy to fabricate (use etching and photolithography).
- Easy to feed (coaxial cable, Microstrip line, etc.) .
- Easy to use in an array or incorporate with other Microstrip circuit elements.
- Patterns are somewhat hemispherical, with a moderate directivity (about 6-8 dB is typical).
- Light weight, smaller size and lesser volume

Disadvantages of Microstrip Antennas:

- Low bandwidth
- Low efficiency
- Low gain

FEEDING TECHNIQUES



REFLECTOR ANTENNAS IS A COMBINATION OF A REFLECTOR AND AN ANTENNA. REFLECTOR ANTENNAS ARE USED TO MODIFY THE RADIATION PATTERN OF AN ANTENNA.

TYPES OF REFLECTOR ANTENNAS:

Depending on the shape of reflector, the following are the types:

Flat Sheet Reflectors

Corner Reflectors

Parabolic Reflectors

Circular Reflectors

Elliptical Reflectors

Semi Circular Reflectors

Corner reflector

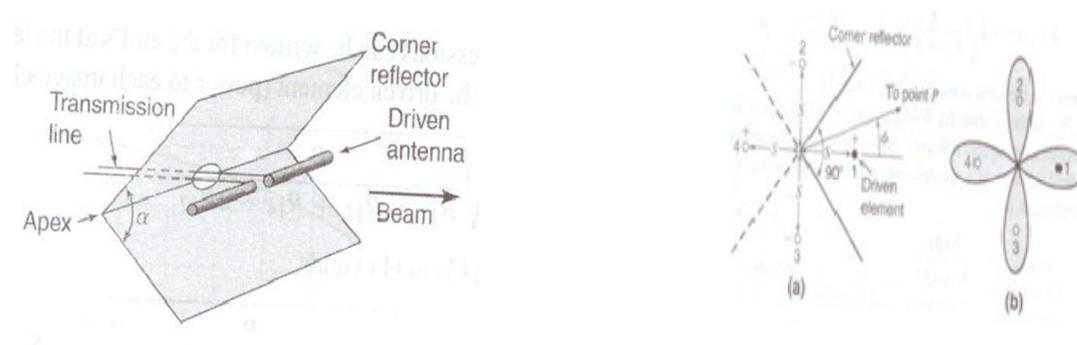


Fig 6.5: Square Corner reflector with images used in the analysis

Two flat reflecting sheets intersecting at an angle or corner as in figure 6.5 form an effective directional antenna. When the corner angle $\alpha=90^\circ$, the sheets intersect at right angles, forming a square-corner reflector. Corner angles both greater or less than 90° can be used although there are practical disadvantages to angles much less than 90° . A corner reflector with $\alpha=180^\circ$ is equivalent to a flat sheet reflector and may be considered as limiting case of the corner reflector.

Assuming perfectly conducting reflecting sheets infinite extent, the method of images can be applied to analyze the corner reflector antenna for angle $\alpha = 180^\circ/n$, where n is any positive integer. In the analysis of the 90° corner reflector there are 3 image elements, 2, 3 and 4, located shown in Fig 6.5. The driven antenna 1 the 3 images have currents of equal magnitude. The phase of the currents in 1 and 4 is same. The phase of the currents in 2 and 3 is the same but 180° out of phase with respect the currents in 1 and 4. All elements are assumed to be $\lambda/2$ long.

At the point P at a large distance D from the antenna. The field intensity is

$$E(\phi) = 2kI_1 [\cos(S_r \cos \phi) - \cos(S_r \sin \phi)] \text{-----} (6.9)$$

Where

I_1 = current in each element

S_r = spacing of each element from the corner, rad
 $= 2\pi S/\lambda$

K = constant involving the distance D,

For arbitrary corner angles, analysis involves integrations of cylindrical functions. The emf V_t at the terminals at the center of the driven element is

$$V_t = I_1 Z_{11} + I_1 R_{1L} + I_1 Z_{14} - 2I_1 Z_{12}$$

Where

Z_{11} = Self-Impedance of driven element

R_{1L} = Equivalent loss resistance of driven element

Z_{12} = Mutual impedance of element 1 and 2

Z_{14} = Mutual impedance of element 1 and 4

If 'P' is the power delivered to the driven element, then from symmetry

$$I_1 = \sqrt{\frac{P}{R_{11} + R_{1L} + R_{14} - 2R_{12}}} \text{-----} (6.10)$$

$$E(\phi) = 2k \sqrt{\frac{P}{R_{11} + R_{1L} + R_{14} - 2R_{12}}} [\cos(S_r \cos \phi) - \cos(S_r \sin \phi)] \text{-----} (6.11)$$

The Field Intensity at 'P' with reflector removed

$$E_{HW}(\phi) = 2k \sqrt{\frac{P}{R_{11} + R_{1L}}} \text{-----} (6.12)$$

The Gain in the field intensity of a square corner reflector antenna over a single $\lambda/2$ antenna

$$G_f(\phi) = \frac{E(\phi)}{E_{HW}(\phi)}$$

$$G_f(\phi) = 2 \sqrt{\frac{R_{11} + R_{1L}}{R_{11} + R_{1L} + R_{14} - 2R_{12}}} \left[\cos(S_r \cos \phi) - \cos(S_r \sin \phi) \right] \text{-----(6.13)}$$

Where the expression in brackets is the pattern factor and the expression included under the radical sign is the coupling factor. The pattern shape is a function of both the angle ϕ , and the antenna-to-corner spacing S . For the 60° corner the analysis requires a total of 6 elements, 1 actual antenna and 5 images as in Fig.6.6

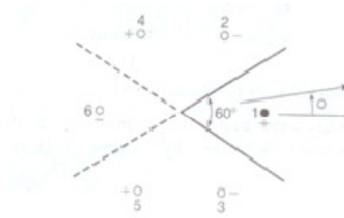


Fig 6.6 : A 60 deg corner reflector with images used in analysis

Parabolic reflectors

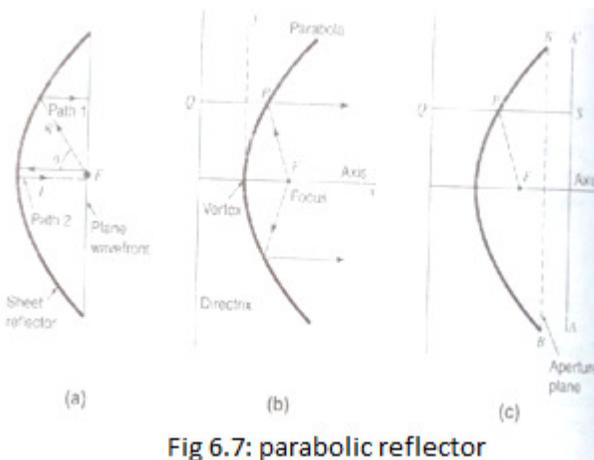


Fig 6.7: parabolic reflector

Suppose that we have a point source and that we wish to produce a plane-wave front over a large aperture by means of a sheet reflector. Referring to Fig. 6.7(a), it is then required that the distance from the source to the plane-wave front via path 1 and 2 be equal or

The parabola-general properties

$$2L = R(1 + \cos \theta) \text{----- (6.14)}$$

$$R = \frac{2L}{1 + \cos \theta} \text{----- (6.15)}$$

Referring to Fig. 6.7(b), the parabolic curve may be defined as follows. The distance from any point P on a parabolic curve to a fixed point F, called the focus, is equal to the perpendicular distance to a fixed line called the directrix. Thus, in Fig.6.7(b), $PF = PQ$. Referring now to Fig.6.7(c), let AA' be a line normal to the axis at an arbitrary distance QS from the directrix. Since $PS = QS - PQ$ and $PF = PQ$, it follows that the distance from the focus to S is

$$PF+PS=PF+QS-PQ=QS$$

Thus, a property of a parabolic reflector is that waves from an isotropic source at the focus that are reflected from the parabola arrive at a line AA' with equal phase. The “image” of the focus is the directrix and the reflected field along the line AA' appears as though it originated at the directrix as a plane wave. The plane BB' (Fig. 6.7c) at which a reflector is cut off is called the aperture plane. A cylindrical parabola converts a cylindrical wave radiated by an in-phase line source at the focus, as in Fig. 6.7a, into a plane wave at the aperture, or a paraboloid-of-revolution converts a spherical wave from an isotropic source at the focus, as in Fig. 6.7b, into a uniform plane wave at the aperture. Confining our attention to a single ray or wave path, the paraboloid has the property of directing or collimating radiation from the focus into a beam parallel to the axis.

The presence of the primary antenna in the path of the reflected wave, as in the above examples, has two principle disadvantages. These are, first, that waves reflected from the parabola back to the primary antenna produce interaction and mismatching. Second, the primary antenna acts as an obstruction, blocking out the central portion of the aperture and increasing the minor lobes. To avoid both effects, a portion of the paraboloid can be used and the primary antenna displaced as in Fig. 6.8. This is called an offset feed.

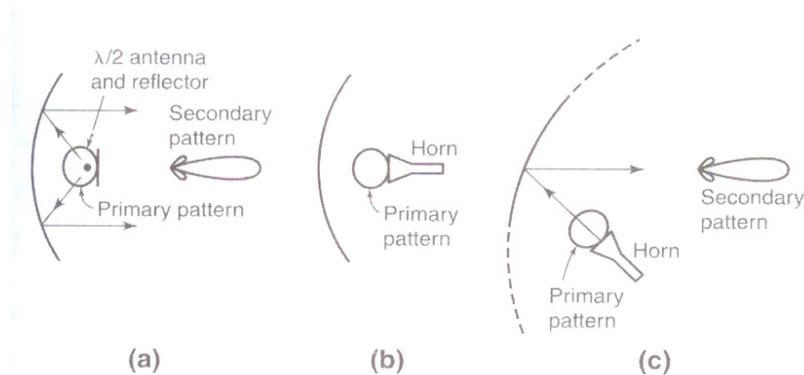


Fig 6.8 : Parabolic reflector with Offset feed

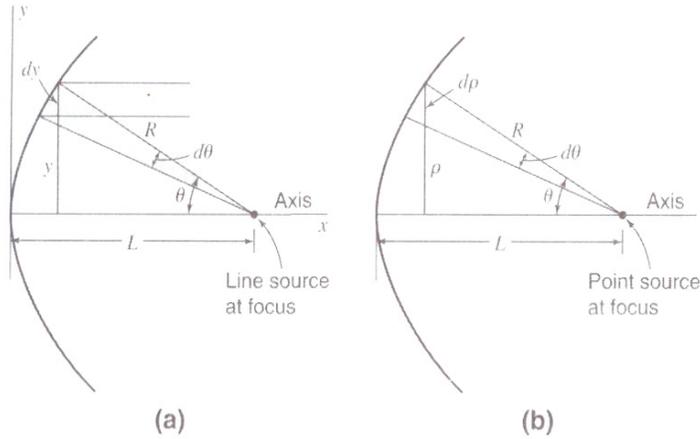


Fig 6.9 : Cross Sections of cylindrical parabola (a) and of paraboloid of revolution (b)

Let us next develop an expression for the field distribution across the aperture of a parabolic reflector. Since the development is simpler for a cylindrical parabola, this case is treated first, as an introduction to the case for a paraboloid. Consider a cylindrical parabolic reflector with line source as in Fig. 6.9a. The line source is isotropic in a plane perpendicular to its axis (plane of page). For a unit distance in the z direction (normal to page in Fig. 6.9a) the power P in a strip of width dy is

$$P = dyS_y \text{----- (6.16)}$$

Where S_y = the power density at y, $W m^{-2}$

$$P = U'd\theta \text{----- (6.17)}$$

U' =the power per unit angle per unit length in the direction

$$S_y dy = U'd\theta$$

$$\frac{S_y}{U'} = \frac{1}{(d/d\theta)(R \sin \theta)}$$

$$R = \frac{2L}{1 + \cos \theta}$$

$$S_y = \frac{1 + \cos \theta}{2L} U'$$

The ratio of the power density

$$\frac{S_{\theta}}{S_0} = \frac{1 + \cos \theta}{2} \text{----- (6.18)}$$

The field intensity ratio in the aperture plane is equal to the square root of the power ratio

$$\frac{E_{\theta}}{E_0} = \sqrt{\frac{1 + \cos \theta}{2}} \text{----- (6.19)}$$

$$P = 2\pi\rho d\rho S\rho \text{----- (6.20)}$$

$$P = 2\pi \sin \theta d\theta U \text{----- (6.21)}$$

Equating Equations 6.20 and 6.21

$$\rho d\rho S\rho = \sin \theta d\theta U$$

$$\frac{S_{\rho}}{U} = \frac{\sin \theta}{\rho(d\rho/d\theta)} \text{----- (6.22)}$$

$$S_{\rho} = \frac{(1 + \cos \theta)^2}{4L^2} U$$

$$\frac{S_{\theta}}{S_0} = \frac{(1 + \cos \theta)^2}{4} \text{----- (6.23)}$$

$$\frac{E_{\theta}}{E_0} = \frac{1 + \cos \theta}{2} \text{----- (6.24)}$$

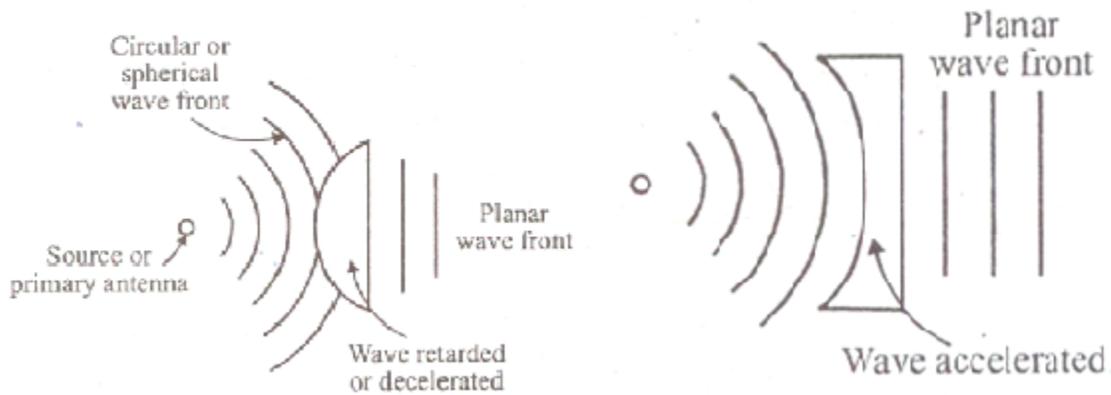
The Log periodic antenna

This is a frequency independent antenna for which the impedance and radiation pattern (and hence the directivity) remains constant as a function of frequency. But in this antenna, the electrical properties like impedance are a logarithmically periodic function of the frequency. i.e. if a graph of 'z' is plotted v/s log f a repetitive variation will be obtained.

One of the design for a log periodic antenna is as shown in fig. 6.10

Lens antenna

Like parabolic reflectors, lens is used to convert circular or spherical wave fronts into planar wave fronts, versa as a receiver. Lens is a medium through which the waves are transmitted or e decelerating medium and accelerating medium. In decelerating system, the velocity with in the medium is less than that of free space velocity. Pure dielectrics like Lucite or polysterene,



impure dielectrics or H-plane metal plates can be used as decelerating mediums. Accelerating system is the one in which the velocity within the medium is more than that of free space velocity. E-plane metal plates are the examples for accelerating types. Lens Antenna with different refractive index are as shown in fig.6.12 and 6.13.

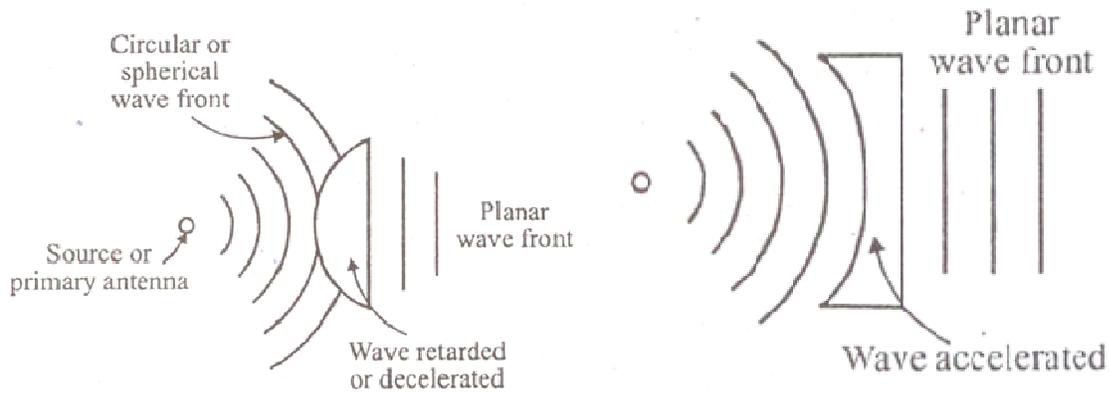


Fig 6.12 : Lens Antenna

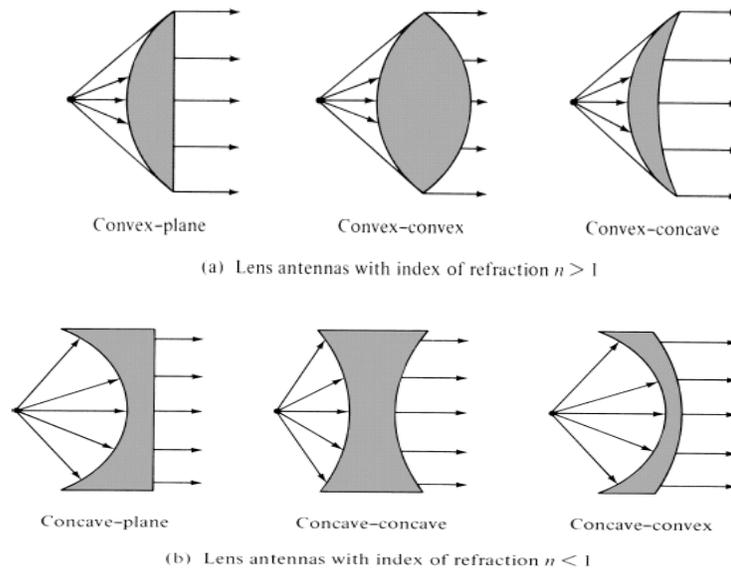


Fig 6.13 : Lens Antenna with different refractive index

Dielectric Lens Antenna

The dielectric material used should have a refractive index more than 1 w.r.t. free space having minimum dielectric losses. Lucite and polystyrene can be used having a refractive index $n=1.5$. The system is constructed

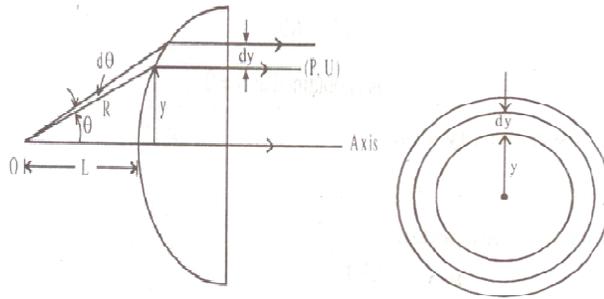


Fig 6.15 : Lens Analysis

Consider the dielectric lens with a primary source at the focus point O as shown in fig.6.15. Let P is the power density and U is radiation intensity at a distance y from the axis. Assuming P and U remain constant within the elemental aperture subtended by $d\theta$ or dy , the power radiated through elemental aperture is

$$dW = 2\pi y \cdot dy \cdot P \quad \text{----- (6.30)}$$

Where $W = \iint U \cdot d\Omega$

$$W = \int_0^{2\pi} \int_{\theta}^{\theta+d\theta} U \sin \theta \, d\theta \, d\phi$$

$$W = U \cdot 2\pi \int_{\theta}^{\theta+d\theta} \sin \theta \, d\theta$$

$$W = 2\pi U \sin \theta \, d\theta \quad \text{----- (6.31)}$$

Equating Equations 6.29 and 6.30

$$2\pi y \cdot dy \cdot P = 2\pi U \sin \theta \, d\theta \quad \text{----- (6.32)}$$

$$\frac{P}{U} = \frac{\sin \theta \, d\theta}{y \, dy} = \frac{\sin \theta}{y \frac{dy}{d\theta}} = \frac{\sin \theta}{y \frac{d}{d\theta}(y)} \quad \text{----- (6.33)}$$

From the geometry,

$$y = R \sin \theta$$

$$y = \frac{L(a-1) \sin \theta}{a \cos \theta - 1} \text{ (6.34)}$$

Substituting above equation in equation 6.33

$$\frac{P}{U} = \frac{\sin \theta}{\frac{L(a-1) \sin \theta}{a \cos \theta - 1} \frac{d}{d\theta} \left(\frac{L(a-1) \sin \theta}{a \cos \theta - 1} \right)}$$

$$P = U \frac{(a \cos \theta - 1)^3}{L^2 (a-1)^2 (a - \cos \theta)} \text{ (6.35)}$$

The power density along the axis is obtained at $\theta = 0$.

$$P_0 = \frac{U}{L^2}$$

The relative power density is

$$\frac{P}{P_0} = \frac{(a \cos \theta - 1)^3}{(a-1)^2 (a - \cos \theta)} \text{ (6.36)}$$

The relative electric field is

$$\frac{E}{E_0} = \sqrt{\frac{(a \cos \theta - 1)^3}{(a-1)^2 (a - \cos \theta)}} \text{ (6.37)}$$

when

$$\theta=0, \frac{E}{E_0} = 1$$

$$\theta=20, \frac{E}{E_0} = 0.7$$

$$\theta=40, \frac{E}{E_0} = 0.14$$

Relative electric field:

Relative Electric field is as shown in fig.6.16

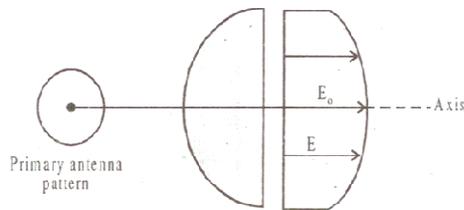


Fig 6.16 : Relative Electric Field

E-Plane Metal Plate Lens

The velocity in between E-Plane Metal Plate is more than the Free space velocity v_0

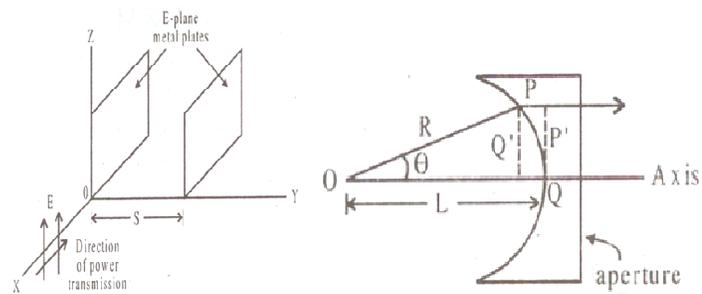


Fig 6.17 : E-Plane Metal Plate Lens

Advantages of Lens Antenna

- Can be used as Wide band Antenna since its shape is independent of frequency.
- Provides good collimation.
- Internal dissipation losses are low, with dielectric materials having low loss tangent.
- Easily accommodate large band width required by high data rate systems.

Quite in-expensive and have good fabrication tolerance

Disadvantages of Lens Antenna

- Bulky and Heavy
- Complicated Design
- Refraction at the boundaries of the lens



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DEPARTMENT OF ECE

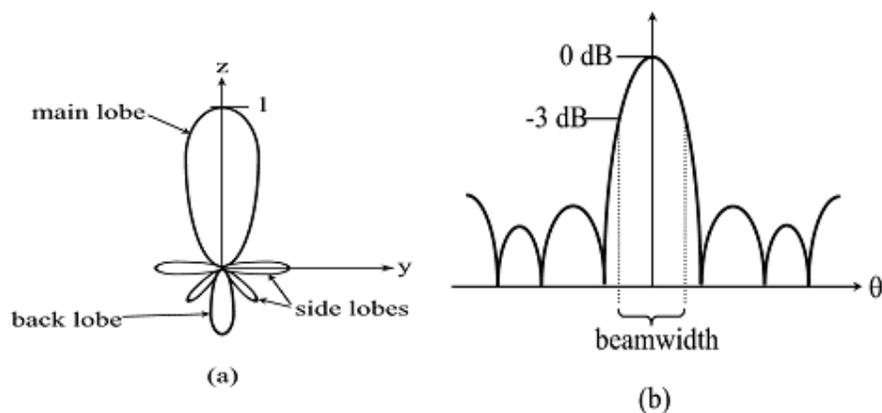
COURSE: ANTENNAS & WAVE PROPAGATION

UNIT-IV

Radiation pattern:

The radiation pattern of antenna is a representation (pictorial or mathematical) of the distribution of the power out-flowing (radiated) from the antenna (in the case of transmitting antenna), or inflowing (received) to the antenna (in the case of receiving antenna) as a function of direction angles from the antenna

Antenna radiation pattern (antenna pattern): It is defined for large distances from the antenna, where the spatial (angular) distribution of the radiated power does not depend on the distance from the radiation source is independent on the power flow direction



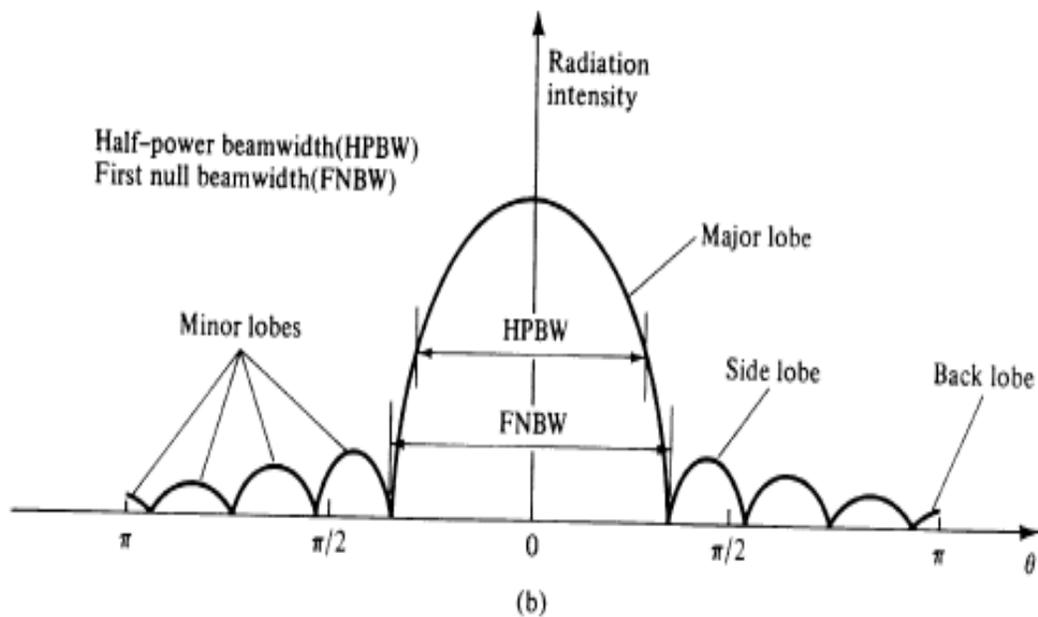
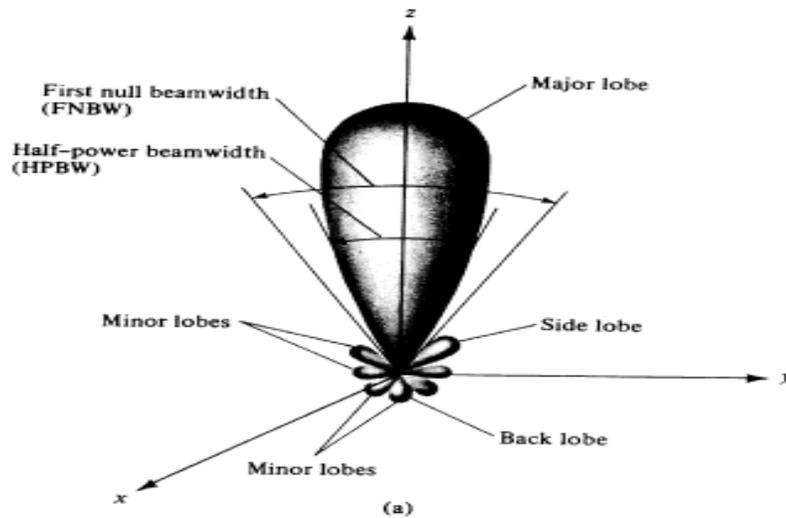
It is clear in Figures a and b that in some very specific directions there are zeros, or nulls, in the pattern indicating no radiation.

The protuberances between the nulls are referred to as lobes, and the main, or major, lobe is in the direction of maximum radiation.

There are also side lobes and back lobes. These other lobes divert power away from the main beam and are desired as small as possible.

Pattern lobe is a portion of the radiation pattern with a local maximum
Lobes are classified as: major, minor, side lobes, back lobes

Point sources and Arrays



Pattern lobes and beam widths

Normalized pattern:

Usually, the pattern describes the normalized field (power) values with respect to the maximum value.

Note: The power pattern and the amplitude field pattern are the same when computed and when plotted in dB.



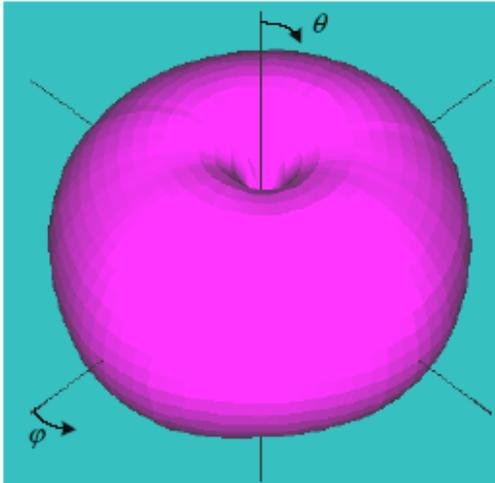


Fig: 3-D pattern

Antenna radiation pattern is 3-dimensional. The 3-D plot of antenna pattern assumes both angles θ and ϕ varying.

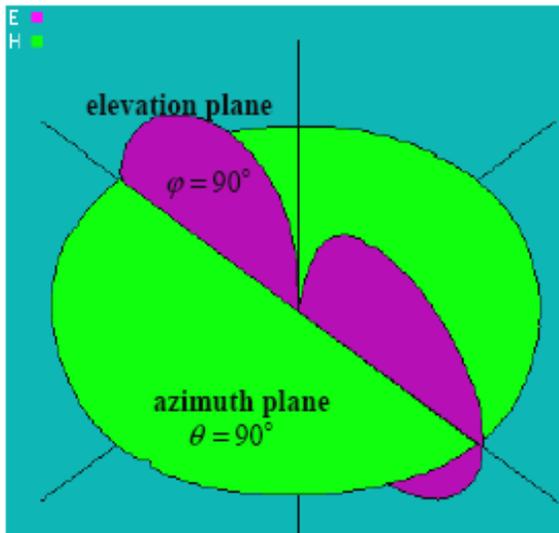


Fig:2-D pattern

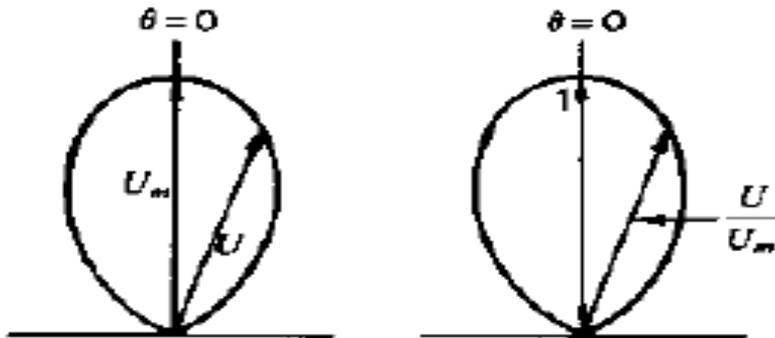
Usually the antenna pattern is presented as a 2-D plot, with only one of the direction angles, θ or ϕ varies.

It is an intersection of the 3-D one with a given plane .Usually it is a $\theta = \text{const}$ plane or a $\phi = \text{const}$ plane that contains the pattern's maximum.



RADIATION INTENSITY

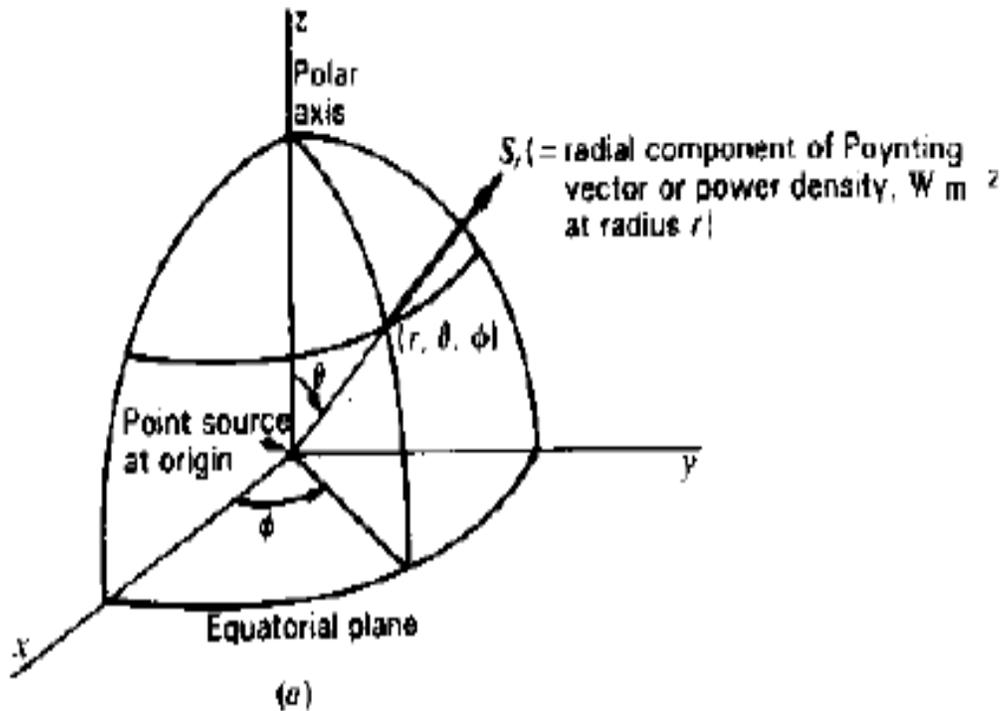
The radiation intensity is total power radiated per unit solid angle and is denoted by U and it is expressed as $U = P/4\pi$.

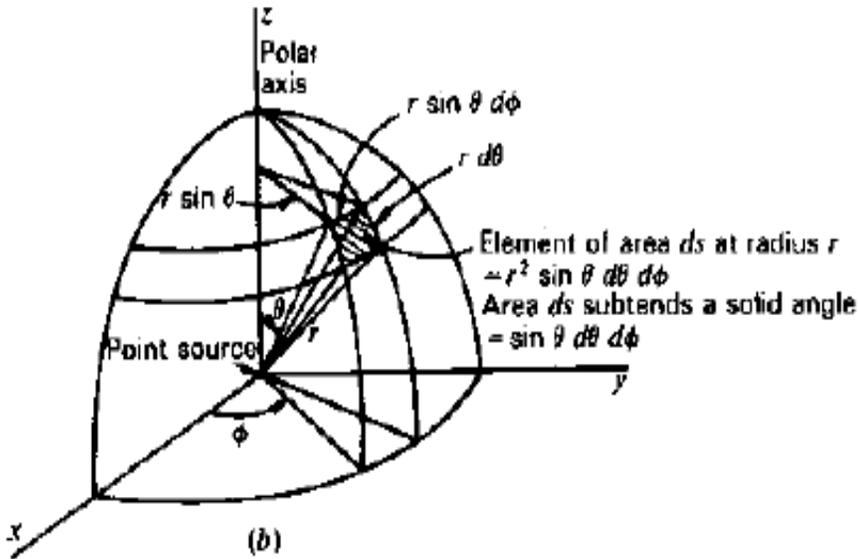


First figure shows radiation intensity of a source and second figure is relative radiation intensity of that source.

POINT SOURCE

A point source is a radiator that has dimensions of a point in space.





POWER PATTERN

The directional property of the antenna is often described in the form of a **power pattern**. The power pattern is simply the effective area normalized to be unity at the maximum.

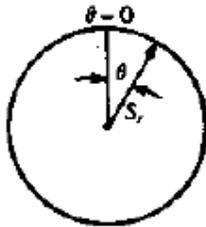


Fig: Power pattern for isotropic source

Power pattern and relative power patterns of a source

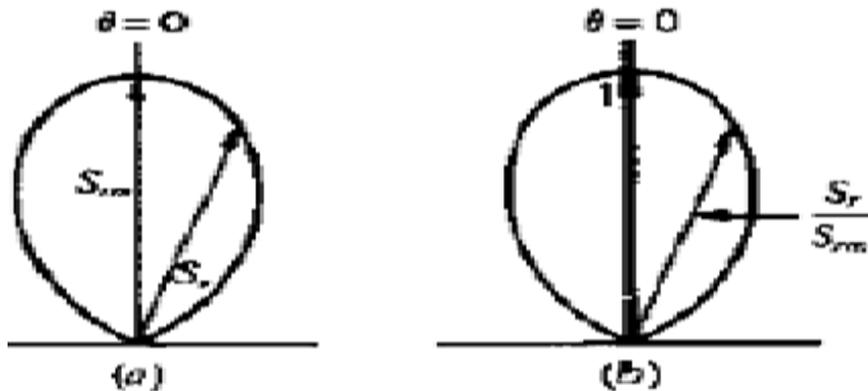


Figure (a) shows power pattern of a source. Figure(b) shows relative power pattern of a same source. Both Patterns have the same shape. The relative power pattern is normalized to a maximum of unity

The radiated energy streams from the source in radial lines.

Time rate of Energy flow/unit area is called as Poynting vector (Power Density)

It is expressed aswatts / square meters.

For a Point source Poynting vector has only radial component S_r

S component in Θ and ϕ directions are zero.

Magnitude of $S = S_r$

Source radiating uniformly in all directions – Isotropic Source.

It is independent of Θ and ϕ .

Graph of S_r at a constant radius as a function of angle is POWER PATTERN

Field pattern

A pattern showing variation of the electric field intensity at a constant radius r as a function of angle(θ , ϕ) is called “**field pattern**”

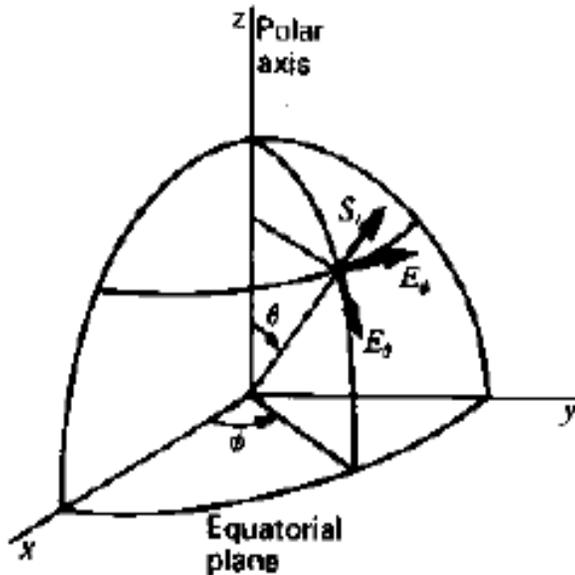


Fig: Relation of poynting vector s and 2 electric field components of a far field

The power pattern and the field patterns are inter-related:

$$P(\theta, \phi) = (1/\eta) * |E(\theta, \phi)|^2 = \eta * |H(\theta, \phi)|^2$$

P = power

E = electrical field component vector

H = magnetic field component vector

$\eta = 377$ ohm (free-space impedance)

The power pattern is the measured (calculated) and plotted received power: $|P(\theta, \phi)|$ at a constant (large) distance from the antenna

The amplitude field pattern is the measured (calculated) and plotted electric (magnetic) field intensity, $|E(\theta, \phi)|$ or $|H(\theta, \phi)|$ at a constant (large) distance from the antenna s

Antenna Arrays

Antennas with a given radiation pattern may be arranged in a pattern line, circle, plane, etc.) to yield a different radiation pattern.

Antenna array - a configuration of multiple antennas (elements) arranged to achieve a given radiation pattern.

Simple antennas can be combined to achieve desired directional effects. Individual antennas are called elements and the combination is an array

Types of Arrays

1. Linear array - antenna elements arranged along a straight line.
2. Circular array - antenna elements arranged around a circular ring.
3. Planar array - antenna elements arranged over some planar surface (example - rectangular array).
4. Conformal array - antenna elements arranged to conform two some non-planar surface (such as an aircraft skin).

Design Principles of Arrays

There are several array design variables which can be changed to achieve the overall array pattern design.

Array Design Variables

1. General array shape (linear, circular, planar)
2. Element spacing.
3. Element excitation amplitude.
4. Element excitation phase.
5. Patterns of array elements.

Types of Arrays

- Broadside: maximum radiation at right angles to main axis of antenna
 - End-fire: maximum radiation along the main axis of antenna
 - Phased: all elements connected to source
 - Parasitic: some elements not connected to source
 - They re-radiate power from other elements
-

Yagi-Uda Array

- Often called Yagi array
- Parasitic, end-fire, unidirectional
- One driven element: dipole or folded dipole
- One reflector behind driven element and slightly longer
- One or more directors in front of driven element and slightly shorter

Log-Periodic Dipole Array

- Multiple driven elements (dipoles) of varying lengths
- Phased array
- Unidirectional end-fire
- Noted for wide bandwidth
- Often used for TV antennas

Monopole Array

- Vertical monopoles can be combined to achieve a variety of horizontal patterns
- Patterns can be changed by adjusting amplitude and phase of signal applied to each element
- Not necessary to move elements
 - Useful for AM broadcasting

Collinear Array

- All elements along same axis
- Used to provide an omnidirectional horizontal pattern from a vertical antenna
- Concentrates radiation in horizontal plane

Broadside Array

- Bidirectional Array
- Uses Dipoles fed in phase and separated by $1/2$ wavelength

End-Fire Array

- Similar to broadside array except dipoles are fed 180 degrees out of phase
 - Radiation max. off the ends
-

Application of Arrays

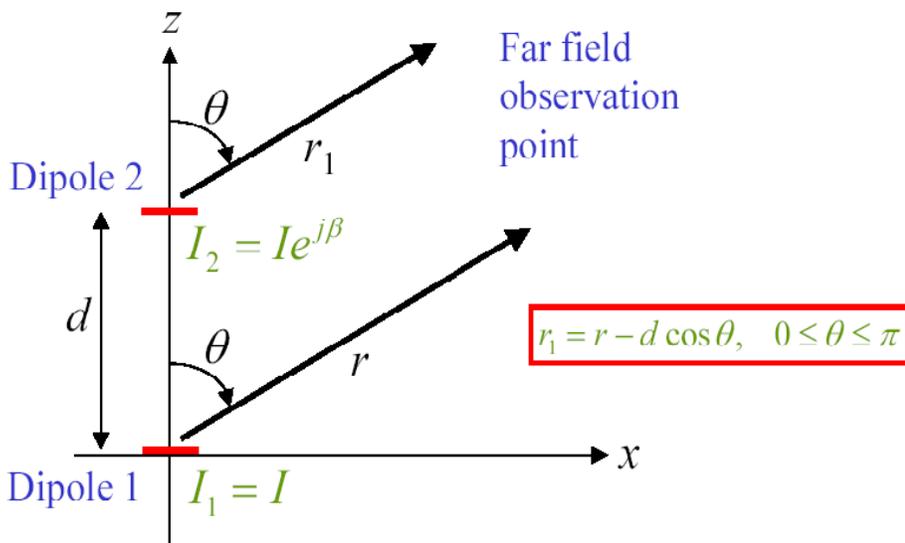
An array of antennas may be used in a variety of ways to improve the performance of a communications system. Perhaps most important is its capability to cancel co channel interferences. An array works on the premise that the desired signal and unwanted co channel interferences arrive from different directions. The beam pattern of the array is adjusted to suit the requirements by combining signals from different antennas with appropriate weighting. An array of antennas mounted on vehicles, ships, aircraft, satellites, and base stations is expected to play an important role in fulfilling the increased demand of channel requirement for these services

ARRAY OF POINT SOURCES

ARRAY is an assembly of antennas in an electrical and geometrical of such a nature that the radiation from each element add up to give a maximum field intensity in a particular direction & cancels in other directions. An important characteristic of an array is the change of its radiation pattern in response to different excitations of its antenna elements.

CASE1:

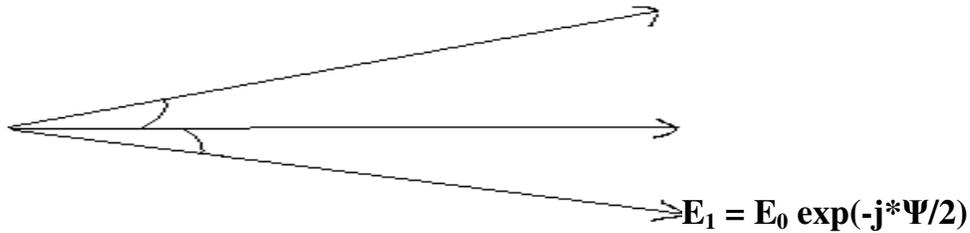
2 isotropic point sources of same amplitude and phase



- **Phase difference** $= \beta d/2 * \cos \theta = 2\pi/\lambda * d/2 * \cos \theta$
 β = propagation constant

and $d_r = \beta d = 2\pi/\lambda * d = \mathbf{Path\ difference}$

$$E_2 = E_0 \exp(j*\Psi/2)$$



The total field strength at a large distance r in the direction θ is :

$$E = E_1 + E_2 = E_0[\exp(j*\Psi/2) + \exp(-j*\Psi/2)]$$

$$\text{Therefore: } E = 2E_0 \cos\Psi/2 \dots\dots\dots (1)$$

$\Psi =$ phase difference between E_1 & E_2 & $\Psi/2 = dr/2 * \cos\theta$

$E_0 =$ amplitude of the field at a distance r by single isotropic antenna

Substituting for Ψ in (1) & normalizing

$$E = 2E_0 \cos(2\pi/\lambda * d/2 * \cos\theta)$$

$$E_{nor} = \cos(dr/2 * \cos\theta)$$

for $d = \lambda/2$

$$E = \cos(\pi/2 * \cos\theta)$$

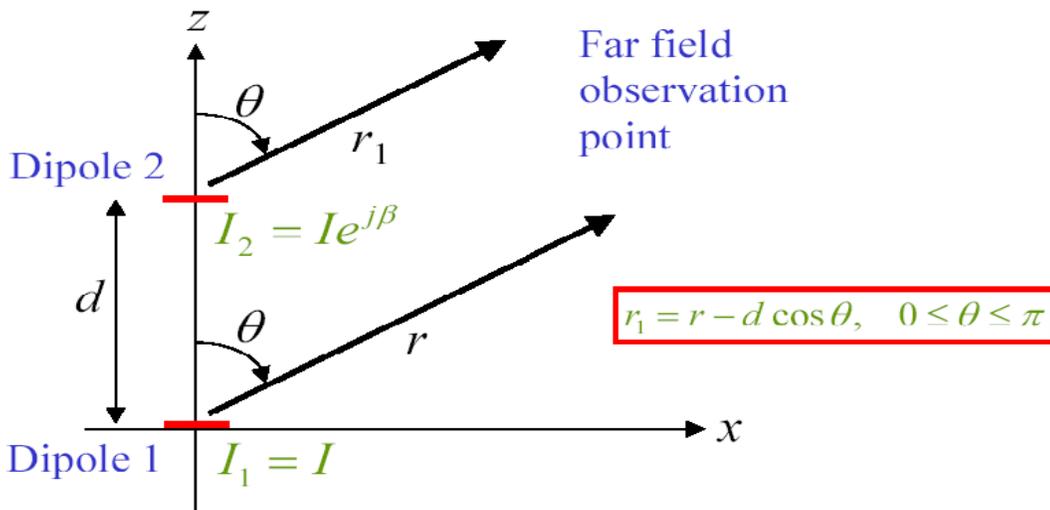
At $\theta = \pi/2$ $E = 1 \dots$ Point of maxima = $\pi/2$ (or) $3\pi/2$

At $\theta = 0$ $E = 0 \dots$ Point of minima = 0 (or) π

At $\theta = \pm\pi/3$ $E = 1/\sqrt{2}$ 3db bandwidth point = $\pm\pi/3$

CASE2:

2 isotropic point sources of same amplitude but opposite phase



The total field strength at a large distance r in the direction θ is :

$$E = E_1 + E_2 = E_0[\exp(j*\Psi/2) - \exp(-j*\Psi/2)]$$

Point sources and Arrays

Therefore: $E = 2jE_0 \text{SIN}(\Psi/2)$ (2)

$\Psi =$ phase difference between E_1 & E_2

$\Psi/2 = dr/2 * \cos\theta$

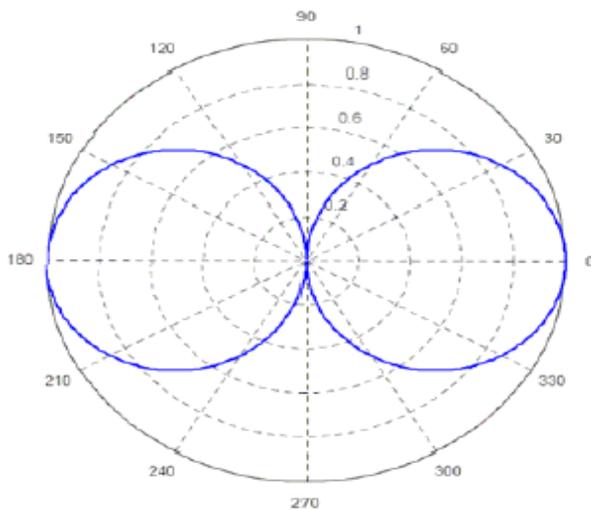
$E_0 =$ amplitude of the field at a distance by single isotropic antenna

At $k=0$ $E=1$ Point of maxima= 0(or) π

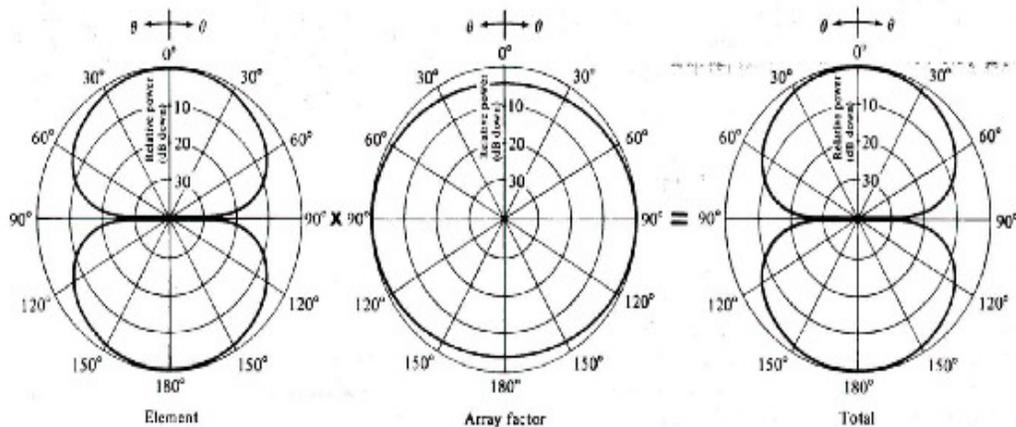
At $k=0, \theta=\pi/2$ $E=0$ Point of minima= $\pi/2$ (or) $-\pi/2$

At $\theta=\pm\pi/3$ $E=1/\sqrt{2}$ 3db bandwidth point= $\pm\pi/3$

END FIRE ARRAY PATTERN



Examples of array patterns using pattern multiplication:



Array pattern of a two-element array of Hertzian dipoles ($\beta = 0^\circ$, and $d = \lambda/4$)

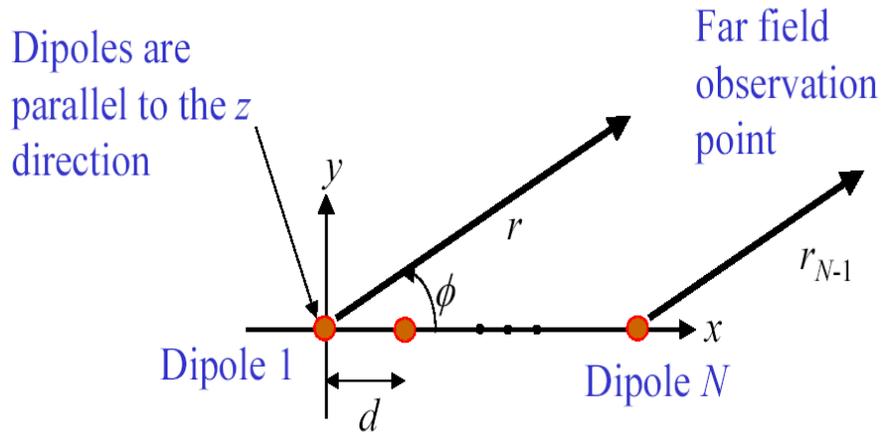
Pattern multiplication:

The total far-field radiation pattern $|E|$ of array (array pattern) consists of the original radiation pattern of a single array element multiplying with the magnitude of the array factor $|AF|$. This is a general property of antenna arrays and is called the principle of pattern multiplication.

Uniformly excited equally spaced linear arrays

Linear arrays of N-isotropic point sources of equal amplitude and spacing:

An array is said to be linear if the individual elements of the array are spaced equally along a line and uniform if the same are fed with currents of equal amplitude and having a uniform phase shift along the line



The total field E at distance point in the direction of ϕ is given by $E = 1 + e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + \dots + e^{j(n-1)\Psi}$ (1)

Where $\Psi =$ total phase difference between adjacent source

$$\Psi = dr \cos \phi + \delta = 2\pi/\lambda * d * \cos \phi + \delta$$

$\delta =$ phase difference of adjacent source

multiplied equation (1) by $e^{j\Psi}$

$$E e^{j\Psi} = e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + \dots + e^{jn\Psi} \quad (3)$$

(1)-(3)

$$E(1 - e^{jn\Psi}) = (1 - e^{j\Psi})$$

$$E = (1 - e^{jn\Psi}) / (1 - e^{j\Psi})$$

$$E = e^{j(n-1)\Psi/2} \{ \sin(n\Psi/2) / \sin(\Psi/2) \}$$

If the phase is referred to the centre point of the array, then E reduces to

$$E = \sin(n\Psi/2) / \sin(\Psi/2)$$

when $\Psi = 0$

$$E = \lim_{\Psi \rightarrow 0} (\sin(n\Psi/2) / \sin(\Psi/2))$$

$$\Psi \rightarrow 0$$

$$E = n E_{\max}$$

$$\Psi = 0 \quad E = E_{\max} = n \dots \dots \dots \text{normalizing}$$

$$E_{\text{norm}} = E/E_{\max} = (1/n)(\sin(n\Psi/2)) / \sin(\Psi/2)$$

CASE 1: LINEAR BROAD SIDE ARRAY

An array is said to be **broadside** if the phase angle is such that it makes maximum radiation perpendicular to the line of array i.e. 90° & 270°

For broad side array $\Psi = 0$ & $\delta = 0$

Therefore $\Psi = dr \cos \phi + \delta = \beta d \cos \phi + 0 = 0 \quad \phi = \pm 90^\circ$

therefore $\phi_{\max} = 90^\circ$ & 270°

Broadside array example for $n=4$ and $d=\lambda/2$

By previous results we have $\phi_{\max} = 90^\circ$ & 270°

Direction of pattern maxima:

$$E = (1/n)(\sin(n\Psi/2)) / \sin(\Psi/2)$$

This is maximum when numerator is maximum i.e. $\sin(n\Psi/2) = 1$

$$n\Psi/2 = \pm(2k+1)\pi/2$$

where $k=0,1,2,\dots$

$K=0$ major lobe maxima

$$K=1 \quad n\Psi/2 = \pm 3\pi/2 \quad \Psi = \pm 3\pi/4$$

Therefore $dr \cos \phi = 2\pi/\lambda * d * \cos \phi = \pm 3\pi/4$

$$\cos \phi = \pm 3/4$$

$$\phi = (\phi_{\max})_{\text{minor lobe}} = \cos^{-1}(\pm 3/4) = \pm 41.4^\circ \text{ or } \pm 138.6^\circ$$

At $K=2 \quad \phi = \cos^{-1}(\pm 5/4)$ which is not possible

Direction of pattern minima or nulls

It occurs when numerator = 0

i.e. $\sin(n\Psi/2) = 0 \quad n\Psi/2 = \pm k\pi$

where $k=1,2,3,\dots$

now using condition $\delta=0$

$$\Psi = \pm 2k\pi/n = \pm k\pi/2 \quad dr \cos \phi = 2\pi/\lambda * d/2 * \cos \phi$$

Substituting for d and rearranging the above term

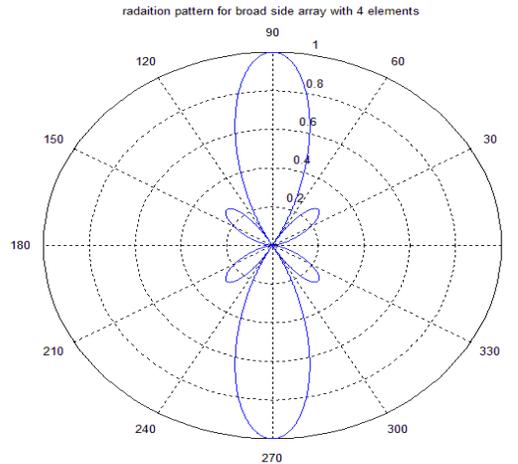
$$\pi \cos \phi = \pm k\pi/2 \quad \cos \phi = \pm k/2$$

therefore $\phi_{\min} = \cos^{-1}(\pm k/2)$

$$K=1 \quad \phi_{\min} = \cos^{-1}(\pm 1/2) = \pm 60^\circ \text{ or } \pm 120^\circ$$

$$K=2 \quad \phi_{\min} = \cos^{-1}(\pm 1) = 0^\circ \text{ or } \pm 180^\circ$$

Beam width is the angle b/w first nulls



From the pattern we see that

Beamwidth between first pair of nulls =BWFN= 60°
 Half power beam width =BWFN/ 2= 30°

CASE2: END FIRE ARRAY

An array is said to be end fire if the phase angle is such that it makes maximum radiation in the line of array i.e. 0° & 180°

For end fire array $\Psi=0$ & $\phi =0^{\circ}$ & 180°

Therefore $\Psi =dr*\cos\phi +\delta$ $\delta= -dr$

The above result indicates that for an end fire array the phase difference b/w sources is retarded progressively by the same amount as spacing b/w the sources in radians.

If $d= \lambda/2$ $\delta= -dr = - 2\pi/\lambda \times \lambda /2= -\pi$

The above result indicates that source 2 lags behind source1 by π radians.

End fire array example for n=4 and d= $\lambda/2$

Direction of maxima

Maxima occurs when $\sin(n\Psi/2)=1$

i.e. $\Psi/2= \pm(2k+1)\pi/2$

where $k=0,1,2,\dots$

$\Psi = \pm(2k+1)\pi/n$ $dr*\cos\phi+\delta= \pm(2k+1)\pi/n$

$\cos\phi= [\pm(2k+1)\pi/n -\delta]/dr$

Therefore $\phi_{\max} = \cos^{-1} \{ [\pm(2k+1)\pi/n - \delta]/dr \}$

By definition For end fire array : $\delta = -dr = -2\pi/\lambda * d$

Therefore $\phi_{\max} = \cos^{-1} \{ [\pm(2k+1)\pi/n - \delta]/ (-2\pi/\lambda * d) \}$

For $n=4$, $d=\lambda/2$ $dr=\pi$ after substituting these values in above equation & solving we get

$$\Phi_{\max} = \cos^{-1} \{ [\pm(2k+1)/4 + 1] \}$$

Where $k=0,1,2,\dots$

For major lobe maxima,

$$\begin{aligned} \Psi = 0 &= dr * \cos\phi + \delta \\ &= dr * \cos\phi - dr \\ &= dr(\cos\phi - 1) \quad \cos\phi_m = 1 \end{aligned}$$

there fore $\phi_m = 0^0$ or 180^0

Minor lobe maxima occurs when $k=1,2,3,\dots$

$$\begin{aligned} K=1 \quad (\phi_{\max})_{\text{minor}1} &= \cos^{-1} \{ [\pm(3)/4 + 1] \} \\ &= \cos^{-1} (7/4 \text{ or } 1/4) \end{aligned}$$

Since $\cos^{-1} (7/4)$ is not possible

$$\text{Therefore } (\phi_{\max})_{\text{minor}1} = \cos^{-1} (1/4) = 75.5$$

$$\begin{aligned} K=2 \quad (\phi_{\max})_{\text{minor}2} &= \cos^{-1} \{ [\pm(5)/4 + 1] \} \\ &= \cos^{-1} (9/4 \text{ or } -1/4) \end{aligned}$$

Since $\cos^{-1} (9/4)$ is not possible

Therefore

$$(\phi_{\max})_{\text{minor}1} = \cos^{-1} (-1/4) = 104.4$$

Direction of nulls:

it occurs when numerator=0

$$\text{i.e. } \sin(n\Psi/2) = 0 \quad n\Psi/2 = \pm k\pi$$

where $k=1,2,3,\dots$

$$\text{Here } \Psi = dr * \cos\phi + \delta = dr(\cos\phi - 1)$$

$$dr = 2\pi/\lambda * \lambda/2 = \pi$$

Substituting for d and n

$$dr(\cos\phi - 1) = \pm 2k\pi/n \quad \cos\phi = \pm k/2 + 1$$

therefore

$$\phi_{\text{null}} = \cos^{-1}(\pm k/2 + 1)$$

$$k=1, \quad \phi_{\text{null}1} = \cos^{-1}(\pm 1/2 + 1) = \cos^{-1}(3/2 \text{ or } 1/2)$$

since $\cos^{-1}(3/2)$ not exist ,

$$\phi_{\text{null}1} = \cos^{-1}(1/2) = \pm 60$$

there fore

$$\phi_{\text{null}1} = \pm 60$$

$$k=2,$$

Point sources and Arrays

$$\phi_{\text{null}2} = \cos^{-1}(\pm 2/2+1)$$

$$= \cos^{-1}(2 \text{ or } 0)$$

since $\cos^{-1}(2)$ not exist ,

$$\phi_{\text{null}2} = \cos^{-1}(0) = \pm 90$$

there fore $\phi_{\text{null}2} = \pm 90^\circ$

$$k=3, \quad \phi_{\text{null}3} = \cos^{-1}(\pm 3/2+1) = \cos^{-1}(5/2 \text{ or } -1/2)$$

since $\cos^{-1}(5/2)$ not exist , $\phi_{\text{null}3} = \cos^{-1}(-1/2) = \pm 120^\circ$

there fore, $\phi_{\text{null}3} = \pm 120^\circ$

$$k=4, \quad \phi_{\text{null}4} = \cos^{-1}(\pm 4/2+1) = \cos^{-1}(3 \text{ or } -1)$$

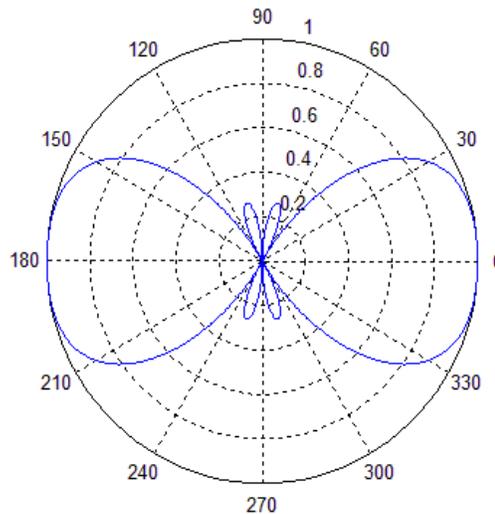
since $\cos^{-1}(3)$ not exist , $\phi_{\text{null}4} = \cos^{-1}(-1) = \pm 180^\circ$

there fore $\phi_{\text{null}4} = \pm 180^\circ$

$$k=5, \quad \phi_{\text{null}5} = \cos^{-1}(\pm 5/2+1) = \cos^{-1}(7/2 \text{ or } -3/2)$$

both values doesn't exist

radiation pattern for endfire side array with 4 elements



$$\text{BWFN} = 60 + 60 = 120^\circ$$

END FIRE ARRAY WITH INCREASED DIRECTIVITY

HANSEN & WOODYARD CONDITION:

It states that a large directivity is obtained by increasing phase change b/w sources so that,

$$\delta = - (dr + \pi/n)$$

$$\text{now, } \Psi = dr * \cos\phi + \delta$$

$$= dr * \cos\phi - (dr + \pi/n)$$

$$= dr(\cos\phi - 1) - \pi/n$$

End fire array with Increased directivity

Example with $n=4$ & $d=\lambda/2$

$$dr = 2\pi/\lambda * \lambda/2 \quad \Psi = \pi (\cos\phi - 1) - \pi/4$$

W.K.T major lobe occurs in the direction $\phi=0^\circ$ or 180°

$$\text{at } 0^\circ \quad E = (1/n)(\sin(n\Psi/2)) / \sin(\Psi/2)$$

$$\text{where } \Psi = \pi (\cos\phi - 1) - \pi/4$$

$$= \pi (\cos 0 - 1) - \pi/4$$

$$= -\pi/4$$

therefore

$$E = (1/4) \sin(-\pi/2) / \sin(-\pi/8) = 0.653$$

At 180°

$$E = (1/n)(\sin(n\Psi/2)) / \sin(\Psi/2)$$

$$\text{where } \Psi = \pi (\cos\phi - 1) - \pi/4$$

$$= \pi (\cos 180 - 1) - \pi/4$$

$$= -9\pi/4$$

therefore

$$E = (1/4) \sin(-9\pi/2) / \sin(-9\pi/8)$$

$$= -0.653$$

MAXIMA DIRECTIONS:

$$\text{by definition } \sin(n\Psi/2) = 1 \quad n\Psi/2 = \pm(2k+1)\pi/2$$

Where $k = 1, 2, 3, \dots$

$$\text{now, } \Psi = \pm(2k+1)\pi/n \quad \pi(\cos\phi - 1) - \pi/4 = \pm(2k+1)\pi/4$$

there fore

$$\cos\phi = \pm(2k+1)/4 + 5/4$$

$$K=1 \quad \cos\phi = \pm(3)/4 + 5/4 = 1/2$$

$$\text{which implies } \phi = \cos^{-1}(1/2) = \pm 60^\circ$$

$$\text{there fore } (\phi_{\max})_{\text{minor1}} = \pm 60^\circ$$

$$\text{Now } E = (1/n)(\sin(n\Psi/2)) / \sin(\Psi/2)$$

$$\text{where } \Psi = \pi (\cos\phi - 1) - \pi/4$$

$$= \pi (\cos 60 - 1) - \pi/4$$

$$= -3\pi/4$$

Now,

$$E = (1/4) \sin(-3\pi/2) / \sin(-3\pi/8)$$

$$= -0.27$$

$$\text{therefore } E = -0.27 \text{ at } \pm 60^\circ$$

$$K=2 \quad \cos\phi = \pm(5)/4 + 5/4 = 0 \text{ \& } 10/4 \text{ which is not possible}$$

$$\text{which implies } \phi = \cos^{-1}(0) = \pm 90^\circ$$

there fore $(\phi_{\max})_{\text{minor}2} = \pm 90^\circ$

Now $E = (1/n)(\sin(n\Psi/2)) / \sin(\Psi/2)$

where $\Psi = \pi (\cos\phi - 1) - \pi/4$

$$= \pi (\cos 90 - 1) - \pi/4$$

$$= -5\pi/4$$

Now, $E = (1/4) \sin(-5\pi/2) / \sin(-5\pi/8) = 0.27$

therefore $E = 0.27$ at $\pm 90^\circ$

$K=3$ $\cos\phi = \pm(7)/4 + 5/4 = -1/2$ & $12/4$ which is not possible

which implies $\phi = \cos^{-1}(-1/2) = \pm 120^\circ$

there fore $(\phi_{\max})_{\text{minor}3} = \pm 120^\circ$

Now $E = (1/n)(\sin(n\Psi/2)) / \sin(\Psi/2)$

where $\Psi = \pi (\cos\phi - 1) - \pi/4$

$$= \pi (\cos 120 - 1) - \pi/4$$

$$= -7\pi/4$$

Now, $E = (1/4) \sin(-7\pi/2) / \sin(-7\pi/8) = \pm 0.653$

therefore $E = \pm 0.653$ at $\pm 120^\circ$

$K=4$ $\cos\phi = \pm(9)/4 + 5/4 = -1$ & $14/4$ which is not possible

which implies $\phi = \cos^{-1}(-1) = \pm 180^\circ$

there fore $(\phi_{\max})_{\text{minor}4} = \pm 180^\circ$

Direction of nulls

$$(\sin(n\Psi/2)) = 0 \quad n\Psi/2 = \pm k\pi$$

Where $k=1, 2, 3, 4, \dots$

now, $\Psi = \pm 2k\pi/n \quad \pi(\cos\phi - 1) - \pi/4$

there fore $\cos\phi = \pm(2k/4) - 5/4$

$K=1$ $\cos\phi = \pm(1)/2 + 5/4 = 3/4$ & $7/4$ which is not possible which

implies $\phi = \cos^{-1}(3/4) = \pm 41.4^\circ$

there fore $\phi_{\text{null}1} = \pm 41.4^\circ$

$K=2$ $\cos\phi = \pm(1) + 5/4 = 1/4$ & $9/4$ which is not possible which

implies $\phi = \cos^{-1}(1/4) = \pm 75.5^\circ$

there fore $\phi_{\text{null}2} = \pm 75.5^\circ$

$K=3$ $\cos\phi = \pm(6/4) + 5/4 = -1/4$ & $11/4$ which is not possible which

implies $\phi = \cos^{-1}(-1/4) = \pm 104.4^\circ$

there fore $\phi_{\text{null}3} = \pm 104.4^\circ$

$K=4$ $\cos\phi = \pm(8/4) + 5/4 = -3/4$ & $13/4$ which is not possible which

implies $\phi = \cos^{-1}(-3/4) = \pm 75.5^\circ$

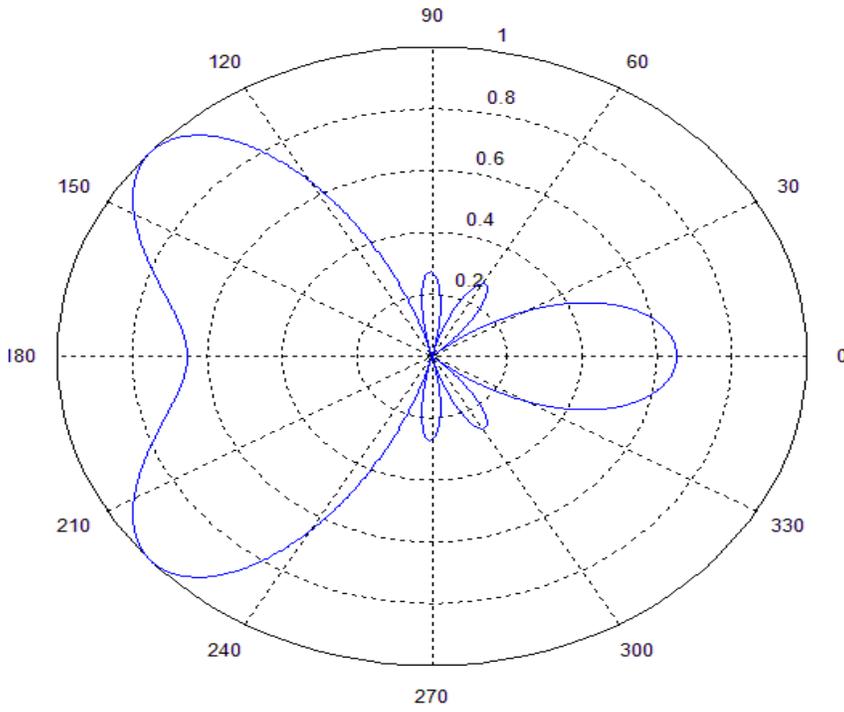
there fore $\phi_{\text{null}4} = \pm 75.5^\circ$

$K=5$ $\cos\phi = \pm(10/4) + 5/4 = -5/4$ & $15/4$

Point sources and Arrays

Both values are not possible

radaiton pattern for endfire side array with increased directivity 4 elements



**G.PULLAIAH COLLEGE OF ENGINEERING AND
TECHNOLOGY, KURNOOL**

DEPARTMENT OF ECE

COURSE: ANTENNAS & WAVE PROPAGATION

UNIT-V

UNIT-5 WAVE PROPAGATION

Ground wave propagation

Space wave propagation

Sky wave propagation

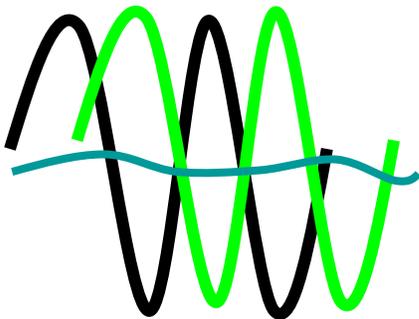
Troposphere Scatter propagation

RADIO PROPAGATION

What is Radio?

Radio is a Transmitter or a Receiver. The Radio Transmitter induces electric and magnetic fields. The electrostatic field Components is $\propto 1/d^3$, induction field components is $\propto 1/d^2$ and radiation field components is $\propto 1/d$. The radiation field has E and B Component. Surface area of sphere centered at transmitter, the field strength at distance $d = E \times B \propto 1/d^2$.

Two main factors affect signal at the Receiver. One is distance (or delay) that results in path attenuation, second is multipath that results in Phase differences



Green signal travels $1/2\lambda$ farther than Black to reach receiver, who sees Blue. For 2.4 GHz, λ (wavelength) = 12.5cm.

Your ability to work with radio is based on 4 factors:

1. Your skill as a radio operator (knowing your regs. etc..);
2. Your equipment and how you use it;
3. The antennas you use;
4. Understanding radio wave propagation.

Antennas:

The antennas are the transducers. The transmitting antenna changes the electrical energy into electromagnetic energy or waves. The receiving antenna changes the electromagnetic energy back into electrical energy. These electromagnetic waves propagate at rates ranging from 150kHz to 300GHz.

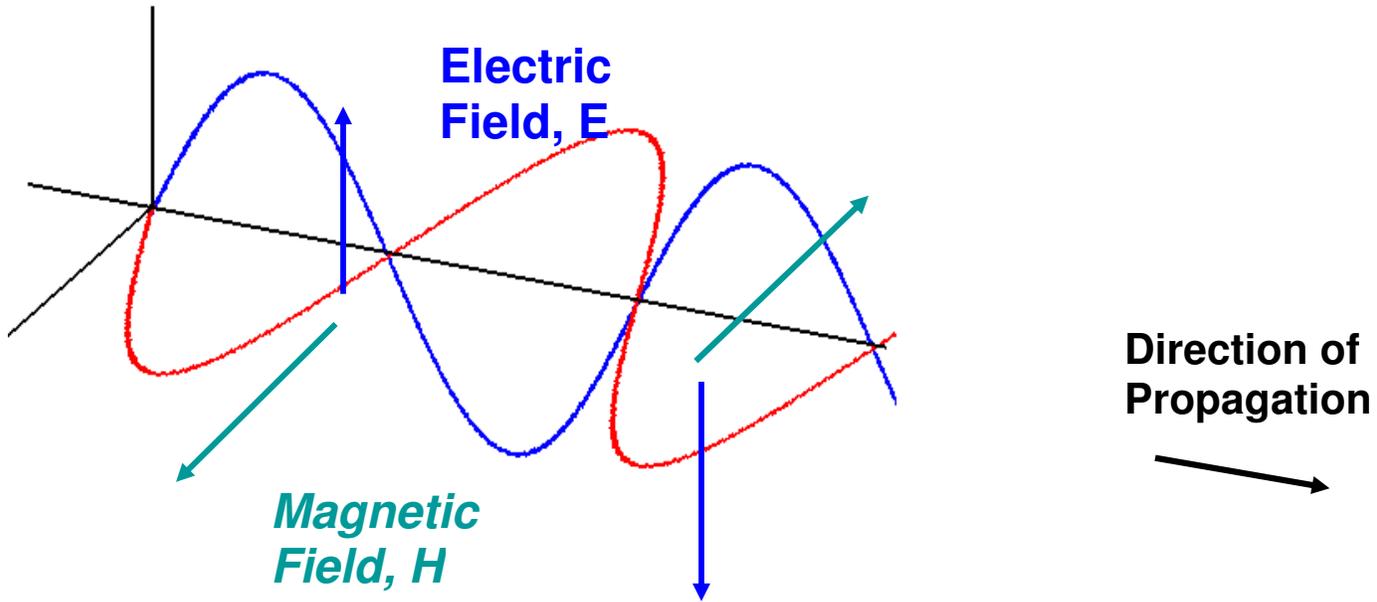
POLARIZATION:

The polarization of an antenna is the orientation of the *electric field with respect to the Earth's surface* and is determined by the physical structure of the antenna and by its orientation. Radio waves from a vertical antenna will usually be vertically polarized and that from a horizontal antenna are usually horizontally polarized.

PROPAGATION

Propagation means how radio waves travel from one point A to another point B. What are the events that occur in the transmission path and how they affect the communications between the points?

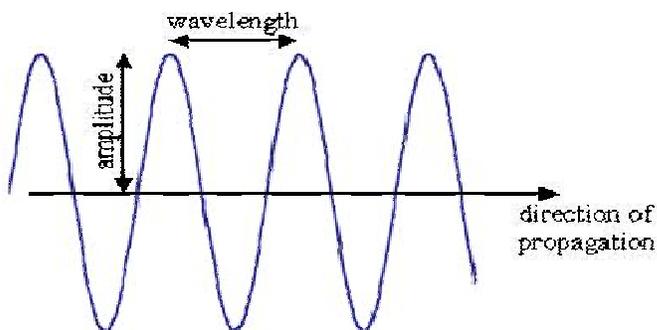
Electromagnetic Waves (EM waves) are produced when the electrons in a conductor i.e antenna wire are made to oscillate back and forth. These waves radiate outwards from the source at the speed of light(300 million meters per second). Electromagnetic Waves are of two types (i)Light Waves (waves we see) (ii)Radio Waves (waves we hear).Both of these EM Waves differ only in frequency and wavelength. EM waves travel in straight lines, unless acted upon by some outside force. They travel faster through a vacuum than through any other medium. As EM waves spread out from the point of origin, they decrease in strength in what is described as an "*inverse square relationship*".



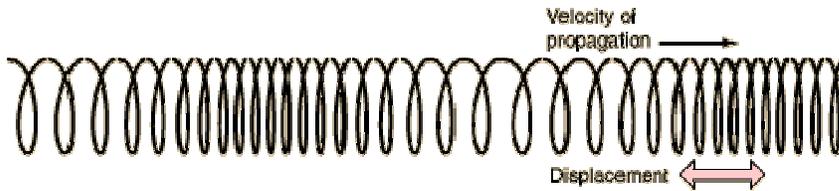
The two fields are at right-angles to each other and the direction of propagation is at right-angles to both fields. The Plane of the Electric Field defines the Polarisation of the wave.

The radio waves can further be classified as Transverse and longitudinal. The Transverse Waves Vibrates from side to side; i.e., at right angles to the direction in which they travel for eg: A guitar string vibrates with transverse motion.

EM waves are always transverse.



For Longitudinal radio waves vibrations are parallel to the direction of propagation. Sound and pressure waves are longitudinal and oscillate back and forth as vibrations are along or parallel to their direction of travel

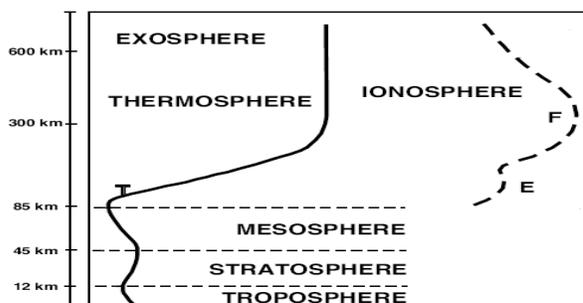


Factors affecting the propagation of radio wave are

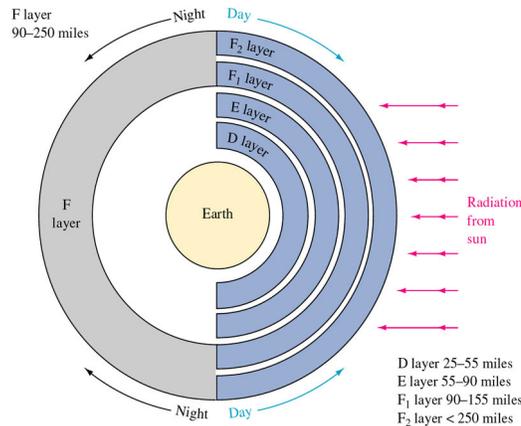
- (i) Spherical shape of the earth:-For Free Space RW travel in straight line. But communication on the earth surface is limited by distance to horizon and requires change in propagation.
- (ii) Atmosphere-Height of about 600km. Is divided into layers. RW near the surface is affected by troposphere. Higher up RW is influenced by ionosphere.
- (iii) Interaction with the objects.

Atmosphere:-

Is divided into Troposphere(earth's surface to about 6.5 mi), Stratosphere(extends from the troposphere upwards for about 23 mi), Ionosphere(extends from the stratosphere upwards for about 250mi) Beyond this layer is Free Space.

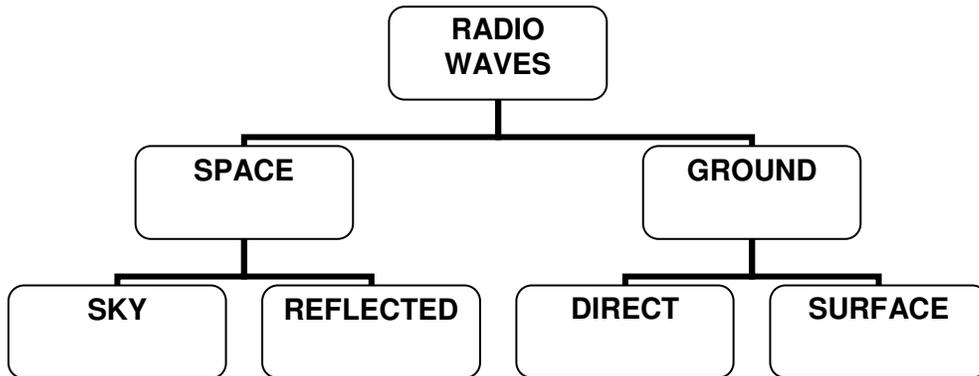


The ionosphere is the uppermost part of the atmosphere and is ionized by solar radiation. Ionization is the conversion of atoms or molecules into an ion by light (heating up or charging) from the sun on the upper atmosphere. Ionization also creates a horizontal set of stratum (layer) where each has a peak density and a definable width or profile that influences radio propagation. The ionosphere is divided into layers.



About 120 km to 400 km above the surface of the Earth is the F layer. It is the top most layer of the ionosphere. Here extreme ultraviolet (UV) (10-100 nm) solar radiation ionizes atomic oxygen (O). The F region is the most important part of the ionosphere in terms of HF communications. The F layer combines into one layer at night, and in the presence of sunlight (during daytime), it divides into two layers, the F₁ and F₂. The F layers are responsible for most sky wave propagation of radio waves, and are thickest and most reflective of radio on the side of the Earth facing the sun. The E layer is the middle layer, 90 km to 120 km above the surface of the Earth. This layer can only reflect radio waves having frequencies less than about 10 MHz. It has a negative effect on frequencies above 10 MHz due to its partial absorption of these waves. At night the E layer begins to disappear because the primary source of ionization is no longer present. The increase in the height of the E layer maximum increases the range to which radio waves can travel by reflection from the layer. The D layer is the innermost layer, 50 km to 90 km above the surface of the Earth. when the sun is active with 50 or more sunspots, During the night cosmic rays produce a residual amount of ionization as a result high-frequency (HF) radio waves aren't reflected by the D layer. The D layer is mainly responsible for absorption of HF radio waves, particularly at 10 MHz and below, with progressively smaller absorption as the frequency gets higher. The absorption is small at night and greatest about midday. The layer reduces greatly after sunset. A common example of the D layer in action is the disappearance of distant AM broadcast band stations in the daytime.

Radio Propagation Modes:



Ground Wave Propagation:-

Propagation of EM wave near earth surface (including troposphere). When the Transmit and Receive antenna are on earth there can be multiple paths for communication. If the Transmit and Receive antenna are in line of sight (LOS) then direct path exist. The propagating wave is called direct wave. When EM wave encounters an interface between two dissimilar media, a part of energy will flow along the interface Known as Surface Wave. At LF and MF this is predominant mode of energy transfer for vertically polarized radiation. Interaction with the objects on ground will manifest as, Reflection, Refraction, Diffraction, Scattering. Waves are collectively called as Space Wave.

FREE SPACE:

Implies an infinite space without any medium or objects that can interact with the EM wave. Antenna is kept in free space and radiation fields are in the form of spherical waves with angular power distribution given by the antenna pattern. It assumes far-field (Fraunhofer region) $d \gg D$ and $d \gg \lambda$, where D is the largest linear dimension of antenna, λ is the carrier wavelength. With no interference and obstructions. The received power at distance d is

$$P_r = K P_t / d^2$$

where P_t is the transmitter power in Watts, a constant factor K depends on antenna gain, a system loss factor, and the carrier wavelength.

$$P_r = P_t G_t G_r \lambda^2 / (4\pi R)^2$$

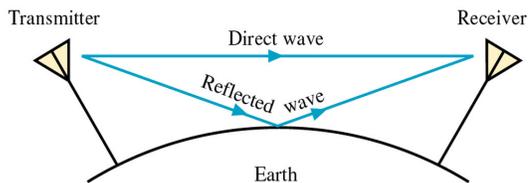
Where P_t =Transmit power, G_t =Transmit gain antenna, G_r =Receive gain antenna

Transfer of electromagnetic energy from transmit antenna to receive antenna take place in a straight line path such communication link is called line of sight link.

The factor $[\lambda / (4\pi R)]^2$ is due propagation and is called free space path loss. It represents the attenuation of the signal due to the spreading of the power as function of distance are 'R'. In decibel units the path loss is expressed as:

$$P_L = 10 \log_{10} (4\pi R / \lambda)^2 \text{ dB}$$

Ground Reflection:



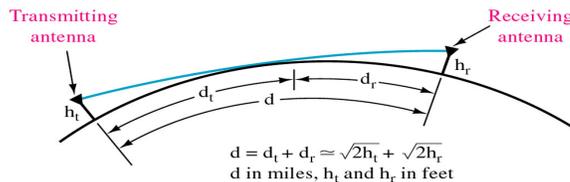
In LOS model, the assumption is that there is only one path for propagation of EM Wave from transmit antenna to receive antenna. The two antennas are kept in free space with no other objects intersecting radiation from transmitter antenna. If two antennas are situated close the ground due to discontinuity in the electrical properties at the air ground interface any wave that falls on the ground is reflected. The amount of reflection depending on factors like angle of incidence, Polarization of wave, Electrical Properties of the Ground i.e conductivity and dielectric constant, the frequency of the propagating wave. Thus, the field at any

point above the ground is a vector sum of the fields due to the direct and the reflected waves.

Direct Wave:-

It is limited to “line-of sight” transmission distances .The limiting factors are antenna height and curvature of earth. The Radio horizon is about 80% greater than line of sight because of diffraction effects. A Part of the signal from the transmitter is bounced off the ground and reflected back to the receiving antenna. If the phase between the direct wave and the reflected wave are not in phase can cause problems

Detune the antenna so that the reflected wave is too weak to receive



To compute the fields of a transmit antenna above an imperfect ground. Used to design of communication links. To select the locations of the transmit and receive antennas and their patterns. Consider a transmit antenna located at point P at a height h_t . Receive antenna located at point Q at a height h_r from the surface of the ground. Let the horizontal distance between the two antenna be d .

The electromagnetic wave from transmit antenna can reach the receive antenna by two possible paths (a) direct path (b) ground reflected path. The total electric field at the field point Q is given by the vector sum of the electric field due to the direct wave and ground reflected wave.

Assumptions:-

1. The transmit antenna and the field points are located in the y-z plane.
2. The transmit antenna is an infinitesimal dipole oriented along the x-axis.

The electric field is of infinitesimal dipole oriented along the x-axis is given by

$$E = -jk\eta(I_0 dl/4\pi)(e^{-jkR}/R)(a_\theta \cos\theta \cos\phi - a_\phi \sin\phi)$$

- R is the distance from the antenna to the field point.

In the y-z plane, $\phi=90^\circ$. Since $\cos 90^\circ = 0$. The θ -component of the electric field is zero. The ϕ -component of the electric field at Q due to the direct wave is given by

$$E_1 = -jk\eta(I_0 dl/4\pi)(e^{-jkR_1}/R_1)$$

The field at Q also has a contribution from the wave that travels via the reflected path PXQ. The location of the point of reflection X depends on h_t , h_r , and d . At X the incident and reflected rays satisfy snell's law of reflection (angle of incidence is equal to angle of reflection). The incident ray PX the reflected ray XQ and the normal to the surface are all contained in the y-z plane. The y-z plane is also known as the plane of incidence. The incident field at X is given by

$$E_i = -jk\eta(I_0 dl/4\pi)(e^{-jkR'_2}/R'_2)$$

- R'2 is the distance from the transmitter to X and the incident E field vector is perpendicular to the plane of incidence. At X the reflection co-efficient, Γ^\perp is given by

$$\begin{aligned} \Gamma^\perp &= E_r/E_i \\ &= (\sin\psi - \sqrt{(\epsilon_r - j\chi) - \cos^2\psi}) / (\sin\psi + \sqrt{(\epsilon_r - j\chi) - \cos^2\psi}) \end{aligned}$$

Electric field is perpendicular to the plane of incidence. At Q is given by

$$E = E_1 + E_2$$

$$E = jk\eta I_0 dl/4\pi(e^{-jkR_1}/R_1 + \Gamma^\perp e^{-jkR_2}/R_2)$$

Field point Q is far away from the transmitter $R_2 \approx R_1$. Total electric field

$$E = E_1 + E_2$$

$$E = jk\eta (I_0 dl/4\pi)(e^{-jkR_1}/R_1) (1+\Gamma^\perp e^{-jk(R_2 - R_1)})$$

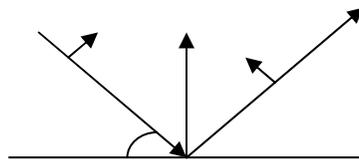
A product of the free space field and an environmental factor, $F \Gamma$ given by

$$F \Gamma = (1+\Gamma^\perp e^{-jk(R_2 - R_1)})$$

The total field at Q due to an infinitesimal dipole at $(0, 0, h_t)$ oriented along the z-direction. The electric field of a z-directed infinitesimal dipole is

$$E = a_0 jk\eta (I_0 d \sin\theta / 4\pi) (e^{-jkR_1}/R_1)$$

Electric field is parallel to the plane of incidence



The electric field is parallel to the plane of incidence and the reflection coefficient, Γ^\perp at X is given by

$$\Gamma_{\parallel} = ((\epsilon_r - j\chi)\sin\psi - \sqrt{(\epsilon_r - j\chi) - \cos 2\psi}) / ((\epsilon_r - j\chi)\sin\psi + \sqrt{(\epsilon_r - j\chi) - \cos 2\psi})$$

The total field at point Q is given by

$$E = jk\eta (I_0 d \sin\theta / 4\pi) (e^{-jkR_1}/R_1) F_{\parallel}$$

$$\text{Where } F_{\parallel} = (1 + \Gamma_{\parallel} e^{-jk(R_2 - R_1)})$$

$$R_1 = \sqrt{d^2 + (h_r - h_t)^2} \approx d\sqrt{1 + (h_r - h_t/d)^2}$$

For $d \gg h_r$ and $d \gg h_t$

Using the first two significant terms in the binomial expansion of $\sqrt{1+x}$,
 $\sqrt{1+x} \approx 1 + x/2$
 for $x \ll 1$;

$$R_1 \approx d[\sqrt{1+1/2*(h_r-h_t/d)^2}]$$

$$R_2 \approx d[\sqrt{1+1/2*(h_r+h_t/d)^2}]$$

The path difference $R_2 - R_1$ is given as $R_2 - R_1 = 2 h_r h_t / d$

For $(h_r h_t / d) \ll \lambda$;

$$\Delta\theta = k(R_2 - R_1) = 4\pi h_r h_t / d\lambda$$

The path difference is small so that $\sin x \approx x$ and $\cos x \approx 1$;

$$e^{-jk(R_2 - R_1)} = \cos(\Delta\theta) - j\sin(\Delta\theta)$$

$$\approx 1 - jk2 h_r h_t / d$$

For low angle of incidence $\Gamma_{\perp} \approx \Gamma_{\parallel} \approx -1$

$$F = F_{\perp} = F_{\parallel} \approx jk2 h_r h_t / d$$

Taking into account the ground reflection, the power received by the receive antenna can be written as

$$P_r = P_t G_t G_r \lambda^2 / (4\pi R)^2 |F|^2$$

For h_r and h_t small compared to d

$$R_1 \approx d$$

Therefore the received power is approximately given by

$$P_r \approx P_t G_t G_r (h_r h_t)^2 / d^4$$

For large d the received power decreases as d^4 . This rate of change of power with distance is much faster than that observed in the free space propagation condition. Taking into account the ground reflection, the power received by the receive antenna can be written as

$$P_r = P_t G_t G_r \lambda^2 / (4\pi R)^2 |F|^2$$

For h_r and h_t small compared to d

$$R_1 \approx d$$

Therefore the received power is approximately given by

$$P_r \approx P_t G_t G_r (h_r h_t)^2 / d^4$$

SURFACE WAVE

Travels directly without reflection on ground. Occurs when both antennas are in LOS

Space wave bend near ground follows a curved path. Antennas must display a very low angle of emission. Power radiated must be in direction of the horizon instead of escaping in sky. A high gain and horizontally polarized antenna is recommended.

If dipole and the field points are on the surface of the earth but separated by a distance d , We have $R_2 = R_1 = d$ and $\psi = 0$

If ground has finite conductivity (typically 10^{-3}S/m - $30 \times 10^{-3} \text{S/m}$) then $\Gamma_{\parallel} = -1$,

The EF due to the direct and ground reflected wave will cancel each other. The EF due to the direct and ground reflected wave is also known as surface wave. Surface wave constitute the primary mode of propagation for frequencies in the range of few KHz-several MHz. In AM broadcast application, A vertical monopole above the ground is used to radiate power in the MW frequency band. The receivers are placed very close to the surface of the earth and hence they receive the broadcast signal via surface wave. Achieve Propagation over hundreds of kilometers. Attenuation factor of the surface wave depends on

1. Distance between the transmitter and receiver.
2. The frequency of the electrical properties of the ground over which the ground propagates. At the surface of the earth the attenuation is also known as the ground wave attenuation factor and is designated as A_{su}

The numerical distance $p = (\pi R / \lambda \chi) \cos b$, where b is the power factor angle

$$b = \tan^{-1}(\epsilon_r + 1 / \chi)$$

Where R is the distance between the transmit and receive antennas and χ is given as

$$\chi = \sigma / \omega \epsilon_0$$

For $\chi \gg \epsilon_r$ the power factor angle is nearly zero and the ground is almost resistive.

For a 1MHz wave propagating over a ground surface with $\sigma = 12 \times 10^{-3} \text{ S/m}$ and $\epsilon_r = 15$ the value of χ is 215.7 and is much greater than ϵ_r .

The power factor angle is 4.25° . At higher frequency 100MHz the value of χ is 2.157 and power factor angle becomes 82.32°

For large numerical distance the attenuation factor decreases by a factor of 10 for every decade i.e 20dB/decade. Thus attenuation is inversely proportional to p and R.

The electric field intensity due to the surface wave is proportional to the product of A_{su} and e^{-jkR}/R . The EF due to the surface wave at large distance from vertically polarized antenna is inversely proportional to the surface of the distance or the power is inversely proportional to R^4 .

The EF of a vertically polarized wave near the surface of the earth have a forward tilt. The magnitude of the wave tilt depends on the conductivity and permittivity of the earth. The horizontal component is smaller than the vertical component and they are not in phase. The EF is elliptically polarized very close to the surface of the earth.

DIFFRACTION

DIFFRACTION is the bending of the wave path when the waves meet an obstruction. The amount of diffraction depends on the wavelength of the wave. Higher frequency waves are rarely diffracted in the normal world. Since light waves are high frequency waves, they are rarely diffracted. However, diffraction in sound waves can be observed by listening to music. When outdoors, behind a solid obstruction, such as a brick wall, hear mostly low notes are heard. This is because the higher notes, having short wave lengths, undergo little or no diffraction and pass by or over the wall without wrapping around the wall and reaching the ears. The low notes, having longer wavelengths, wrap around the wall and reach the ears. This leads to the general statement that lower frequency waves tend to diffract more than higher frequency waves. Broadcast band(AM band) radio waves (lower frequency waves) often travel over a mountain to the opposite side from their source because of diffraction, while higher frequency TV and FM signals from the same source tend to be stopped by the mountain.

Diffraction, results in a change of direction of part of the wave energy from the normal line-of-sight path making it possible to receive energy around the edges of an obstacle. Although diffracted RF energy is usually weak, it can still be detected by a suitable receiver. The principal effect of diffraction extends the radio range beyond the visible horizon. In certain cases, by using high power and very low frequencies, radio waves can be made to encircle the Earth by diffraction.

Mechanism for diffraction

Diffraction arises because of the way in which waves propagate; this is described by the Huygens-Fresnel Principle and the principle of superposition of waves. The propagation of a wave can be visualized by considering every point on a wavefront as a point source for a secondary spherical waves. The wave displacement at any subsequent point is the sum of these secondary waves. When waves are added together, their sum is determined by the relative phases as well as the amplitudes of the individual waves so that the summed amplitude of the waves can have any value between zero and the sum of the individual amplitudes. Hence, diffraction patterns usually have a series of maxima and minima.

There are various analytical models which allow the diffracted field to be calculated, including the Kirchoff-Fresnel diffraction equation which is derived from wave equation, the Fraunhofer diffraction approximation of the Kirchoff equation which applies to the far field and the Fresnel diffraction approximation which applies to the near field. Most configurations cannot be solved analytically, but can yield numerical solutions through finite element and boundary element methods.

It is possible to obtain a qualitative understanding of many diffraction phenomena by considering how the relative phases of the individual secondary wave sources vary, and in particular, the conditions in which the phase difference equals half a cycle in which case waves will cancel one another out.

The simplest descriptions of diffraction are those in which the situation can be reduced to a two-dimensional problem. For water waves, this is already the case; water waves propagate only on the surface of the water. For light, we can often neglect one direction if the diffracting object extends in that direction over a distance far greater than the wavelength. In the case of light shining through small circular holes we will have to take into account the full three dimensional nature of the problem.

Effect of diffraction of waves:

- Speeddoes not change
- Frequency..... does not change
- Wavelength..... does not change
- Amplitudedecreases

If diffraction is due to mountain or a hill, Knife edge diffraction model is used to study the properties of the diffracted ray, and if is due to a building, rounded surface diffraction model is used.

Knife Edge diffraction model:

In EM wave propagation knife-edge effect or edge diffraction is a redirection by diffraction of a portion of the incident radiation that strikes a well-defined obstacle such as a mountain range or the edge of a building.

The knife-edge effect is explained by Huygens- Fresnel principle which states that a well-defined obstruction to an electromagnetic wave acts as a secondary source, and creates a new wave front. This new wave front propagates into the geometric shadow area of the obstacle.

TROPOSPHERIC PROPAGATION:

The lowest part of the earth's atmosphere is called the troposphere. Typically, the troposphere extends from the surface of the earth to an altitude of approximately 9 km at the poles and 17 km at the equator. This upper boundary is referred to as the tropopause and is defined as the point at which the temperature in the atmosphere begins to increase with height. Within the troposphere, the temperature is found to decrease with altitude at a rate of approximately 7°C per km. The earth's weather system is confined to the troposphere and the fluctuations in weather parameters like temperature, pressure and humidity cause the refractive index of the air in this layer to vary from one point to another. It is in this context that the troposphere assumes a vital role in the propagation of radio waves at VHF (30-300 MHz) and UHF (300-3000 MHz) frequencies. The meteorological conditions therefore influence the manner in which radio wave propagation occurs in the troposphere both on a spatial and temporal scale.

Refractive Index, Refractivity and Modified Refractivity

[“Transhorizon Radiowave Propagation due to Evaporation Ducting, The Effect of Tropospheric Weather Conditions on VHF and UHF Radio Paths Over the Sea”, S D Gunashekar, D R Siddle and E M Warrington]

In general, the refractive index, n , of the troposphere decreases with altitude. To simplify the mathematics involved variations in the horizontal are neglected and horizontal homogeneity of the refractive index of the troposphere is assumed in most discussions on this topic. A typical value for n at sea level is 1.000350. A few meters above sea level, this might decrease to a value such as 1.000300. For all practical purposes, at this scale, this change in the refractive index is negligibly small, with hardly any visible deviation. However, immediately above the surface of the sea, it is often this small (but rapid) change in the refractive index profile that facilitates the formation of meteorological phenomena called evaporation ducts. A convenient way of expressing these unwieldy numbers is to use the concept of refractivity instead. Refractivity, N , is defined as follows:

$$N = (n-1) \times 10^6$$

So, for example, when $n = 1.000350$, $N = 350$.

A well-known approximation for refractivity N is given below

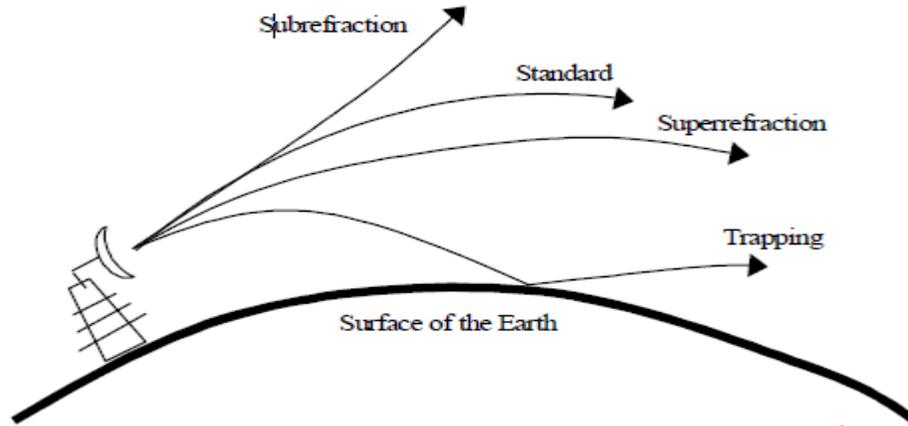
$$N = \frac{77.6}{T} \left(P + \frac{4810 * e}{T} \right)$$

where P = total atmospheric pressure (in mb);

T = atmospheric temperature (in K);

e = water vapour pressure (in mb).

All three terms, P, T and e fall with height in an exponential manner, resulting in a corresponding decrease in N with height. A standard atmosphere, therefore is one in which the refractivity varies with altitude according to equation. Using Snell's law, a radio ray projected into the atmosphere will have to travel from a denser to rarer medium and will refract downwards towards the surface of the earth. The curvature of the ray, however, will still be less than the earth's curvature. The gradient of refractivity in this case generally varies from 0 to -79 N-units per kilo. When the refractivity gradient varies from -79 to -157 N-units per kilo, a super refractive condition is said to prevail in the troposphere and the ray will refract downwards at a rate greater than standard but less than the curvature of the earth . A refractivity gradient that is even less than -157 N-units per kilo will result in a ray that refracts towards the earth's surface with a curvature that exceeds the curvature of the earth. This situation is referred to as trapping and is of particular importance in the context of evaporation ducts. Finally, if the refractivity gradient is greater than 0 N units per kilo, a sub refractive condition exists and a radio ray will now refract upwards, away from the surface of the earth. Depending on the existing conditions in the troposphere, a radio wave will undergo any of the types of refraction: sub refraction, standard refraction, super refraction or trapping. Figure 1 illustrates the four refractive conditions discussed above.



While dealing with radio propagation profiles, the curved radio rays are replaced with linear rays for the purpose of geometric simplicity. To account for drawing radio rays as straight lines, the earth radius has to be increased. The radius of this virtual sphere is known as the effective earth radius and it is approximately equal to four-thirds the true radius of the earth (i.e. roughly 8500 km). A more classical form of representing n is that of modified refractivity, M . In this case, the surface of the earth is represented by a flat plane and the radio rays are constituted by curves that are determined by Snell's law and the corresponding value of M at each point along the radio link. The following is the expression for M

$$M = N + \left(\frac{h}{a}\right) * 10^6$$

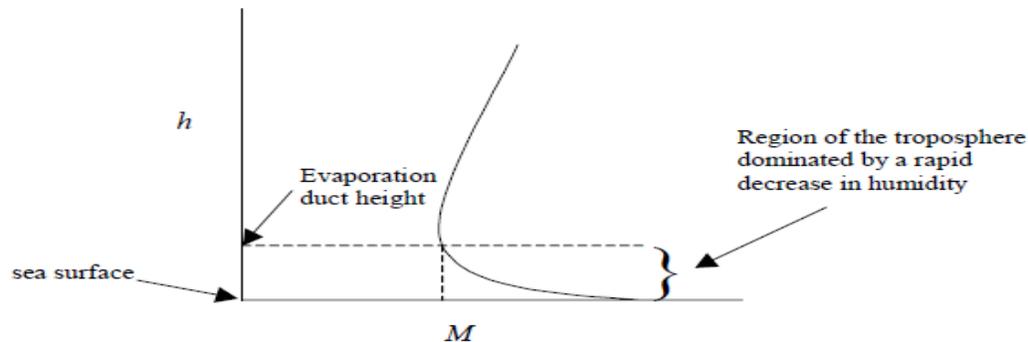
$$N + 0.157h,$$

where N = refractivity (in N-units), h = height above sea level (in s), a = radius of the earth (in s).

Formation of Evaporation Ducts

The air that is in immediate contact with the sea surface is saturated with water vapour (i.e. the relative humidity is 100%). As the height increases, the water vapour pressure in the atmosphere rapidly decreases until it reaches an ambient value at which it remains more or less static for a further increase in height. Therefore, for the first few s above the surface of the sea, it is the water vapour pressure, e , in the expression for N that dominates. This rapid decrease in e causes a steep fall in N .

This is reflected in the modified refractivity, M , which also correspondingly decreases. (The height term h , which increases, is more than offset by the rapidly decreasing N term). This behaviour can be seen in the graph of h vs M



as that portion of the curve with a strong negative M gradient. Therefore, despite the fact that the height h is increasing, it is the sharp fall in the water vapour pressure, e , that contributes to the rapid decrease in M .

Once e has reached its ambient value at a given height, a further rise in altitude does not cause a substantial change in the humidity of the troposphere. Thus, as h increases further, N decreases more (since air pressure and temperature both decrease with height). But this decrease in N is very small over large height increments. Consequently, despite a decreasing N term, it is the h term that starts to dominate in the expression for M . Thus, M now gradually increases with height, and can be seen as the portion of the curve that has a positive M gradient.

The point at which the M gradient changes from negative to positive is referred to as the evaporation duct height (or thickness), and is a practical and realistic measure of the strength of the evaporation duct.

Evaporation Ducts and the Troposphere

By virtue of their nature of formation, evaporation ducts are nearly permanent features over the sea surface. Typically, the height of an evaporation duct is of the order of only a few s; however, this can vary considerably with geographical location and changes in atmospheric parameters such as humidity, air pressure and temperature. In the lower regions of the troposphere where the earth's weather is confined, these parameters do, in fact, fluctuate significantly. The turbulent nature of the atmosphere contributes to its unpredictability and a variable atmosphere, in turn, is one of the major causes of unreliable wireless communications. Depending on their location and the prevailing climate, evaporation duct heights may vary from a few meters to few tens of meters. Additionally, it is observed that calm sea conditions are more conducive for the creation of ducts. As a consequence of

sporadic meteorological phenomena, evaporation duct heights undergo significant spatial and temporal variations. Evaporation ducts are weather-related phenomena; their heights cannot easily be measured directly using instruments like refractometers and radiosondes. At best, the height of an evaporation duct can be deduced from the bulk meteorological parameters that are representative of the ongoing physical processes at the air-sea boundary. The dependence of evaporation ducts on the physical structure of the troposphere signifies that changing weather conditions can indeed result in alterations in radio wave propagation.

Evaporation Ducts and Radio wave Propagation

Over the years, much research has been undertaken to explain the mechanism of radio wave propagation in evaporation ducts. A key reason why evaporation ducts are so important for radio communications is because they are often associated with enhanced signal strengths at receivers. An evaporation duct can be regarded as a natural waveguide that steers the radio signal from the transmitter to a receiver that may be situated well beyond the radio horizon. The drop in the refractive index of the atmosphere within the first few meters above the surface of the sea causes incident radio waves to be refracted towards the earth more than normal so that their radius of curvature becomes less than or equal to that of the earth's surface. The sudden change in the atmosphere's refractivity at the top of the duct causes the radio waves to refract back into the duct, and when it comes in contact with the surface of the sea, it gets reflected upwards again. The waves then propagate long ranges by means of successive reflections (refractions) from the top of the duct and the surface of the earth.

Since the top of an evaporation duct is not 'solid' (as in the case of an actual waveguide), there will be a small but finite amount of energy leakage into the free space immediately above the duct. However, despite this escape of energy, radio waves are still capable of travelling great distances through the duct, with relatively small attenuation and path loss. The ducting effect often results in radio signals reaching places that are beyond the radio horizon with improved signal strengths. This naturally has far reaching implications on practical radio propagation patterns. For this reason, evaporation ducts and their impact on radio wave propagation have been studied extensively over the years. Numerous statistical models have been proposed to describe evaporation ducts and compute the duct heights under different atmospheric conditions.

The presence of evaporation ducts might not always indicate enhanced signal strengths. For instance, if there is an unwanted distant transmitter also located within the duct, then there is always the possibility of the system under

consideration being susceptible to signal interference and interception. This is dependent on the location of the radio paths being investigated. Another scenario that might arise is the interference between the various propagation modes that exist within the evaporation duct itself. Depending on the separation of the transmitter and receiver and the prevailing atmospheric conditions, there could be destructive interference between the direct and reflected rays, the latter of which is comprised of the various multiple hop (one-hop, two-hop, and so on) propagation modes. Additionally, signal degradation may also occur if there is destructive interference between various modes that arrive at the receiver after refraction from different heights in the troposphere. All these situations could possibly cause key problems in the domain of cellular mobile communication systems in littoral regions. Thus, in addition to aiding radio wave propagation, evaporation ducts could also be principal limiting factors in beyond line of sight over-the-sea UHF propagation.

IONOSPHERE PROPAGATION

The **ionosphere** is a part of the upper atmosphere, from about 85 km to 600 km altitude, comprising portions of the mesosphere, thermosphere, and exosphere, distinguished because it is ionized by solar radiation. It plays an important part in atmospheric electricity and forms the inner edge of the magnetosphere. It has practical importance because, among other functions, it influences radio wave Propagation to distant places on the earth.

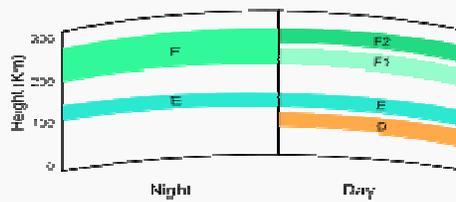
In a region extending from a height of about 90 km to over thousands of kms, most of the molecules of the atmosphere are ionized by radiation from the Sun. This region is called the *ionosphere*

At greater heights- intensity of ionizing radiation is very high, few molecules are available for ionization, ionization density is *low*

As height decreases- more molecules are available due to reduced atmospheric pressure, ionization density is higher (closer to the earth)

But as height decreases further, ionization density decreases though more molecules are available since the energy in the ionizing radiation has been used up to create ions.

Hence, ionization is different at different heights above the earth and is affected by time of day and solar activity



Ionospheric layers.

At night the F layer is the only layer of significant ionization present, while the ionization in the E and D layers is extremely low. During the day, the D and E layers become much more heavily ionized, as does the F layer, which develops an additional, weaker region of ionisation known as the F₁ layer. The F₂ layer persists by day and night and is the region mainly responsible for the refraction of radio waves.

D layer

The D layer is the innermost layer, 60 km to 90 km above the surface of the Earth. Ionization here is due to Lyman series alpha hydrogen radiation at a of 121.5 nanometer (nm).. In addition, with high solar activity hard X rays (wavelength < 1 nm) may ionize (N₂, O₂). During the night cosmic rays produce a residual amount of ionization. Recombination is high in the D layer, the net ionization effect is low, but loss of wave energy is great due to frequent collisions of the electrons (about ten collisions every msec). As a result high-frequency (HF) radio waves are not reflected by the D layer but suffer loss of energy therein. This is the main reason for absorption of HF radio waves, particularly at 10 MHz and below, with progressively smaller absorption as the frequency gets higher. The absorption is small at night and greatest about midday. The layer reduces greatly after sunset; a small part remains due to galactic cosmic rays. A common example of the D layer in action is the disappearance of distant AM broadcast band stations in the daytime.

During solar proton events , ionization can reach unusually high levels in the D-region over high and polar latitudes. Such very rare events are known as Polar Cap Absorption (or PCA) events, because the increased ionization significantly enhances the absorption of radio signals passing through the region. In fact, absorption levels can increase by many tens of dB during intense events, which is enough to absorb most (if not all) transpolar HF radio signal transmissions. Such events typically last less than 24 to 48 hours.

E layer

The E layer is the middle layer, 90 km to 120 km above the surface of the Earth. Ionization is due to soft X-ray (1-10 nm) and far ultraviolet (UV) solar radiation ionization of molecular oxygen(O₂). Normally, at oblique incidence, this layer can only reflect radio waves having frequencies lower than about 10 MHz and may contribute a bit to absorption on frequencies above. However, during intense Sporadic E events, the E_s layer can reflect frequencies up to 50 MHz and higher. The vertical structure of the E layer is primarily determined by the competing effects of ionization and recombination. At night the E layer rapidly disappears because the primary source of ionization is no longer present. After sunset an increase in the height of the E layer maximum increases the range to which radio waves can travel by reflection from the layer.

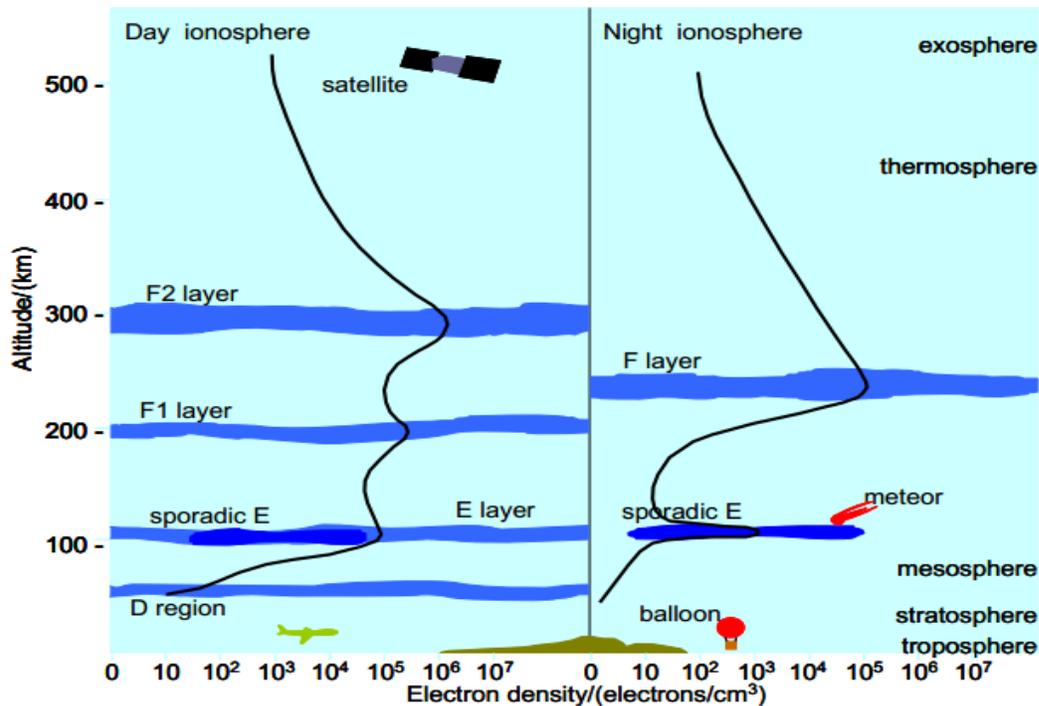
E_s

The E_s layer (sporadic E-layer) is characterized by small, thin clouds of intense ionization, which can support reflection of radio waves, rarely up to 225 MHz. Sporadic-E events may last for just a few minutes to several hours. Sporadic E propagation makes radio amateurs very excited, as propagation paths that are generally unreachable can open up. There are multiple causes of sporadic-E that are still being pursued by researchers. This propagation occurs most frequently during the summer months when high signal levels may be reached. The skip distances are generally around 1,000 km (620 mi). Distances for one hop propagation can be as close as 900 km [500 miles] or up to 2,500 km (1,600 mi). Double-hop reception over 3,500 km (2,200 mi) is possible.

F layer

The F layer or region, also known as the Appleton layer extends from about 200 km to more than 500 km above the surface of Earth. It is the densest point of the ionosphere, which implies signals penetrating this layer will escape into space. At higher altitudes the amount of oxygen ions decreases and lighter ions such as hydrogen and helium become dominant, this layer is the topside ionosphere . Here extreme ultraviolet (UV, 10–100 nm) solar radiation ionizes atomic oxygen. The F layer consists of one layer at night, but during the day, a deformation often forms in the profile that is labeled F₁. The F₂ layer remains by day and night responsible for most skywave propagation of radio waves, facilitating high frequency (HF, or shortwave) radio communications over long distances.

Day and night structure of ionosphere:



VARIATIONS IN THE IONOSPHERE [Integrated Publishing, Electrical Engineering Training Series]

Because the existence of the ionosphere is directly related to radiations emitted from the sun, the movement of the Earth about the sun or changes in the sun's activity will result in variations in the ionosphere. These variations are of two general types:

- (1) those which are more or less regular and occur in cycles and, therefore, can be predicted in advance with reasonable accuracy, and
- (2) those which are irregular as a result of abnormal behavior of the sun and, therefore, cannot be predicted in advance. Both regular and irregular variations have important effects on radio wave propagation.

Regular Variations

The regular variations that affect the extent of ionization in the ionosphere can be divided into four main classes: daily, seasonal, 11-year, and 27-day variations.

DAILY. - Daily variations in the ionosphere are a result of the 24-hour rotation of the Earth about its axis. Daily variations of the different layers (fig. 2-14) are summarized as follows:

The D layer reflects VLF waves; is important for long range VLF communications; refracts lf and mf waves for short range communications; absorbs HF waves; has little effect on vhf and above; and disappears at night. In the E layer, ionization depends on the angle of the sun. The E layer refracts HF waves during the day up to 20 megahertz to distances of about 1200 miles. Ionization is greatly reduced at night. Structure and density of the F region depend on the time of day and the angle of the sun. This region consists of one layer during the night and splits into two layers during daylight hours.

- Ionization density of the F1 layer depends on the angle of the sun.

Its main effect is to absorb hf waves passing through to the F2 layer.

- The F2 layer is the most important layer for long distance HF communications.

It is a very variable layer and its height and density change with time of day, season, and sunspot activity.

SEASONAL. - Seasonal variations are the result of the Earth revolving around the sun; the relative position of the sun moves from one hemisphere to the other with changes in seasons. Seasonal variations of the D, E, and F1 layers correspond to the highest angle of the sun; thus the ionization density of these layers is greatest during the summer. The F2 layer, however, does not follow this pattern; its ionization is greatest in winter and least in summer, the reverse of what might be expected. As a result, operating frequencies for F2 layer propagation are higher in the winter than in the summer.

ELEVEN-YEAR SUN SPOT CYCLE. - One of the most notable phenomena on the surface of the sun is the appearance and disappearance of dark, irregularly shaped areas known as SUNSPOTS. The exact nature of sunspots is not known, but scientists believe they are caused by violent eruptions on the sun and are characterized by unusually strong magnetic fields. These sunspots are responsible for variations in the ionization level of the ionosphere. Sunspots can, of course, occur unexpectedly, and the life span of individual sunspots is variable; however, a

regular cycle of sunspot activity has also been observed. This cycle has both a minimum and maximum level of sunspot activity that occur approximately every 11 years.

During periods of maximum sunspot activity, the ionization density of all layers increases. Because of this, absorption in the D layer increases and the critical frequencies for the E, F1, and F2 layers are higher. At these times, higher operating frequencies must be used for long distance communications.

27-DAY SUNSPOT CYCLE. - The number of sunspots in existence at any one time is continually subject to change as some disappear and new ones emerge. As the sun rotates on its own axis, these sunspots are visible at 27-day intervals, the approximate period required for the sun to make one complete rotation.

The 27-day sunspot cycle causes variations in the ionization density of the layers on a day-to-day basis. The fluctuations in the F2 layer are greater than for any other layer. For this reason, precise predictions on a day-to-day basis of the critical frequency of the F2 layer are not possible. In calculating frequencies for long-distance communications, allowances for the fluctuations of the F2 layer must be made.

Irregular Variations

Irregular variations in ionospheric conditions also have an important effect on radio wave propagation. Because these variations are irregular and unpredictable, they can drastically affect communications capabilities without any warning.

The more common irregular variations are sporadic E, sudden ionospheric disturbances, and ionospheric storms.

SPORADIC E. - Irregular cloud-like patches of unusually high ionization, called sporadic E, often form at heights near the normal E layer. Exactly what causes this phenomenon is not known, nor can its occurrence be predicted. It is known to vary significantly with latitude, and in the northern latitudes, it appears to be closely related to the aurora borealis or northern lights.

At times the sporadic E is so thin that radio waves penetrate it easily and are returned to earth by the upper layers. At other times, it extends up to several hundred miles and is heavily ionized.

These characteristics may be either harmful or helpful to radio wave propagation. For example, sporadic E may blank out the use of higher, more favorable ionospheric layers or cause additional absorption of the radio wave at some frequencies. Also, it can cause additional multipath problems and delay the arrival times of the rays of rf energy.

On the other hand, the critical frequency of the sporadic E is very high and can be greater than double the critical frequency of the normal ionospheric layers. This condition may permit the long distance transmission of signals at unusually high frequencies. It may also permit short distance communications to locations that would normally be in the skip zone.

The sporadic E can form and disappear in a short time during either the day or night. However, it usually does not occur at the same time at all transmitting or receiving stations.

SUDDEN IONOSPHERIC DISTURBANCES. - The most startling of the ionospheric irregularities is known as a **SUDDEN IONOSPHERIC DISTURBANCE (SID)**. These disturbances may occur without warning and may prevail for any length of time, from a few minutes to several hours. When SID occurs, long distance propagation of hf radio waves is almost totally "blanked out." The immediate effect is that radio operators listening on normal frequencies are inclined to believe their receivers have gone dead.

When SID has occurred, examination of the sun has revealed a bright solar eruption. All stations lying wholly, or in part, on the sunward side of the Earth are affected. The solar eruption produces an unusually intense burst of ultraviolet light, which is not absorbed by the F2, F1, and E layers, but instead causes a sudden abnormal increase in the ionization density of the D layer. As a result, frequencies above 1 or 2 megahertz are unable to penetrate the D layer and are usually completely absorbed by the layer.

IONOSPHERIC STORMS. - Ionospheric storms are disturbances in the Earth's magnetic field. They are associated, in a manner not fully understood, with both solar eruptions and the 27-day intervals, thus corresponding to the rotation of the sun.

Scientists believe that ionospheric storms result from particle radiation from the sun. Particles radiated from a solar eruption have a slower velocity than ultraviolet light waves produced by the eruption. This would account for the 18-hour or so time difference between a sid and an ionospheric storm. An ionospheric storm that

is associated with sunspot activity may begin anytime from 2 days before an active sunspot crosses the central meridian of the sun until four days after it passes the central meridian. At times, however, active sunspots have crossed the central region of the sun without any ionospheric storms occurring. Conversely, ionospheric storms have occurred when there were no visible spots on the sun and no preceding SID. As you can see, some correlation between ionospheric storms, sid, and sunspot activity is possible, but there are no hard and fast rules. Ionospheric storms can occur suddenly without warning.

The most prominent effects of ionospheric storms are a turbulent ionosphere and very erratic sky wave propagation. Critical frequencies are lower than normal, particularly for the F2 layer. Ionospheric storms affect the higher F2 layer first, reducing its ion density. Lower layers are not appreciably affected by the storms unless the disturbance is great. The practical effect of ionospheric storms is that the range of frequencies that can be used for communications on a given circuit is much smaller than normal, and communications are possible only at the lower working frequencies.